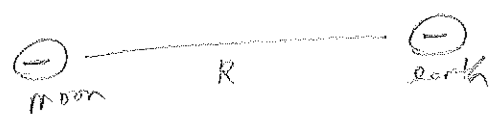
Review

- charges q_1 produce electric field $\vec{E}_1(\vec{r}\,)$
- electric field exerts force on charge q_2: $\vec{F}_{21} = q_2 \vec{E}_1(\vec{r}_2)$

$\therefore$ charges exert forces on other charges (Coulomb's)

$$\vec{F}_{12} = \frac{K q_1 q_2}{r_{12}^2} \vec{r}_{12}$$

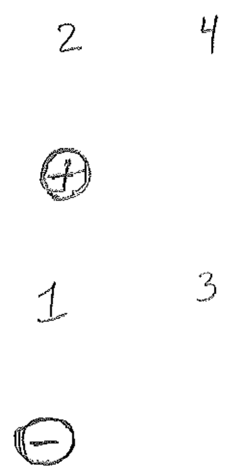
[Why introduce $\vec{E}$ field at all?
Coulomb's law $\Rightarrow$ action at a distance (instantaneous)



If electron on moon suddenly moves, R changes & F on elect on earth changes. Immediately?
No speed of light delay. Takes awhile for $\vec{E}$ caused by $\ominus$ on moon to change. Meanwhile $\ominus$ on earth continues to respond to old $\vec{E}$. $\vec{E}$ carries "memory"
(required by special relativity)

Superposition principle

the electric field caused by a collection of charges is the vector sum of the field produced by each.



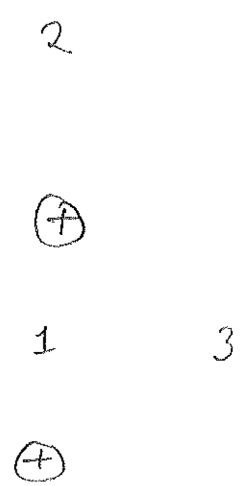
[Q: net field at 1, 2, 3, 4]

A →
B ←
C ↑
D ↓
E none

1. D
2. C
3. D
4. A

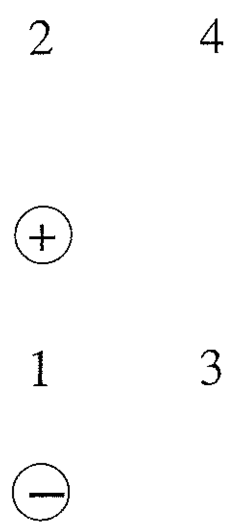
[4 is tricky + only possible given the choices]

[Can omit this as it is covered in 1st prob. set.]



1. E
2. C
3. A

[phet.colorado.edu/en/simulation/charges-and-fields]



A →

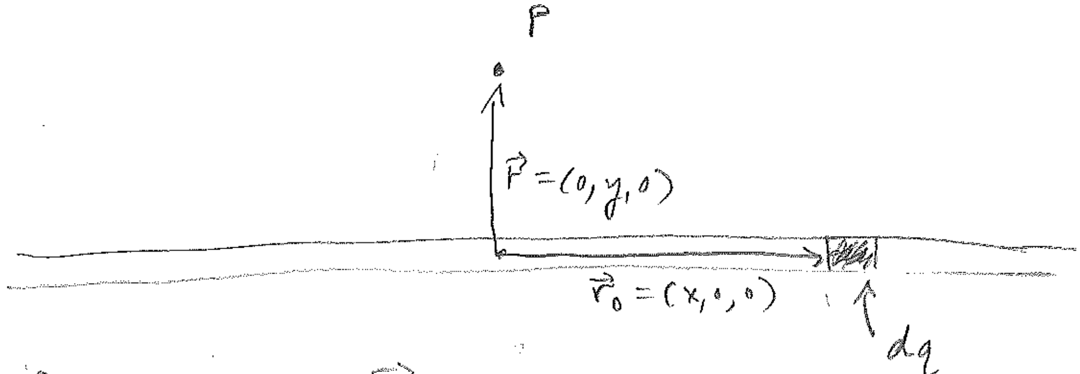
B ←

C ↑

D ↓

E no direction

Infinite line of uniformly distributed charge



electric field at P = $\vec{E}(\vec{r})$

= vector sum (integral) of infinitesimal fields $d\vec{E}$ produced by infinitesimal charge dq

$$d\vec{E}(\vec{r}) = \frac{K\, dq\, (\vec{r} - \vec{r}_0)}{|\vec{r} - \vec{r}_0|^3}$$

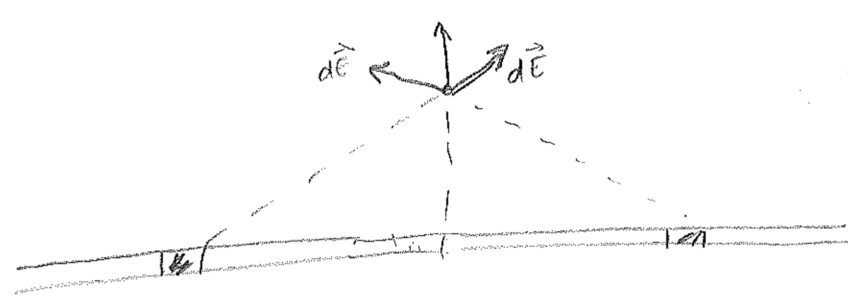
$$\vec{r} - \vec{r}_0 = (-x, y, 0)$$

$$|\vec{r} - \vec{r}_0| = \sqrt{x^2 + y^2}$$

Let λ = charge per unit length $= \frac{dq}{dx}$ "lambda"

$$\Rightarrow dq = \lambda\, dx$$

$$\vec{E}(r) = \int d\vec{E} = \int_{-\infty}^{\infty} \frac{K \lambda\, dx\, (-x, y, 0)}{(x^2 + y^2)^{3/2}}$$



x-components cancel

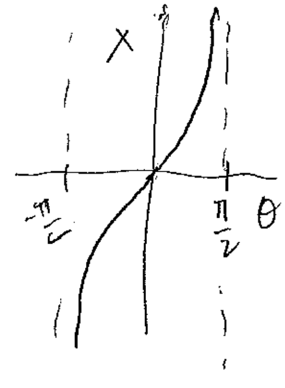
$$E_y = \int_{-\infty}^{\infty} \frac{K\lambda\, y\, dx}{(x^2+y^2)^{3/2}} \qquad \left(\begin{array}{c} y \text{ held constant} \\ \text{in integration over } x \end{array}\right)$$

Trig substitution: $x = y\tan\theta$

$$dx = y\sec^2\theta\, d\theta$$

$$x^2+y^2 = y^2(1+\tan^2\theta) = y^2\sec^2\theta$$

$$x \to \pm\infty \implies \theta \to \pm\frac{\pi}{2}$$



$$E_y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{K\lambda y\,(y\sec^2\theta)\,d\theta}{y^3\sec^3\theta}$$

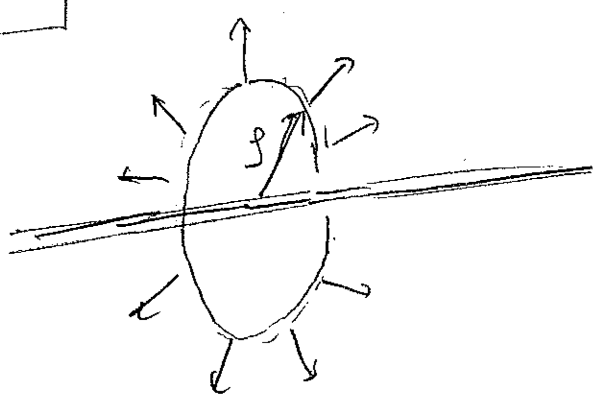
$$= \frac{K\lambda}{y}\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta\, d\theta}_{\sin\theta\,\big|_{-\pi/2}^{\pi/2}} = \frac{K\lambda}{y}\left[1-(-1)\right]$$

$$= \frac{2K\lambda}{y}$$

If asked:
Guide to trig subst

$1-\sin^2\theta = \cos^2\theta$
$\sin^2\theta+\cos^2\theta = 1$
$\tan^2+1 = \sec^2\theta$
$\sec^2-1 = \tan^2\theta$

If	then
a^2-x^2	$x = a\sin\theta$
x^2+a^2	$x = a\tan\theta$
x^2-a^2	$x = a\sec\theta$



$$E(\rho) = \frac{2K\lambda}{\rho}$$

where ρ = distance from wire

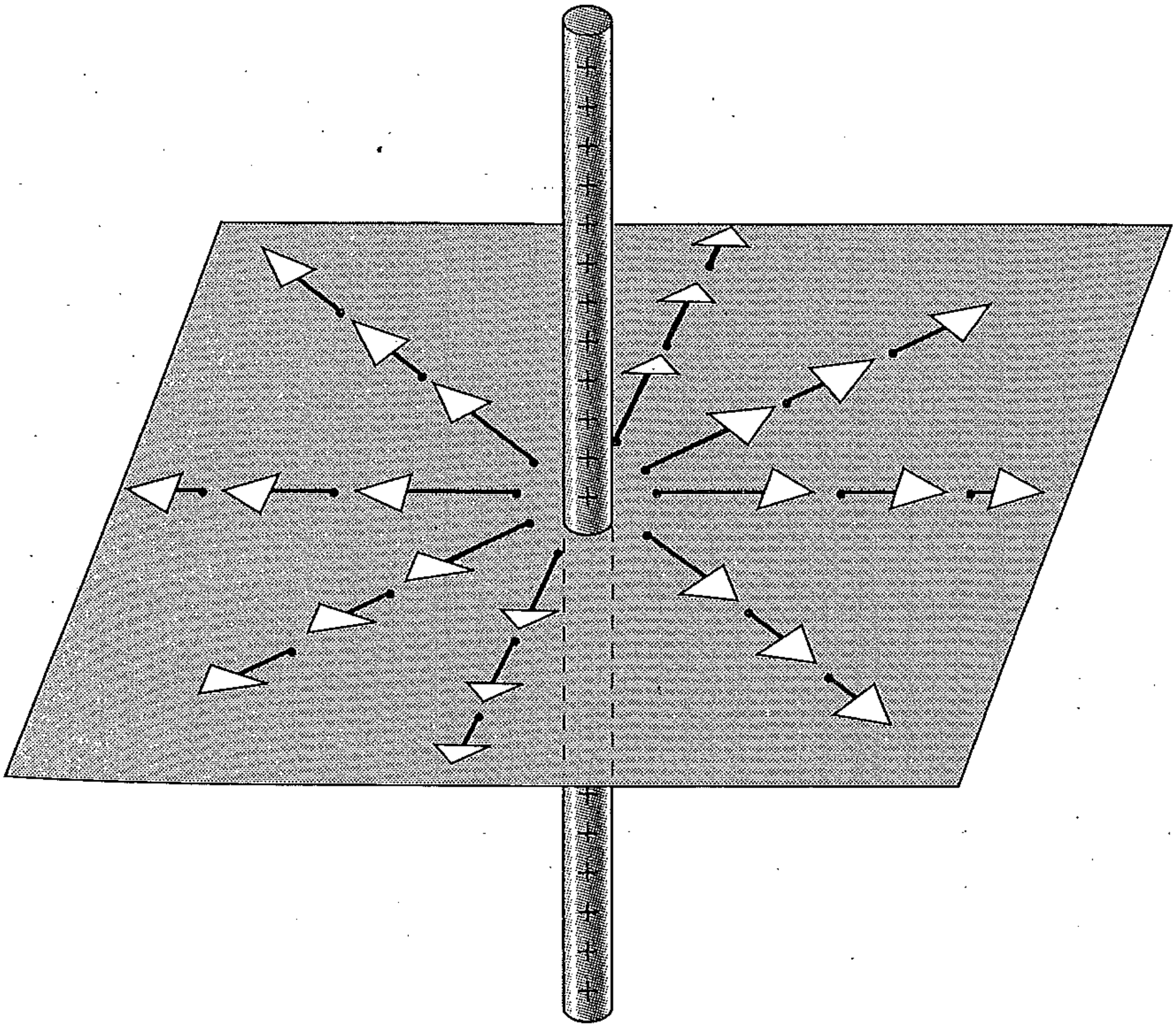
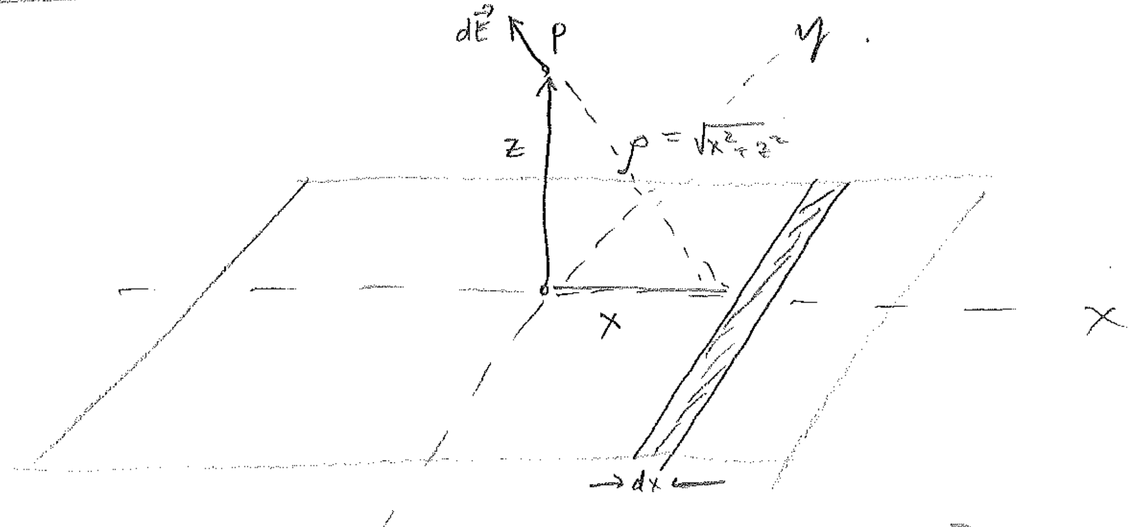


FIGURE 26-7. Electric field due to a positively charged rod. The field has cylindrical symmetry about the axis of the rod.

Infinite sheet of uniformly distributed charge



field at P = vector sum of infinitesimal fields $d\vec{E}$ produced by infinitesimal strips of charge

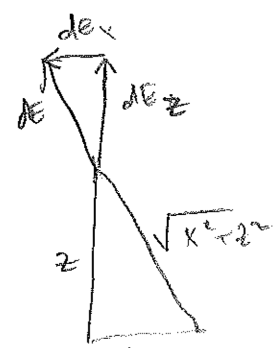
Let σ = charge per unit area of the sheet $= \dfrac{dq}{dx\,dy}$

Linear charge density of a strip of width dx is

$$\lambda = \frac{dq}{dy} = \sigma\, dx$$

The magnitude of the field produced by the strip is

$$|d\vec{E}| = \frac{2K\lambda}{\rho} = \frac{2K\sigma\, dx}{\sqrt{x^2+z^2}}$$



As before the x-components cancel out.

$$\frac{dE_z}{|d\vec{E}|} = \frac{z}{\sqrt{x^2+z^2}} \Rightarrow dE_z = \frac{2K\sigma z\, dx}{x^2+z^2}$$

$$E_z = \int dE_z = 2K\sigma z \int_{x=-\infty}^{x=\infty} \frac{dx}{x^2+z^2}$$

(z held constant)

$$x = z \tan\theta$$
$$dx = z \sec^2\theta \, d\theta$$
$$x^2+z^2 = z^2(1+\tan^2\theta) = z^2 \sec^2\theta$$

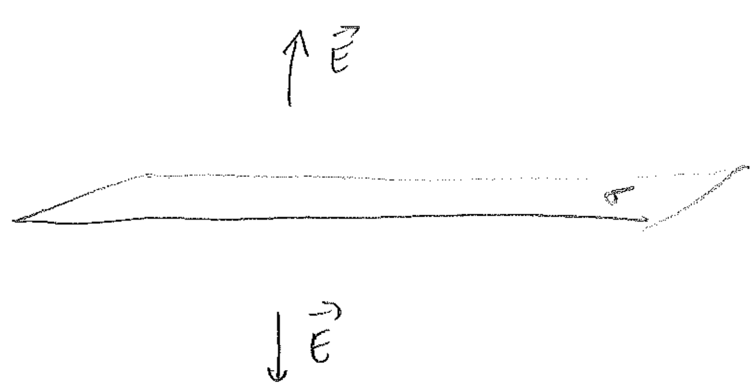
$$x \to \pm\infty \Rightarrow \theta \to \pm\frac{\pi}{2}$$

$$E_z = 2K\sigma z \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\theta}{z} = 2K\sigma z \Big[\underbrace{\frac{\pi}{2z} - \left(-\frac{\pi}{2z}\right)}_{\frac{\pi}{z}} \Big]$$

$$E_z = 2\pi K\sigma$$ independent of z!

Recall $K = \frac{1}{4\pi\epsilon_0}$ so this is usually written $\boxed{E_z = \frac{\sigma}{2\epsilon_0}}$

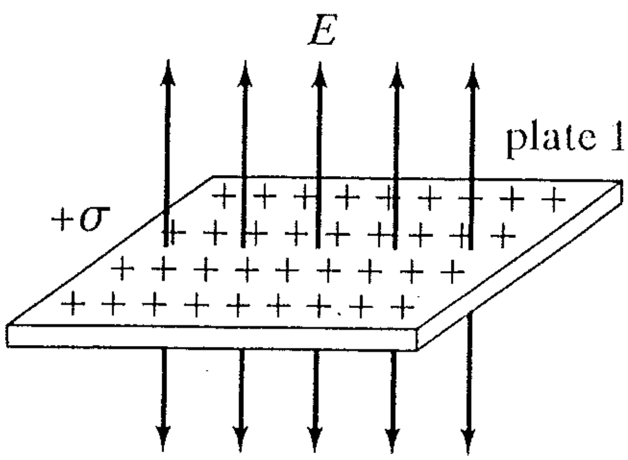
electric field above the sheet of charge.

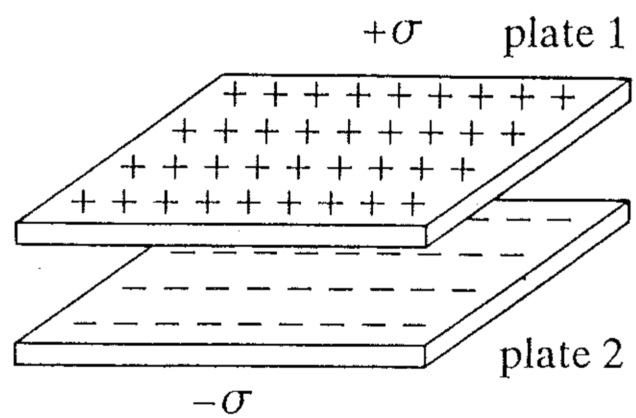


$$E_z = \begin{cases} \frac{\sigma}{2\epsilon_0}, & \text{above sheet} \\ -\frac{\sigma}{2\epsilon_0}, & \text{below sheet} \end{cases}$$

~~After collecting problem sets of these problems: this can be moved forward past D's & past ET-2 if necessary~~

The charge per unit area is $+\sigma$ on plate 1 and $-\sigma$ on plate 2. The magnitude of the electric field associated with plate 1 is $\sigma/2\epsilon_0$.





When the two plates are placed parallel to one another, the magnitude of the electric field is

(a) σ/ϵ_0 between, zero outside

(b) σ/ϵ_0 between, $\pm\sigma/2\epsilon_0$ outside

(c) zero both between and outside

(d) $\pm\sigma/2\epsilon_0$ both between and outside

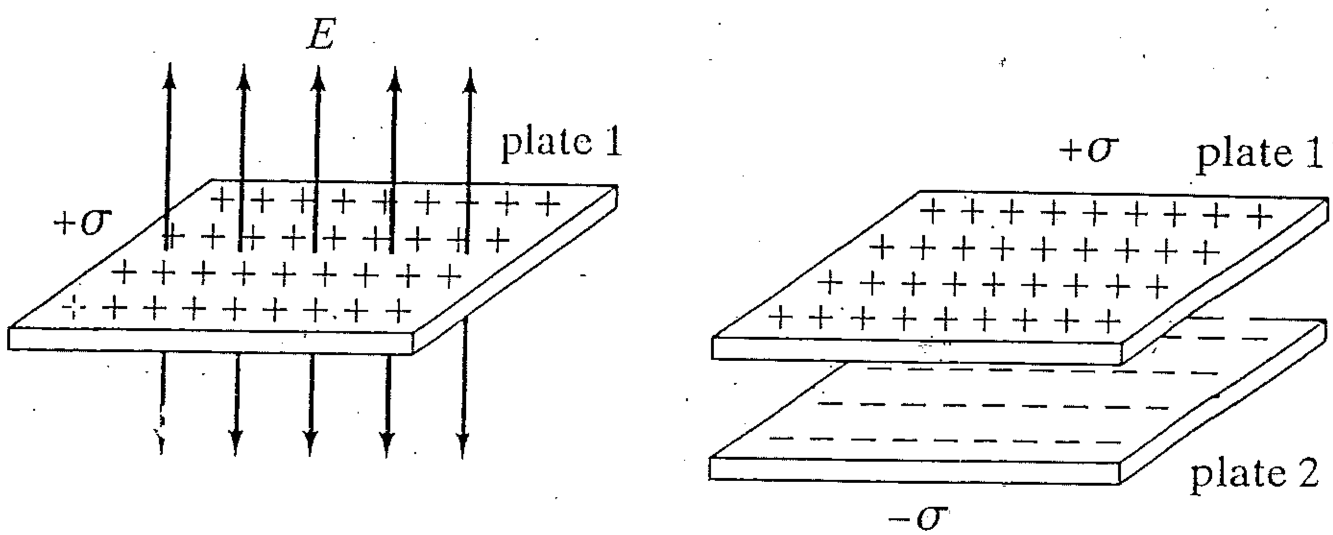
(e) none of the above

The 2 difficulties ppl face are:
(1) realizing that bottom plate creates a field above the top plate
(2) seeing that $\uparrow\uparrow = \uparrow$ & $\uparrow\downarrow = 0$
we gonna do this explicitly

[After answering this, draw diagram]

C6

The charge per unit area is $+\sigma$ on plate 1 and $-\sigma$ on plate 2. The magnitude of the electric field associated with plate 1 is $\sigma/2\epsilon_0$.



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(c) zero both between and outside

(d) $\pm\sigma/2\epsilon_0$ both between and outside

(e) none of the above

<u>Review</u>

Point charge $\quad |\vec{E}| = \frac{Kq}{r^2} = \frac{q}{4\pi\epsilon_0 r^2}$

Line charge $\quad |\vec{E}| = \frac{2K\lambda}{\rho} = \frac{\lambda}{2\pi\epsilon_0 \rho}$ $\quad \rho$ = distance from nearest point on line of charge

Surface charge $\quad |\vec{E}| = \frac{\sigma}{2\epsilon_0}$