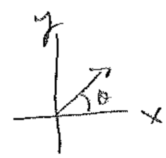
Review of vectors

$\vec{V}$ (magnitude and direction)

magnitude = length of vector $= |\vec{V}| = V$ (scalar)

direction = angles w.r.t. coordinate axes



$\alpha\vec{V}$ has same direction as $\vec{V}$ but different magnitude

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unit vector $\hat{V}$ ← "hat" is parallel to $\vec{V}$ but has length 1.

$$\hat{V} = \frac{\vec{V}}{V} \quad \Rightarrow \quad |\hat{V}| = \frac{|\vec{V}|}{V} = 1$$

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components

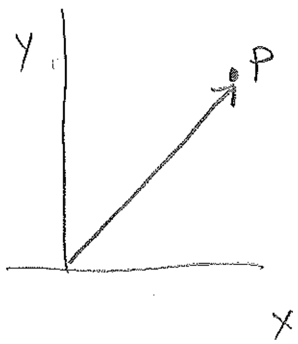
$$\vec{V} = (V_x, V_y, V_z) = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$\alpha\vec{V} = (\alpha V_x, \alpha V_y, \alpha V_z)$$

$$\hat{V} = \left(\frac{V_x}{V}, \frac{V_y}{V}, \frac{V_z}{V} \right)$$

Position vector, $\vec{r}$



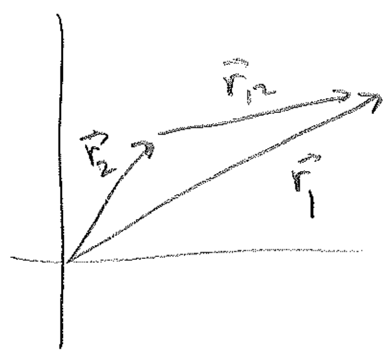
$\vec{r}$ = position of P relative to origin

$$\vec{r} = (x, y, z)$$

$$|r| = \sqrt{x^2+y^2+z^2}$$

Relative position

$\vec{r}_{12}$ = position of $\vec{r}_1$ relative to $\vec{r}_2$



$\vec{r}_2 + \vec{r}_{12} = \vec{r}_1$ (head to tail rule)

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$

$$= (x_1, y_1, z_1) - (x_2, y_2, z_2)$$

$$= (x_1 - x_2,\ y_1 - y_2,\ z_1 - z_2)$$

$$|\vec{r}_{12}| = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

Field

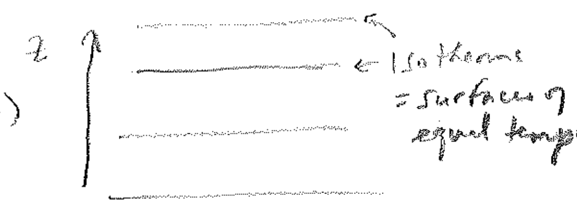
a quantity that has a value at each point in space,
i.e. a function of position

[e.g. temperature, about 70°, but higher toward ceiling; see temp map]

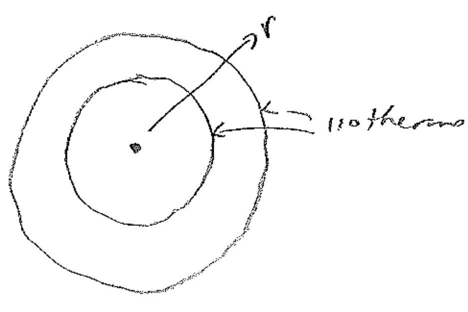
Scalar field

$T(x, y, z) = T(\vec{r})$

[If T only varies vertically:] $T = T(z)$

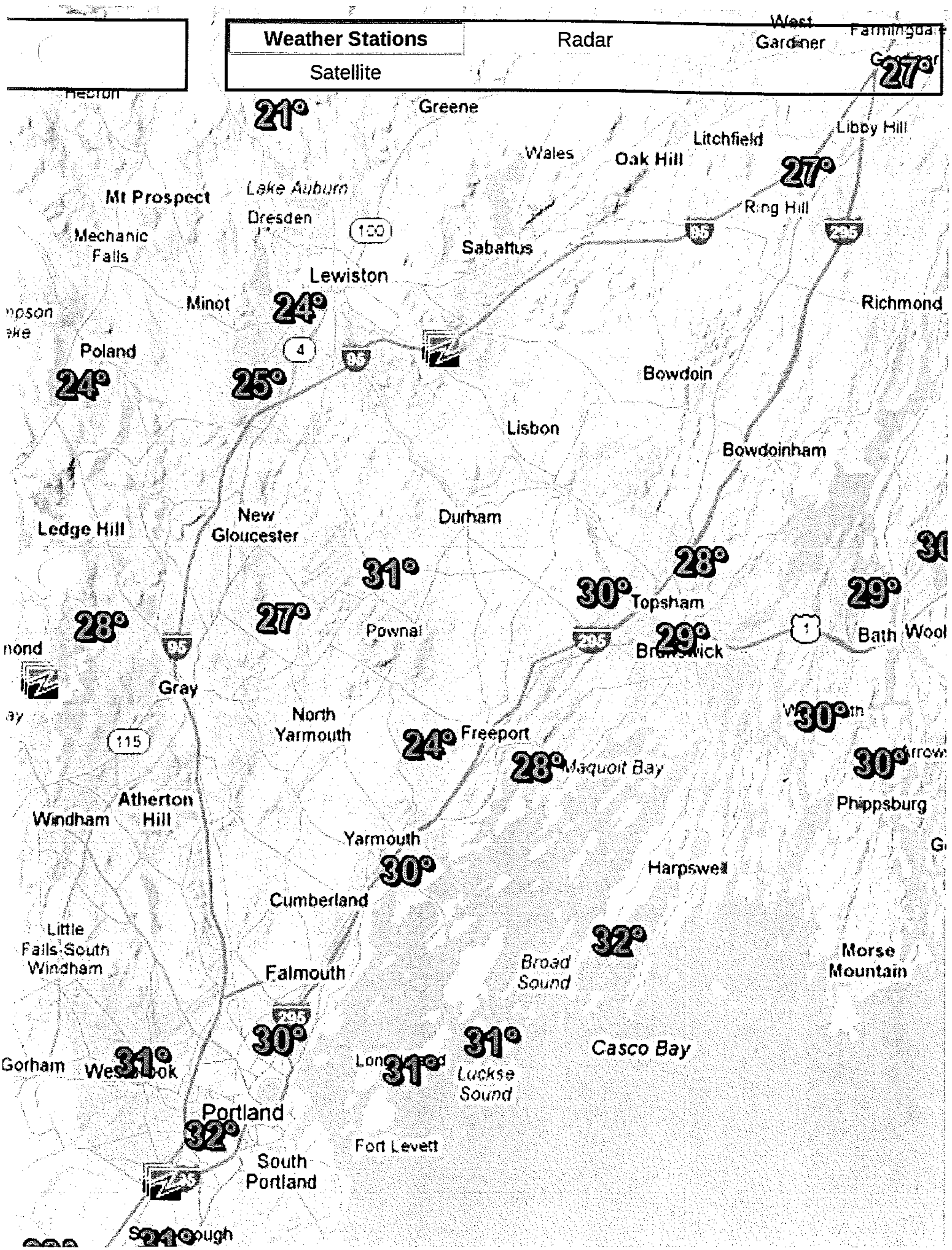


[If T only varies as the distance from a source] $T = T(r)$

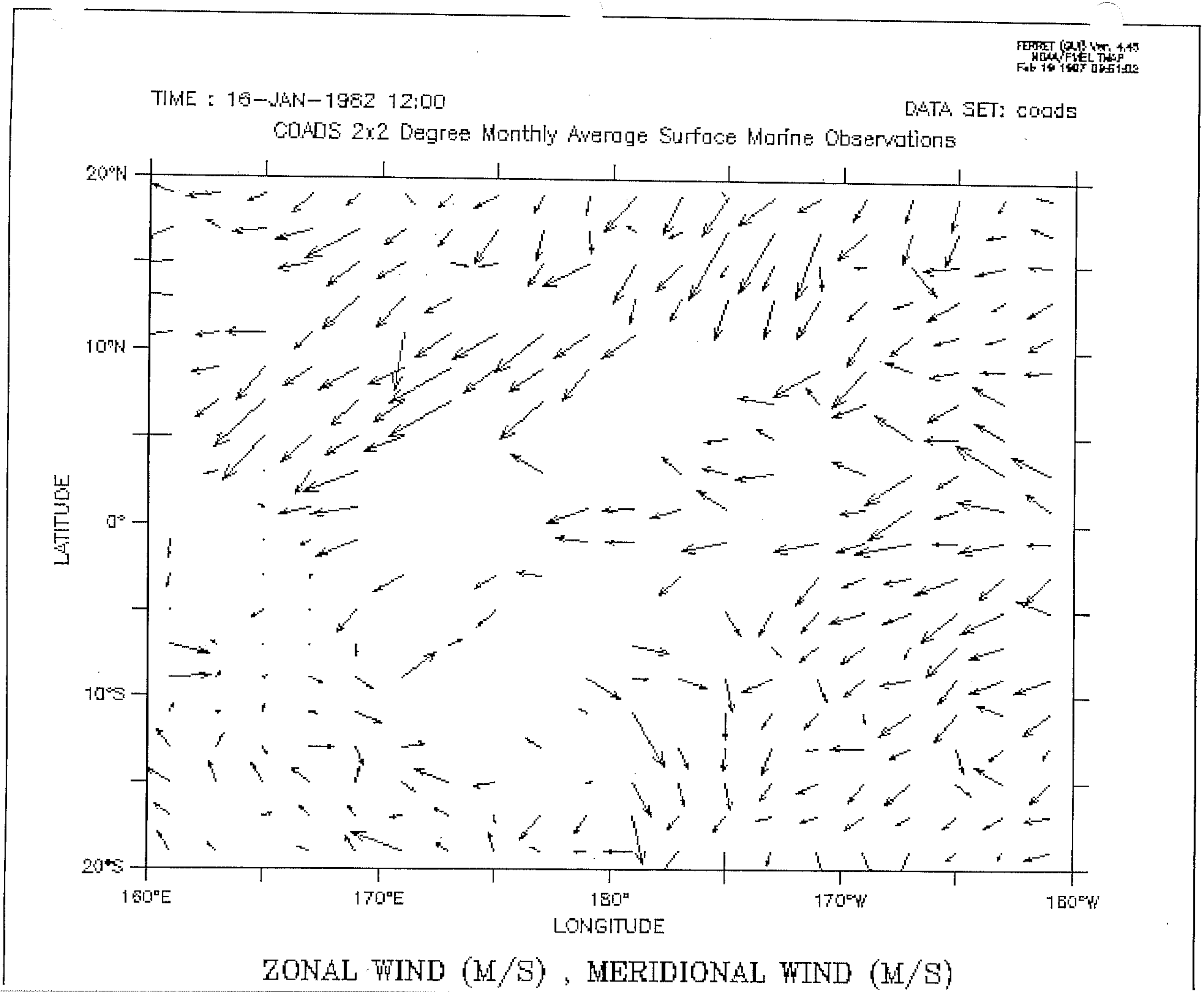


vector field

wind velocity $\vec{V}(\vec{r})$



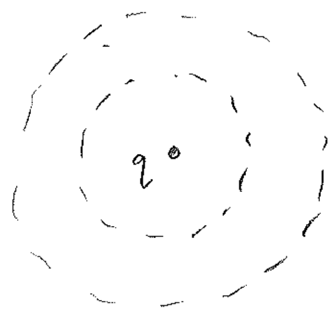
Weather Stations
Radar
Satellite
West Gardiner
27°
21°
Greene
Wales
Oak Hill
Litchfield
Libby Hill
27°
Ring Hill
Mt Prospect
Lake Auburn
Dresden
Mechanic Falls
100
Sabattus
95
295
Lewiston
Minot
24°
Richmond
Poland
4
95
24°
25°
Bowdoin
Lisbon
Bowdoinham
Ledge Hill
New Gloucester
Durham
28°
31°
30°
Topsham
29°
28°
27°
Pownal
295
29°
Bath
95
Gray
North Yarmouth
115
24°
Freeport
30°
28°
Maquoit Bay
30°
Atherton Hill
Windham
Phippsburg
Yarmouth
30°
Harpswell
Cumberland
32°
Little Falls-South Windham
Broad Sound
Morse Mountain
Falmouth
295
30°
31°
Casco Bay
Gorham
31°
31°
Luckse Sound
Portland
32°
Fort Levett
South Portland


FERRET (GUI) Ver. 4.45
NOAA/PMEL TMAP
Feb 19 1997 09:51:02
TIME : 16-JAN-1982 12:00
DATA SET: coads
COADS 2x2 Degree Monthly Average Surface Marine Observations
20°N
10°N
0°
10°S
20°S
LATITUDE
160°E
170°E
180°
170°W
160°W
LONGITUDE
ZONAL WIND (M/S) , MERIDIONAL WIND (M/S)

Electric field

An electric charge q creates an electric field in the space surrounding it

For a stationary charge



[more intense near it, less intense further away]

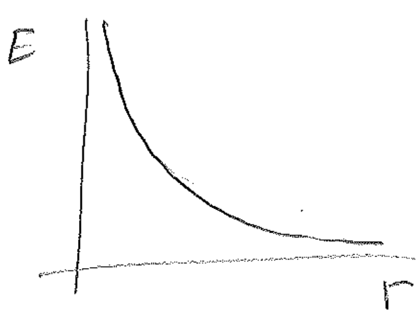
strength $E(\vec{r}) = \frac{Kq}{r^2}$

$K = \text{const} = 8.99 \times 10^9 \ \frac{N \cdot m^2}{C^2}$

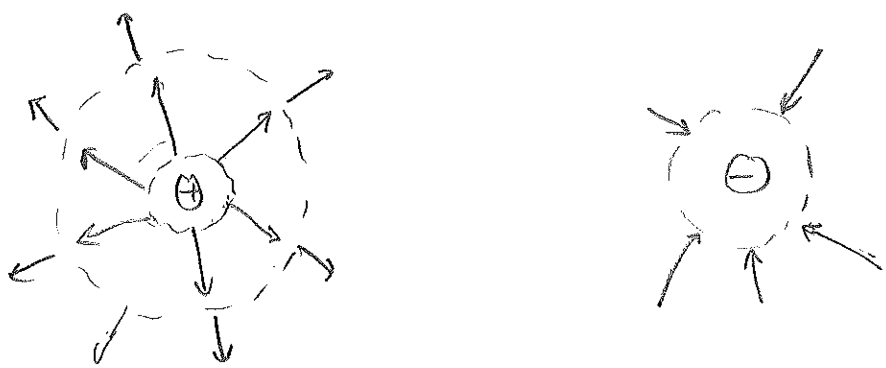
$K = \frac{1}{4\pi\epsilon_0}$

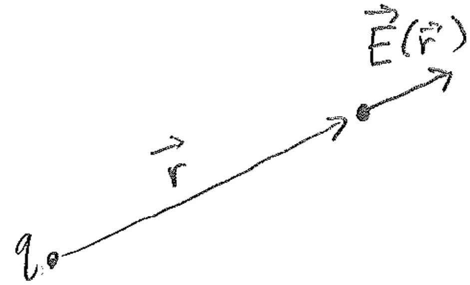
ϵ_0 = permittivity of vacuum

$E \uparrow$ w/ $q \uparrow$ and $E \downarrow$ as $\frac{1}{r^2}$



An electric field is a vector field $\vec{E}(\vec{r})$ pointing radially away from a positive charge & toward a negative charge





[place tail of $\vec{E}$ vector at location $\vec{r}$]

$$\vec{E}(\vec{r}) = \left(\frac{Kq}{r^2}\right)\hat{r}$$

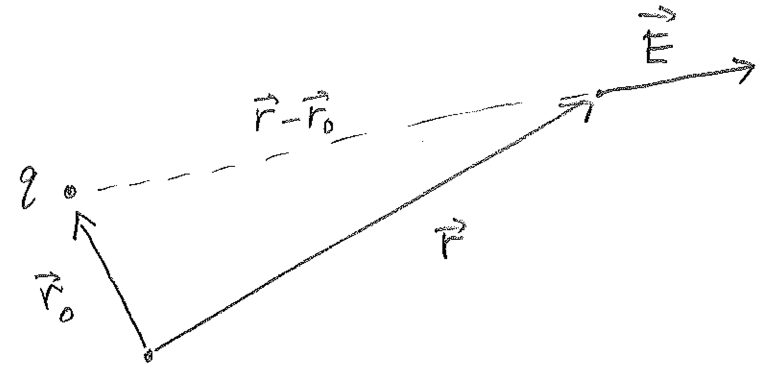
[also works for $q < 0$]

Recall $\hat{r} = \frac{\vec{r}}{r}$

$$= \frac{Kq}{r^2}\frac{\vec{r}}{r}$$

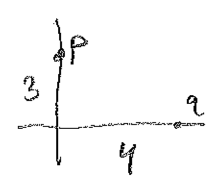
$$= \frac{Kq}{r^3}\vec{r} = \left(\frac{Kqx}{r^3}, \frac{Kqy}{r^3}, \frac{Kqz}{r^3}\right)$$

Field produced at $\vec{r}$ by a point charge q at $\vec{r}_0$



$$\vec{E}(\vec{r}) = \frac{Kq\,(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$$

[example

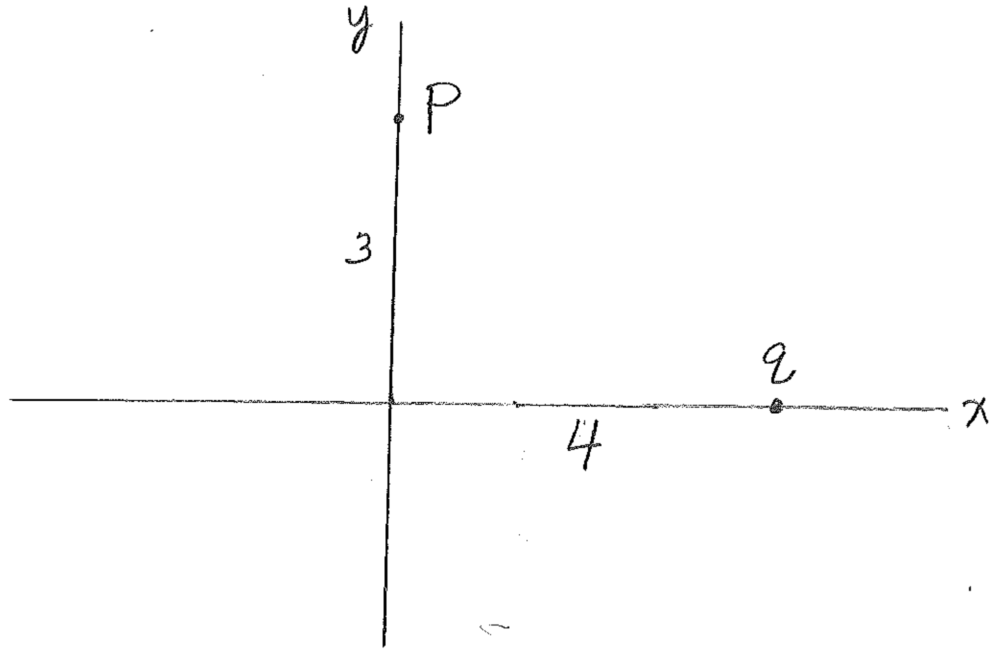


$\vec{r} = (0, 3, 0)$

$\vec{r}_0 = (4, 0, 0)$

$\vec{r} - \vec{r}_0 = (-4, 3, 0)$

$$\vec{E} = \frac{Kq\,(-4, 3, 0)}{125}$$



The y-component of $\vec{E}$ at point P is:

A) $\frac{4Kq}{125}$ B) $\frac{3Kq}{25}$ c) $\frac{3Kq}{343}$ d) $\frac{3Kq}{125}$ e) None of these

[Ans: d]

The x-component of $\vec{E}$ at point P is:

(same choices)

[Ans: e]

2010
First term
63% D

[can do this 2 ways]

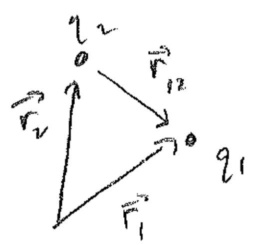
Electrostatic (or Coulomb) force

A charge produces an electric field,
but also responds to the fields produced by other charges.

An electric field $\vec{E}$ exerts on a charge q the force

$\vec{F} = q\vec{E}$ [same direction for $q > 0$
opp. " " $q < 0$]

Let q_1, q_2 be point charges at $\vec{r}_1, \vec{r}_2$



Let $\vec{F}_{12} \equiv$ force exerted on q_1 by (the field produced by) q_2

$= q_1 \vec{E}_2(\vec{r}_1)$

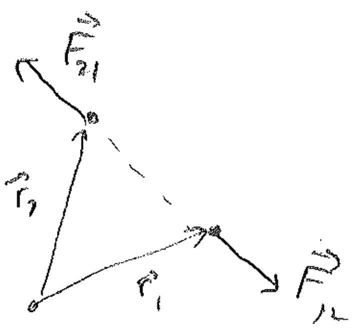
$$\vec{E}_2(\vec{r}_1) = \frac{K q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}$$

$$\Rightarrow \boxed{\vec{F}_{12} = \frac{K q_1 q_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}}$$ Coulomb's law

Let $\vec{F}_{21}$ = force exerted on q_2 by (field caused by) q_1

$$= q_2 \vec{E}_1(\vec{r}_2)$$

$$= \frac{K q_2 q_1 \vec{r}_{21}}{|\vec{r}_{21}|^3}$$



$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1 = -(\vec{r}_1 - \vec{r}_2) = -\vec{r}_{12}$$

① Since $\vec{r}_{12} = -\vec{r}_{21}$, forces are equal + opposite $\vec{F}_{12} = -\vec{F}_{21}$

(Newton's 3rd)

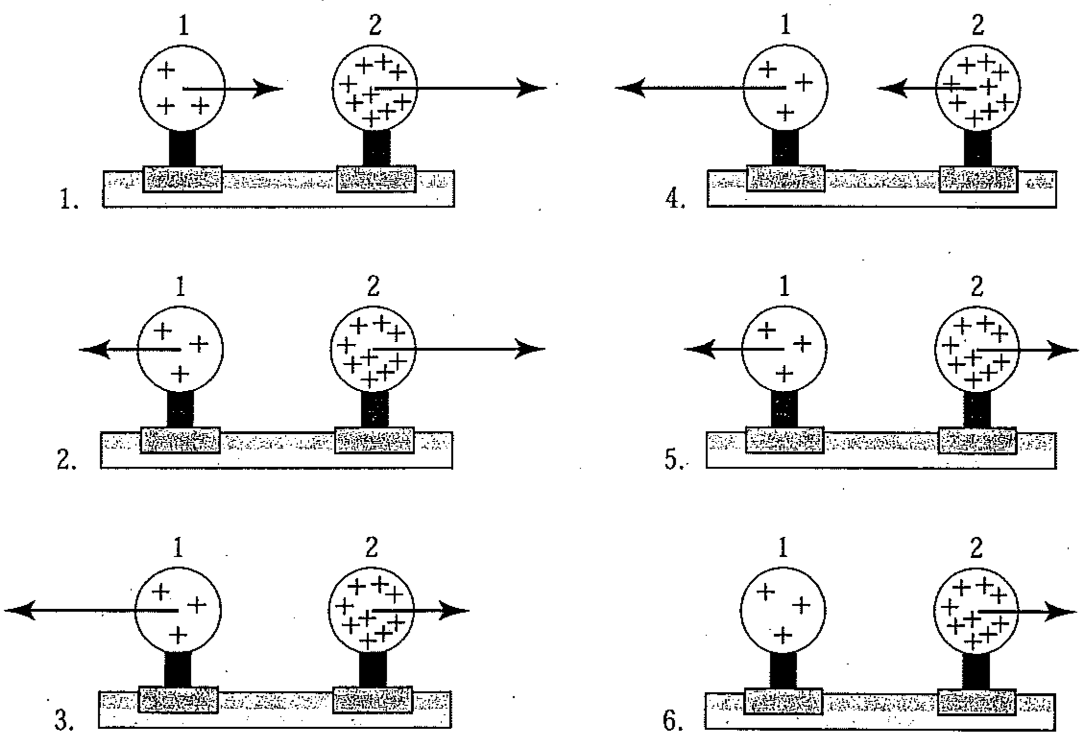
② $\vec{F}_{12}$ depends on product of charges: likes repel, opposites attract

③ inverse square (like gravity) $F_{12} = \frac{K q_1 q_2}{r_{12}^2}$

④ units of K make sense $\frac{N \cdot m^2}{C^2}$

optional (did in 2019)

Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces:



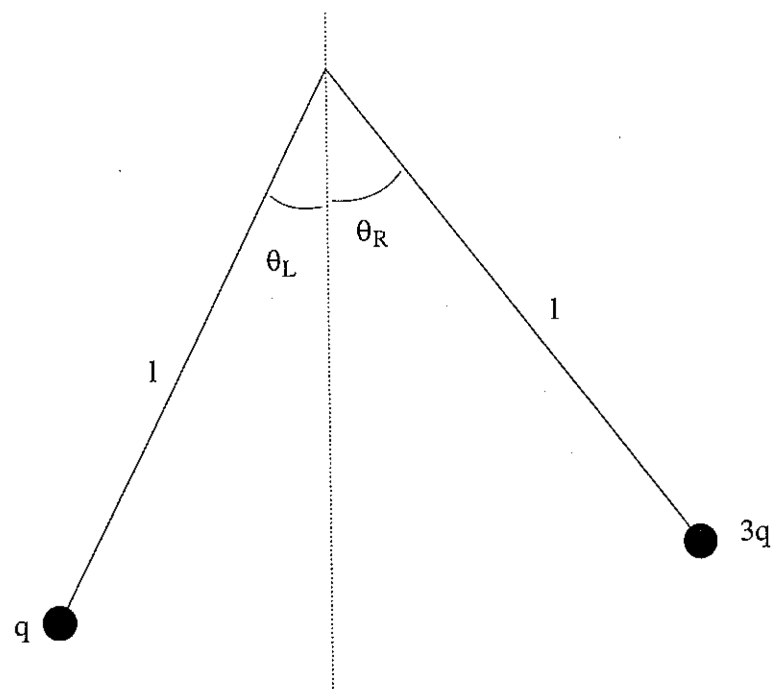
7. none of the above

5. 612
3. 332

optional
did not do

Question:

Two charged spheres of equal mass m are suspended by two threads of equal length l. The left sphere carries a charge q, the right sphere carries a charge $3q$. What is the relation between the two angles θ_L and θ_R?



(a) Not enough information

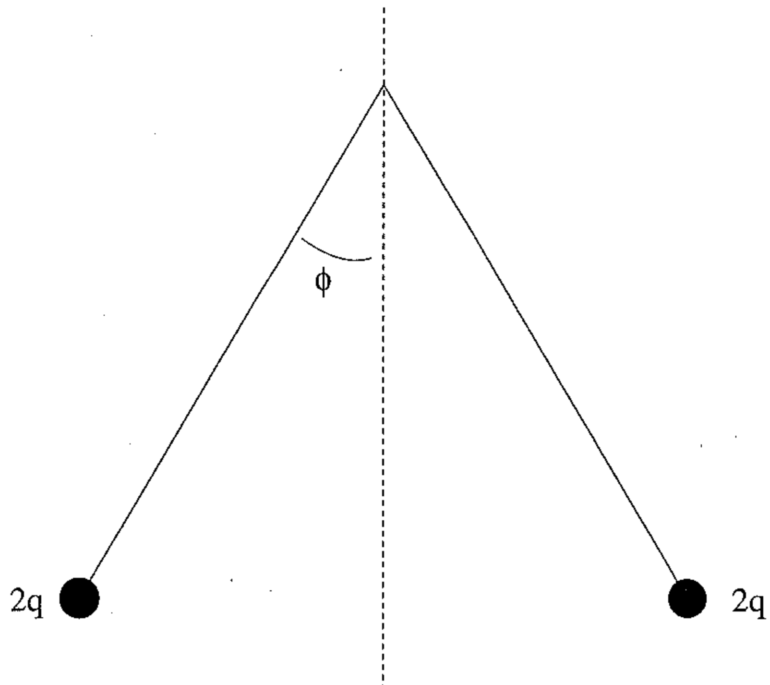
(b) $\theta_L < \theta_R$

(c) $\theta_L = \theta_R$

(d) $\theta_L > \theta_R$

Question:

Now you briefly touch the two conducting spheres, so that afterwards both spheres carry a charge $2q$. How does the new angle ϕ compare with the old angle θ?



(a) Not enough information

(b) $\phi < \theta$

(c) $\phi = \theta$

(d) $\phi > \theta$

A hydrogen atom is composed of a nucleus containing a single proton, about which a single electron orbits. The electric force between the two particles is 2.3×10^{39} greater than the gravitational force! If we can adjust the distance between the two particles, can we find a separation at which the electric and gravitational forces are equal?

1. Yes, we must move the particles farther apart.
2. Yes, we must move the particles closer together.
3. no, at any distance