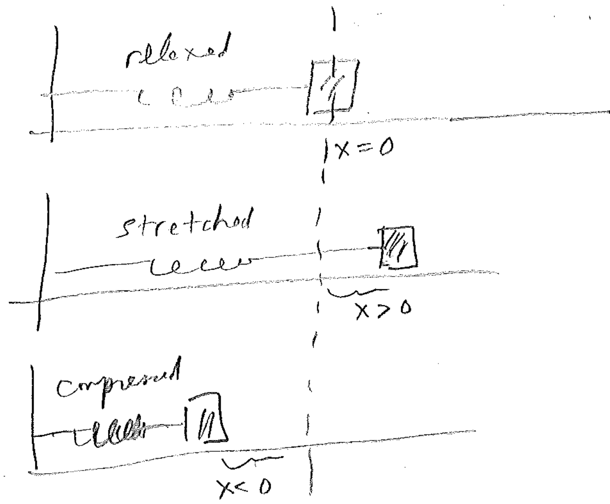
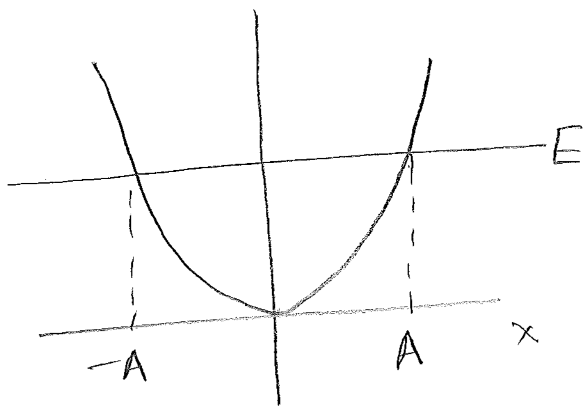
N11 oscillatory motion (any vibrating system)

Let $x = x_0$ at relaxed position and choose $x_0 = 0$



Potential energy $V(x) = \frac{1}{2} k_s x^2$ (choose $x_0 = 0$)



Let $\pm A$ be turning points

mass oscillates between A and -A

A = amplitude of oscillation

$$E = V(\pm A) = \frac{1}{2} k_s A^2$$

Q: What is maximum speed of mass in its oscillation

$$\frac{1}{2} m v_{max}^2 = E = \frac{1}{2} k_s A^2$$

$$v_{max} = \sqrt{\frac{k_s}{m}}\, A$$

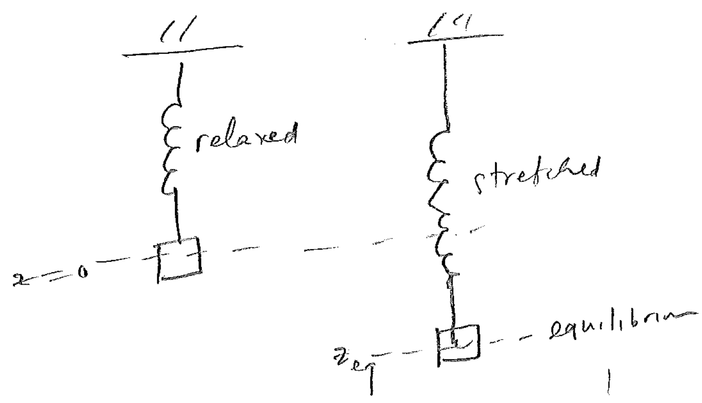
Let T = period of oscillation
= time to go from A to -A and back.

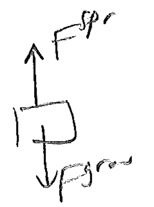
$F_x^{spr.} = -\frac{dV}{dx} = -k_s x$ Hooke's law

Restoring force: if $x > 0$, then $F_x < 0$
if $x < 0$, then $F_x > 0$
if $x = 0$, then $F_x = 0$

k_s large $\Rightarrow$ stiff spring

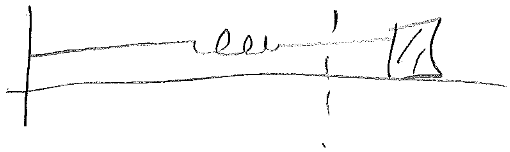
<u>Hanging mass</u>





$F_z^{net} = -k_s z - mg$

At equilibrium $F_z^{net} = 0 = -k_s z_{eq} - mg \Rightarrow \underline{z_{eq} = -\frac{mg}{k_s}}$



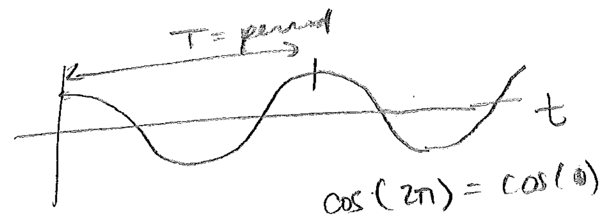
[remind self of N2 + C1]

$$F_{net,x} = -kx \stackrel{!}{=} m a_x = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x$$

Can't directly integrate because don't know $x(t)$

Guess: $x = A\cos(\omega t)$



$\cos(2\pi) = \cos(0)$

so

$\omega T = 2\pi$

$\omega = \frac{2\pi}{T}$

$\omega \equiv$ angular frequency $\left(\frac{\text{rad}}{\text{sec}}\right)$

See if this works

$$\frac{dx}{dt} = -A\omega \sin(\omega t)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t)$$

these are equal if

$$-\frac{k}{m}x = -\frac{k}{m} A\cos(\omega t)$$

$\omega^2 = \frac{k}{m}$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Note: $x = A\sin(\omega t)$ also works

Most general $x = A\cos(\omega t + \theta)$ for arbitrary θ

ω = angular frequency = $\frac{\text{rad}}{\text{sec}}$

f = frequency = $\frac{\text{cycles}}{\text{sec}}$ = Hertz

But T = sec/cycle so $\boxed{f = \frac{1}{T}}$

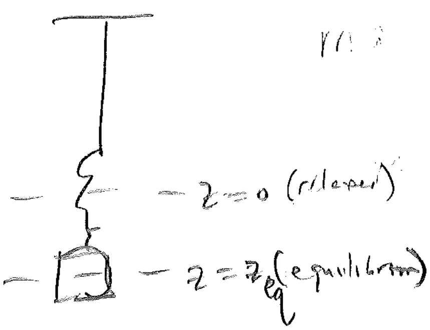
so $\omega = 2\pi f$, $f = \frac{\omega}{2\pi}$

N11.T1 C
N11.T2 E
N11.T3 D
N11.T4 B
N11.T5 F

$\hookrightarrow \omega = \frac{2\pi}{.6} = 10$

[demo?] N11-4

Hanging mass: z = distance from relaxed position



$$F_{net,z} = -kz - mg$$

Recall $F_{net,z}=0$ when $z = z_{eq} = -\frac{mg}{k}$

so $-mg = kz_0$

so

$$F_{net,z} = -k(z - z_{eq})$$

$$m\frac{d^2 z}{dt} = -k(z - z_{eq})$$

Let z' = distance from equilibrium position

$$= z - z_{eq}$$

$$\frac{dz'}{dt} = \frac{dz}{dt}$$

$$\frac{d^2z'}{dt^2} = \frac{d^2z}{dt^2}$$

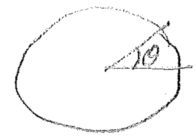
$$\Rightarrow \quad m\frac{d^2z'}{dt^2} = -kz'$$

Same eqn as before

$$z' = A\cos(\omega t + \theta)$$

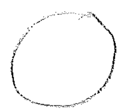
$$z = z_{eq} + A\cos(\omega t + \theta)$$

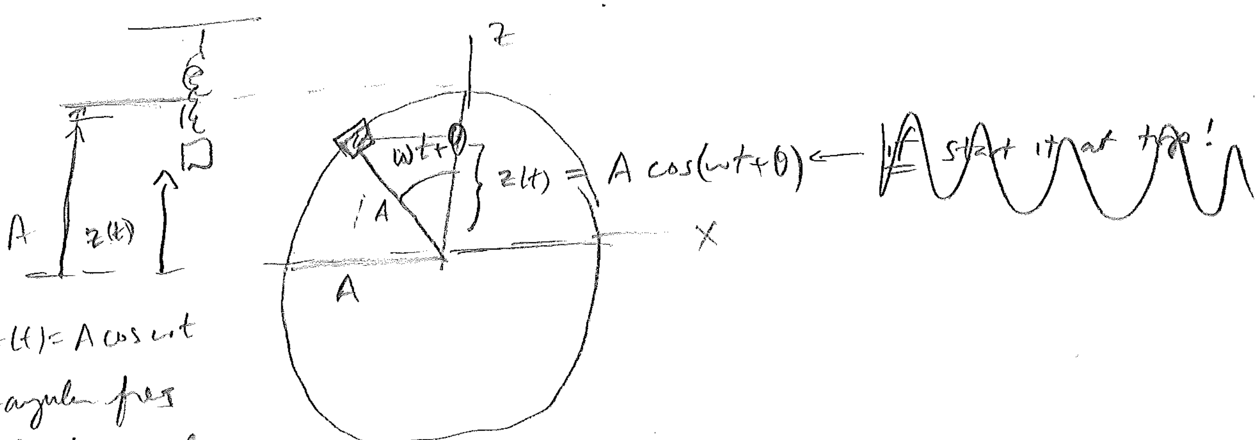
Analogy w/ circular motion

Recall ω = angular speed = $\frac{rad}{sec}$  $\omega = \frac{d\theta}{dt}$

T = period = time for one revolution = $\frac{2\pi}{\omega}$

f = freq = $\frac{\text{revolutions (cycles)}}{sec} = \frac{1}{T}$

DEMO! show it  first then sideways (setting ≈ 7.5)



$z(t) = A\cos\omega t$

ω = angular freq

T = time for one cycle

$f = \frac{1}{T} = \frac{cyc}{sec}$

If start it at angle θ from vertical then

$$z(t) = A\cos(\omega t + \theta)$$

↑ initial phase

$\omega = \sqrt{\frac{k}{m}}$ N11-6

How determine A, θ? initial conditions

Suppose initial position is x_0 & initial velocity is V_{0x} then

(more useful $x(0)$ and $v_x(0)$ throughout)

$x_0 = A\cos\theta \longrightarrow \cos\theta = \frac{x_0}{A}$

$V_x = \frac{dx}{dt} = -A\omega\sin(\omega t+\theta)$

$V_{0x} = -A\omega\sin\theta \longrightarrow \sin\theta = \frac{-V_{0x}}{A\omega}$

then $\frac{V_{0x}}{x_0} = \frac{-A\omega\sin\theta}{A\cos\theta} = -\omega\tan\theta$ so $\theta = \arctan\left(-\frac{V_{0x}}{\omega x_0}\right)$

$\Rightarrow \theta = \arctan\left(-\frac{V_{0x}}{\omega x_0}\right)$

gives 2 answers; need to try it in x_0 & see which works

$\left.\begin{array}{l} \frac{x_0}{A} = \cos\theta \\ -\frac{V_{0x}}{A\omega} = \sin\theta \end{array}\right\}$

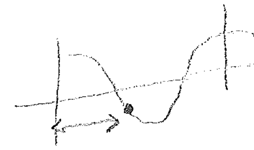
$\cos^2\theta + \sin^2\theta = 1$

$\frac{x_0^2}{A^2} + \frac{V_{0x}^2}{A^2\omega^2} = 1$

$A = \sqrt{x_0^2 + \frac{V_{0x}^2}{\omega^2}}$

Do N11.X7

Probably also N11T.7, 11



$\omega = \sqrt{\frac{100}{1}} = 10 \frac{rad}{sec}$

$x_0 = -0.10\,m$

$v_0 = -0.50\,m/s$

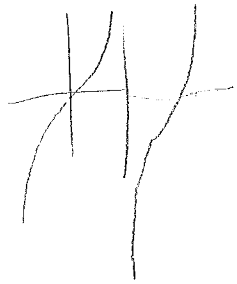
$\tan\theta = -\left(-\frac{5}{10}\right) =$

$\theta = -26^\circ$ or 153°; $\underline{\underline{333^\circ}}$

$\cos(-26^\circ) > 0$

$\cos(153^\circ) < 0$

$A = 0.112\,m$

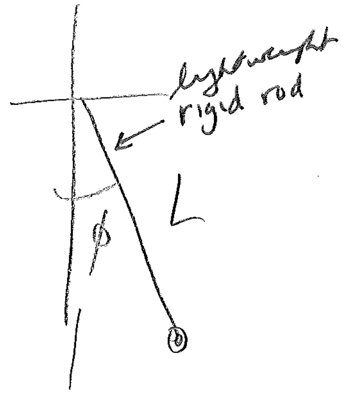


11

Exercise N~~12~~X.7: An 1.0 kg object hangs from a spring whose spring constant is 100 N/m. You take the mass, pull it down 10 cm, and then give it an initial downward speed of 0.50 m/s. What is are the values of A and θ here?

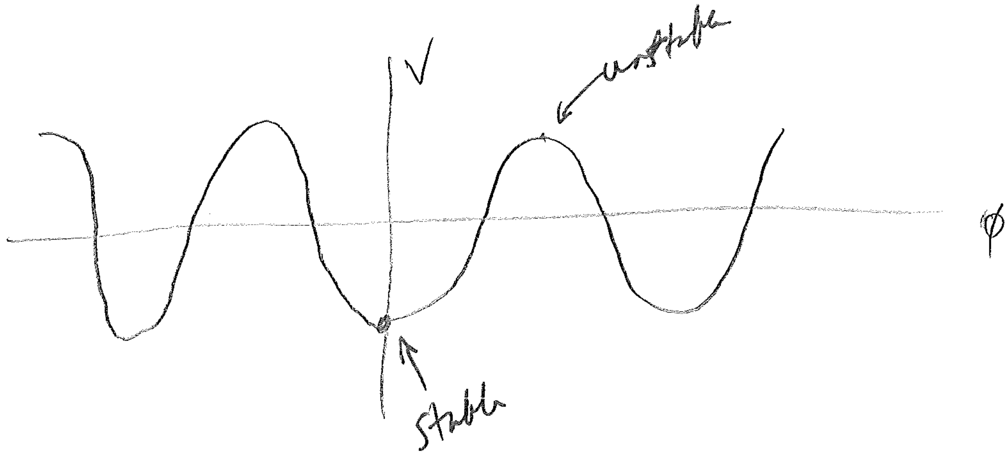
Exercise N12X.8: A 68-kg friend of yours goes bungee jumping, and you watch. You notice that after the jump, the friend oscillates once up and down in about 6.0 s. Estimate the effective spring constant of the bungee cord.

Simple pendulum [Can do this on last day]



$V^{grav} = mgz$

$= -mgL\cos\phi$



[DEMO: pendulum
w/2 legs fixed]

[can do this later w/ oscillation]

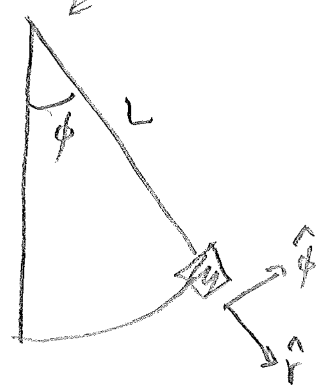
~~N8-21~~

~~N8-2-8~~

N11-8

Pendulum

(let $\phi = 0$ be equilibrium)

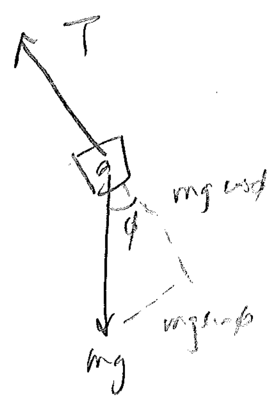


can skip

$$\left[\begin{array}{l} \vec{r} = L\hat{r} \\ \vec{v} = \underbrace{L\frac{d\phi}{dt}}_{v_\phi}\hat{\phi} \end{array}\right]$$

$$\vec{a} = \underbrace{\frac{dv_\phi}{dt}}_{a_\phi}\hat{\phi} - \underbrace{\frac{v_\phi^2}{L}}_{a_r}\hat{r}$$

Forces acting



$$\vec{F}^{net} = F_r^{net}\hat{r} + F_\phi^{net}\hat{\phi}$$

$$F_r^{net} = mg\cos\phi - T = ma_r = -\frac{mv_\phi^2}{L}$$

solve for T

$$T = mg\cos\phi + \frac{m}{L}v_\phi^2$$

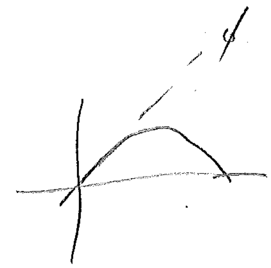
$$F_\phi^{net} = -mg\sin\phi = ma_\phi = m\frac{dv_\phi}{dt} = mL\frac{d^2\phi}{dt}$$

$$\Rightarrow \frac{d^2\phi}{dt} + \frac{g}{L}\sin\phi = 0$$

differential equation describing how ϕ changes in time [solve later]

~~Earlier (NII) derived pendulum eqn~~

$$\frac{d^2\phi}{dt^2} = -\frac{g}{l}\sin\phi$$



~~From 211:~~ $\sin\phi \approx \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!}$

eg even $\phi = 30^\circ \Rightarrow$ $\phi = 0.52$, $\sin\phi = 0.50$ (5% accuracy)

Small angle approximation

$$\frac{d^2\phi}{dt^2} \approx -\frac{g}{l}\phi$$

$$\Rightarrow \phi = A\cos(\omega t + \theta) \qquad \omega = \sqrt{\frac{g}{l}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

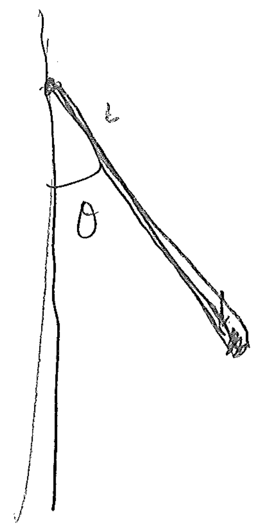
but large angle

[pendulum

[DEMO: pendulum:
normal modes
chaos]

optional

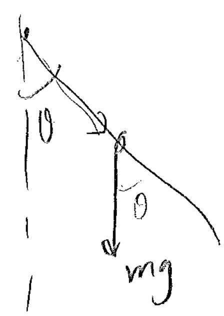
Redo pendulum using torques...



rod of length L pivots about a point.

Calc torque about pivot

$\vec{\tau} = \vec{r} \times \vec{F}_{grav}$



$|\tau| = \frac{L}{2} mg \sin\theta \quad \otimes$

$\vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{L} = I\vec{\omega}$

I = moment of inertia of rod. with respect to axis thru the end

$= \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$

$= \frac{1}{3} mL^2$

$\omega = \frac{d\theta}{dt}, \quad \vec{\omega} = \frac{d\theta}{dt} \odot$

$\vec{L} = I \frac{d\theta}{dt} \odot$

$\frac{d\vec{L}}{dt} = I \frac{d^2\theta}{dt^2} \odot$

$\frac{d\vec{L}}{dt} = \vec{\tau}$

$I \frac{d^2\theta}{dt^2} \odot = -\frac{L}{2} mg \sin\theta \otimes \Rightarrow$

$\frac{d^2\theta}{dt} = -\frac{3}{2}\frac{g}{L}\sin\theta$

$\approx -\frac{3}{2}\frac{g}{L}\theta$

$\omega = \sqrt{\frac{3}{2}\frac{g}{L}}$

N11T.1 If you double the amplitude of a harmonic oscillator, the oscillator's period

A. Decreases by a factor of 2.
B. Decreases by a factor of $\sqrt{2}$.
C. Does not change.
D. Increases by a factor of $\sqrt{2}$.
E. Increases by a factor of 2.
F. Changes by some other factor (specify).

N11T.2 If you double the amplitude of a harmonic oscillator, the object's maximum speed does what? (Use the answers for problem N11T.1.)

N11T.3 If you double the spring constant of a harmonic oscillator, the oscillation frequency does what? (Use the answers for problem N11T.1.)

N11T.4 A glider on an air track is connected by a spring to the end of the air track. If it takes 0.30 s for the glider to travel the distance of 12 cm from one turning point to the other, its amplitude is

A. 12 cm
B. 6 cm
C. 24 cm
D. 36 cm
E. 3.6 cm
F. We are not given enough information to answer.

N11T.5 Consider the glider described in problem N11T.4. Its phase rate is

A. $0.30\ s^{-1}$
B. $0.15\ s^{-1}$
C. $0.60\ s^{-1}$
D. $3.77\ s^{-1}$
E. $0.096\ s^{-1}$
F. Some other result (specify)

N11T.6 A glider on an air track is connected by a spring to the end of the air track. If it is pulled 3.5 cm in the $+x$ direction away from its equilibrium point and then released from rest at $t = 0$, what is the initial phase θ?

A. 0
B. $\pi/4$
C. $\pi/2$
D. π
E. $3\pi/2$
F. Some other result (specify)

N11T.7 A glider on an air track is connected by a spring to the end of the air track. If it is pulled 3.5 cm in the $-x$ direction away from its equilibrium point and then released from rest at $t = 0$, what is the initial phase θ?

A. 0
B. $\pi/4$
C. $\pi/2$
D. π
E. $3\pi/2$
F. Some other result (specify)

N11T.8 A mass hanging from the end of a spring has a phase rate of $\omega = 6.3\ \mathrm{s}^{-1}$ (≈ 1 cycle/s). Let's define $t = 0$ to be when the mass passes $x = 0$ going up. If its speed as it passes is 1.0 m/s, what is its amplitude A?

A. 0
B. 0.16 m
C. 1.0 m
D. 6.3 m
E. We are not given enough information to answer.
F. Some other result (specify).

N11T.9 To double the period of a pendulum, you need to multiply its length by a factor of

A. $\frac{1}{2}$
B. 2
C. $\sqrt{\frac{1}{2}}$
D. $\sqrt{2}$
E. 4
F. Some other result (specify)

Exercise N11X.7

A 1.0-kg object hangs from a spring whose spring constant is 100 N/m. You take the mass, pull it down 10 cm, and then give it an initial downward speed of 0.50 m/s. What are the values of A and θ here?

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