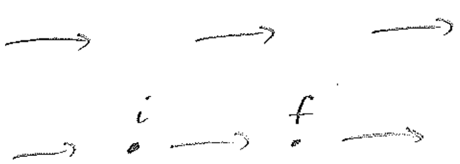
Electrostatic potential energy

[HRK 5e, Ch 28-1, 28-2]

Consider an electric field, constant in time (static) and constant in space (uniform)

[e.g. created by a plate or a pair of plates]



The field exerts force $\vec{F} = q\vec{E}$ on a charge q.

A positive charge q at i will accelerate toward f w/ $\vec{a} = \frac{1}{m}\vec{F}$.

[Can also think in terms of energy]

Starting from rest, its kinetic energy will increase: $K_f > K_i$

Two ways to calculate $\Delta K = K_f - K_i$

The work-energy theorem (Phys 1130) says:

$$\Delta K = K_f - K_i = \underbrace{\int_i^f \vec{F}\cdot d\vec{r}}_{\text{work done by the force}} = \int_i^f q\vec{E}\cdot d\vec{r} \qquad (*)$$

Cons of energy:

Let U = electrostatic potential energy of charge [

Total mechanical energy $K+U$ is conserved, so U must decrease

[NB don't use E for energy]

$$K_f + U_f = K_i + U_i$$

$$K_f - K_i = -(U_f - U_i) \qquad (**)$$

Putting (*) & (**) together, we find

Definition of electrostatic potential energy (only differences in potential energy are physically meaningful)

$$\boxed{\Delta U = U_f - U_i = -q\int_i^f \vec{E}\cdot d\vec{r}}$$

How to do a line integral in 3 easy steps

(a) choose a path from i to f

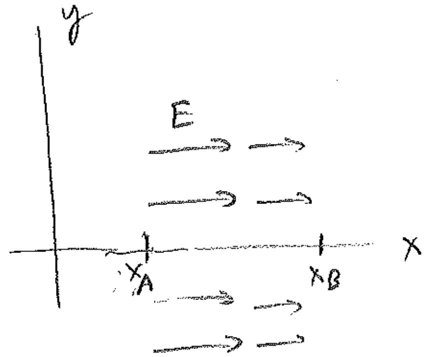
(Note: the result will not depend on the path chosen because the electrostatic force is "conservative")

(b) do the dot product

$$\vec{E}\cdot d\vec{r} = E_x\,dx + E_y\,dy + E_z\,dz$$

(c) finally, do the integral

Special case: uniform electric field [produced by infinite plates]



$\vec{E} = (E, 0, 0)$

Find difference in potential energy between A and B

$$U_B - U_A = -q\int_A^B \vec{E}\cdot d\vec{r}$$

choose path from x_A to x_B

$\vec{r} = (x, 0, 0)$

$d\vec{r} = (dx, 0, 0)$

$\vec{E}\cdot d\vec{r} = E\,dx$

$$U_B - U_A = -q\int_{x_A}^{x_B} E\,dx = -qE\int_{x_A}^{x_B} dx = -qE(x_B - x_A)$$

Note $U_B - U_A < 0$ for $x_B > x_A$, i.e. $U_B < U_A$

If charge has speed v_A at x_A, what is its speed v_B at x_B?

Use conservation of mechanical energy

$$E_{mech} = K_B + U_B = K_A + U_A$$

$$K_B = K_A + U_A - U_B$$

$$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + qE(x_B - x_A)$$

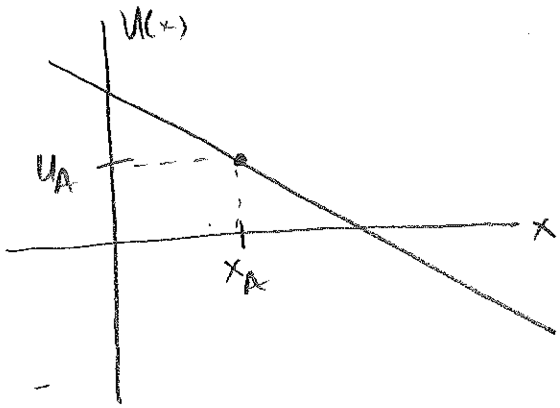
$$v_B = \sqrt{v_A^2 + \frac{2qE}{m}(x_B - x_A)}$$

Let $U(x)$ = potential energy at arbitrary point x

$$U(x) - U_A = -q\int_{x_A}^{x} \vec{E}\cdot d\vec{r}$$

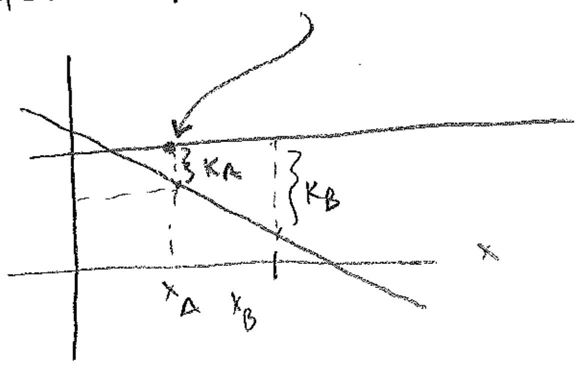
$$U(x) = U_A - \underbrace{qE}_{\text{negative slope}}(x - x_A)$$

[let's graph this]

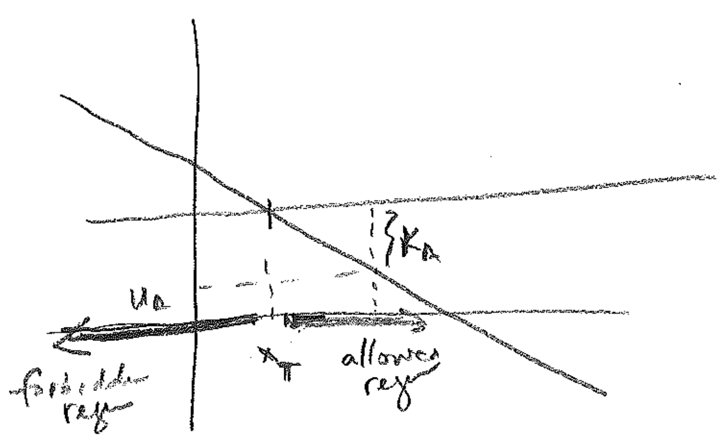


If charge has kinetic energy K_A at x_A
then $E_{mech} = K_A + U_A$

[but this is constant]
so horizontal line



If $U(x) = E$, then $K = 0 \Rightarrow v = 0 \Rightarrow x = x_T$ (turning point)



If $U(x) > E$, then x is in a forbidden region

If $U(x) \leq E$, then x is in an allowed region

(Potential energy diagrams)