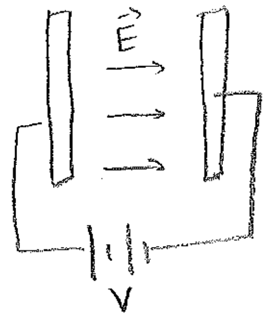
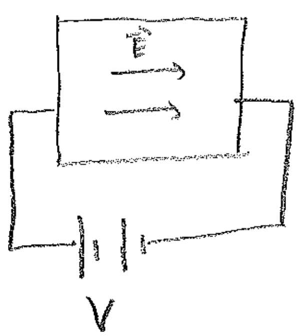
Electrical conductors



An electron placed <u>in vacuum</u> between parallel plates will accelerate w/

$$\vec{a} \propto \vec{E}$$

$$\left(\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}\right)$$



A conductor is a solid material (e.g. metal) w/ a certain density n of conduction electrons

- fixed lattice of positive nuclei
- tightly bound core electrons
- "sea" of conduction electrons that move relatively freely through the conductor (maintaining local neutrality, i.e. others move in to take their place
- each atom contributes 1 or more conduction e^-'s

e.g. copper: $n = 8.5 \times 10^{28} \frac{e^-}{m^3}$ $\quad \left[\simeq \frac{\#\,\text{atoms}}{m^3}\right]$

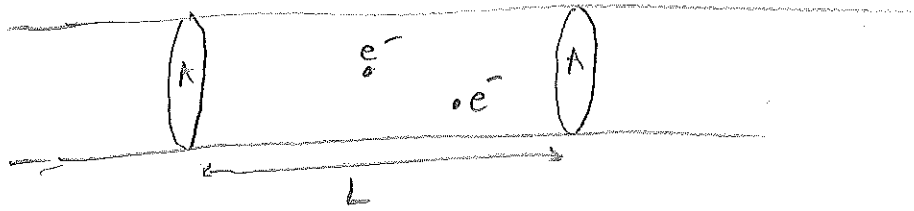
In the electric field created by a battery the conduction electrons move w/ <u>constant drift velocity</u>

$$\vec{v}_d \propto \vec{E}$$

[due to collisions w/ impurities, accelerating electrons soon reach a terminal velocity, like objects falling w/ a drag force

$v_d \sim$ few $\frac{cm}{hr}$; actual random velocities much higher but avg to zero if $\vec{E} = 0$]

Consider a conductor in the shape of a cylinder w/ cross-section A



How much charge due to conduction electrons in length L?

$$\Delta q = (\text{charge per electron})\left(\frac{\#\text{ conduction electrons}}{\text{volume}}\right)(\text{volume})$$

$$= (-e)(n)(A\,L)$$

If conduction electrons move w/ speed v_d, how long Δt until all of them pass through a plane bisecting the conductor?

$$v_d\,\Delta t = L \quad\Rightarrow\quad \Delta t = \frac{L}{v_d}$$

What is the current through the plane?

$$i = \frac{\Delta q}{\Delta t} = \frac{-neAL}{\left(\frac{L}{v_d}\right)} = -neAv_d$$

Define current density $j = \dfrac{\text{current}}{\text{cross-sectional area}} = \dfrac{i}{A} = -nev_d$

Current density is a vector pointing in direction of current flow

$$\vec{j} = -ne\,\vec{v}_d$$

$\vec{j}$ opposite $\vec{v}_d$ because electrons are negatively charged.

We learned that $\vec{V}_d \propto -e\vec{E}$

$$\Rightarrow \quad \vec{J} \propto \vec{E}$$

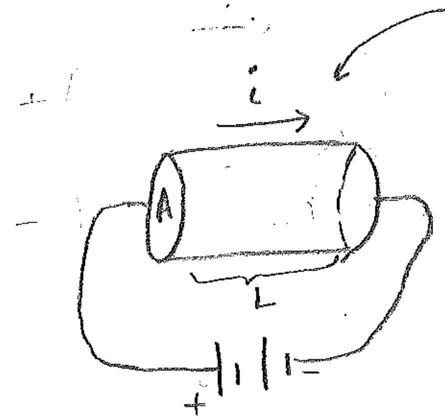
Define conductivity σ as the constant of proportionality

$$\vec{J} = \sigma \vec{E}$$

σ depends on the material of the conductor

Also define
Resistivity $\rho = \frac{1}{\sigma}$

$$\Rightarrow \quad \vec{E} = \rho \vec{J}$$



i = constant at all points along circuit (steady state) due to charge conservation (no charge buildup)

If resistor has constant cross-section A,

then j = constant throughout $= \frac{i}{A}$

Since $\vec{E} = \rho\vec{j}$, $\vec{E}$ is also uniform through resistor

$\Rightarrow$ Voltage difference $\Delta V = -\int \vec{E}\cdot d\vec{r}$

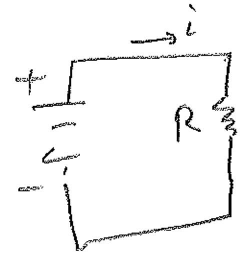
$$= EL$$

$$= \rho j L$$

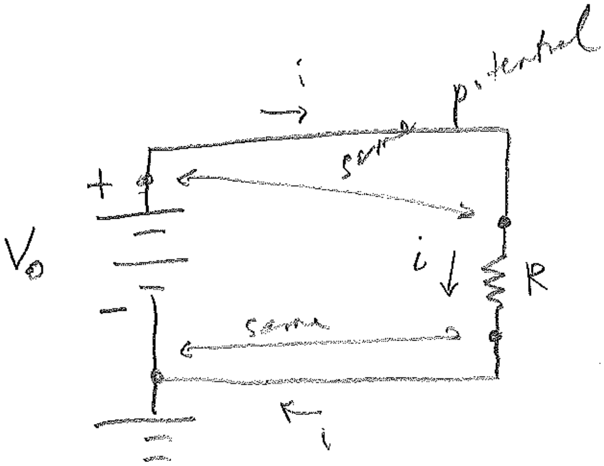
$$= \frac{\rho i}{A} L$$

$\Rightarrow$ voltage across the resistor $V = i\left(\frac{\rho L}{A}\right)$

__Ohm's law__: $V = iR$ where resistance $R = \frac{\rho L}{A}$



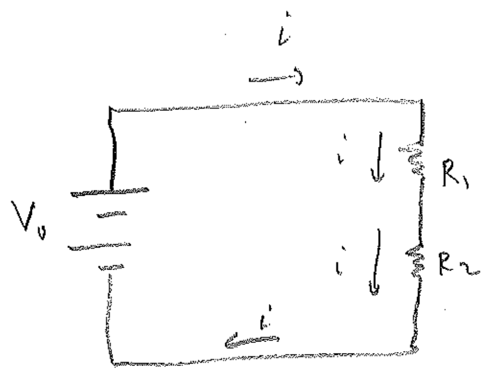
$V = iR$ is the "voltage drop" across a resistor



The potential difference or "voltage drop" across a resistor is $\Delta V = iR$

$V_0 = \Delta V \Rightarrow i = \frac{V_0}{R}$

[the wires in a circuit are conductors w/ very low resistance ($\rho \approx 0$) so voltage drop (for given i) is very small.
Thus voltage at top of R is same as voltage at positive terminal of battery & the voltage at bottom is $\approx$ neg. terminal of batty]

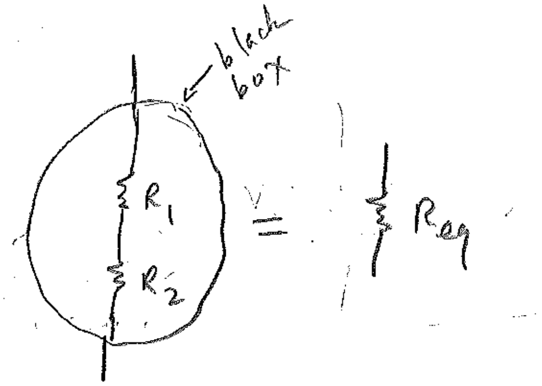


$\Delta V_1 = iR_1$

$\Delta V_2 = iR_2$

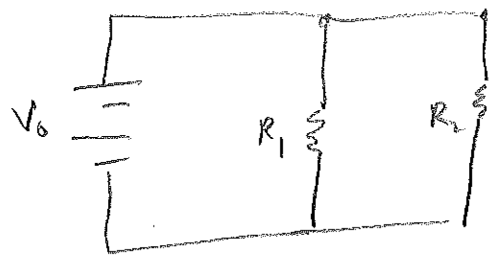
$$V_0 = \Delta V_1 + \Delta V_2 = iR_1 + iR_2 = i(R_1 + R_2)$$

$$i = \frac{V_0}{R_1 + R_2}$$



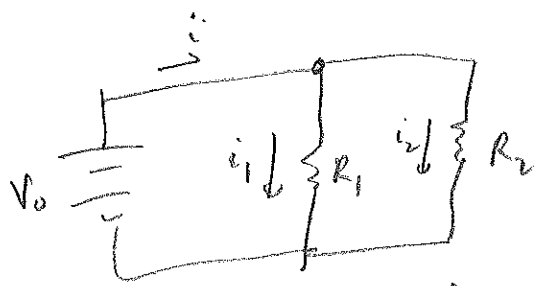
Two resistors connected <u>in series</u> are equivalent to a single resistor w/ resistance

$$R_{eq} = R_1 + R_2$$



Voltage drop across each resistor connected in parallel is the same

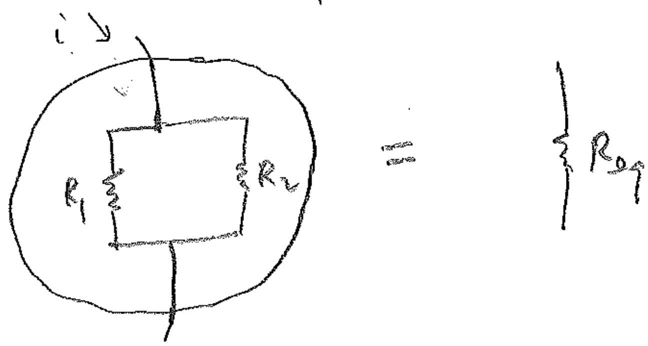
$$\Delta V_1 = \Delta V_2 = V_0$$



Kirchoff's current law $\Rightarrow$ $i = i_1 + i_2$

Ohm $\Rightarrow$ $i_1 = \dfrac{V_0}{R_1}$, $i_2 = \dfrac{V_0}{R_2}$

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_0 = \frac{1}{R_{eq}} V_0$$



Two resistors connected in parallel are equivalent to a single resistor R_{eq} where

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$