Projectile motion (w/o air resistance)
(combine things we've done before)

gravity is the only force acting $\vec{F}_g = m\vec{g} = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}$ $\downarrow m\vec{g}$

$\vec{F}_{net} = m\vec{a}$ so

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$ const acceleration

[weight + mass
$wt = mg$
$1\ kg \Rightarrow 2.2\ lbs = (1kg)(9.8\ m/s^2) = 9.8N$
$lb = 4.45N$]

$$\vec{v} = \int \vec{a}\,dt = \begin{pmatrix} v_{0x} \\ v_{0y} \\ -gt + v_{0z} \end{pmatrix}$$

$$\vec{r} = \int \vec{v}\,dt = \begin{pmatrix} v_{0x}t + x_0 \\ v_{0y}t + y_0 \\ -\frac{1}{2}gt^2 + v_{0z}t + z_0 \end{pmatrix}$$

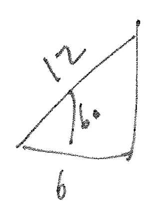
Stress:
motion in different directions is independent.

[N10 T.3]$_c$ ← all got this

→ talked abt equiv principle $m_{grav} = m_{inertial}$

[N10 T.1]$_T$ ← split vote ! !!

[N10 T.5]$_c$ ← didn't have time in 2010



LAB PREP PROBLEM

$$z = h_0 + (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$x = (v_0 \cos\theta)\, t$$

$h_0 = 0$

$$z = 0 \Rightarrow t = \frac{2 v_0 \sin\theta}{g} \Rightarrow x = \frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

$$= \frac{v_0^2}{g}(\sin 2\theta)$$

$h_0 \neq 0$

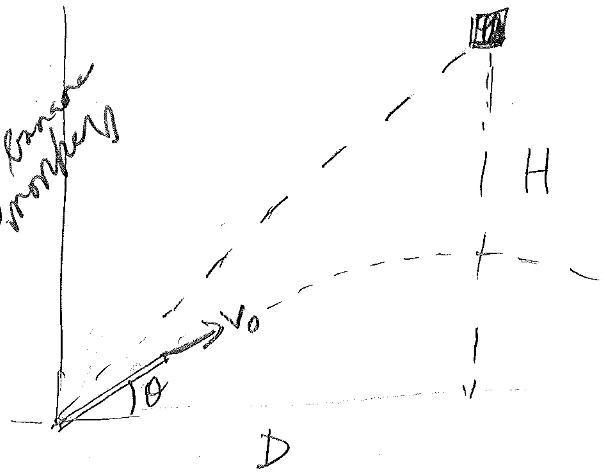
$$t = \frac{v_0 \sin\theta \pm \sqrt{v_0^2 \sin^2\theta + 2 h_0 g}}{g}$$

$$\cancel{x = \frac{v_0^2 \sin\theta \cos\theta}{g}\left[1 \pm \sqrt{1 + \frac{2 h_0 g}{v_0^2 \cos^2\theta}}\,\right]}$$

$$x = (v_0 \cos\theta)\, t$$

N10-2

throwing banana to monkeys



Show
YouTube: ~~Monkey hunter~~ ~~[illegible]~~
(~~[illegible]~~ !)
MIT-Tech TV
on Cenco

in Fall 2010 just did this & explained results

"monkey & gun"

Gun fires at $t=0$. monkey drops at same time

$$x_m = D$$

monkey trajectory: $z_m(t) = H - \frac{1}{2}gt^2$

projectile trajectory $x_p(t) = V_0 \cos\theta \, t$

$$z_p(t) = V_0 \sin\theta \, t - \frac{1}{2}gt^2$$

projectile crosses monkey path when $x_p(t_1) = D$

$$V_0 \cos\theta \, t_1 = D$$

$$t_1 = \frac{D}{V_0 \cos\theta}$$

projectile hits monkey $\underline{\text{if}}$ $z_m(t_1) = z_p(t_1)$

$$H - \cancel{\frac{1}{2}gt_1^2} = V_0 \sin\theta \, t_1 - \cancel{\frac{1}{2}gt_1^2}$$

$$H = V_0 \sin\theta \frac{D}{(V_0 \cos\theta)}$$

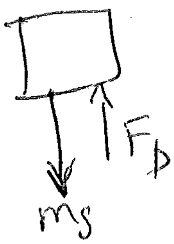
$$H = \tan\theta \, D$$

$$\frac{H}{D} = \tan\theta$$

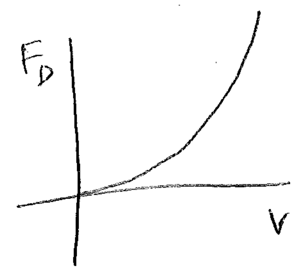
ie gun is aimed at monkey

show web video afterwards

Free fall w/ air resistance



$$F_D \approx c_2 v^2$$



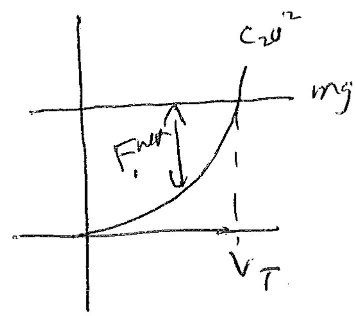
$$F_z^{net} = -mg + c_2 v_z^2$$

when $v_z = -v_T$, $F_z^{net} = 0$

$$-mg + c_2 v_T^2 = 0$$

$$v_T^2 = \frac{mg}{c_2}$$

$$c_2 = \frac{mg}{v_T^2}$$



Approach to terminal velocity starting at rest

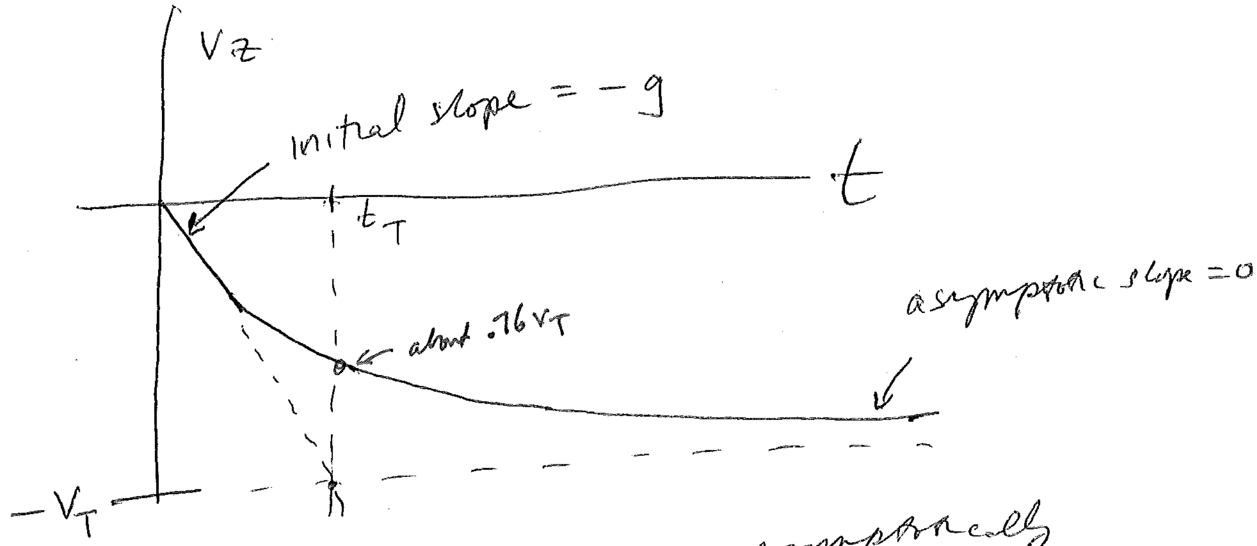
$$F_z^{net} = ma_z = -mg + \left(\frac{mg}{v_T^2}\right) v_z^2$$

$$= -mg\left(1 - \frac{v_z^2}{v_T^2}\right)$$

$$a_z = -g\left(1 - \frac{v_z^2}{v_T^2}\right)$$

When $v_z = 0$, $a_z = -g$ (free fall)

when $v_z = -v_T$, $a_z = 0$



V_z asymptotically approaches $-V_T$

Define t_T as time to reach V_T assuming no air ~~resistance~~ resistance

$V_z = -gt$ so

$-V_T = -g t_T$ so $t_T = \dfrac{V_T}{g}$

so person falling $V_T \simeq 60\,m/s$

$g \simeq 10\,m/s^2$

$t_T \simeq 6\,s$

Just for fun

$$a_z = \frac{dv_z}{dt} = -g\left(1 - \frac{v_z^2}{v_T^2}\right)$$

Let $y = -\frac{v_z}{v_T}$

Let $x = \frac{t}{t_T}$

Then $\frac{dy}{dx} = 1 - y^2$

$$\frac{dy}{1-y^2} = dx$$

$$\operatorname{arctanh} y = x$$

$$y = \tanh x = \frac{1-e^{-2x}}{1+e^{-2x}}$$

$$v_z = -v_T\left(\frac{1-e^{-\frac{2t}{t_T}}}{1+e^{-\frac{2t}{t_T}}}\right)$$

If $t = t_T$ then $\frac{1-e^{-2}}{1+e^{-2}} \approx 0.76$

$$v_z \approx -0.76\, v_T$$

If $t = 2t_T$ then $\frac{1-e^{-4}}{1+e^{-4}}$

$$v_z = -.96\, v_T$$

N10T.1 You are driving 5 ft or so behind a pickup truck (don't actually try such a stupid tailgating stunt, of course!). A jostled crate tips off the back of the truck with only a very small backward velocity. The crate will not hit your car until after it hits the road, regardless of your speed, true (T) or false (F)? (Ignore air resistance.)

N10T.2 A person standing in the cabin of a jet plane drops a coin. This coin hits the floor of the cabin at a point directly below where it was dropped (as seen in the cabin) no matter how fast the plane is moving, T or F?

N10T.3 A tennis ball is dropped from rest at the exact same instant and height that a bullet is fired horizontally. Which hits the ground first (ignoring air resistance)?

A. The bullet hits first.
B. The ball hits first.
C. Both hit at the same time.

N10T.4 As a projectile moves along its parabolic trajectory, which of the following remain constant (ignoring air resistance, and defining z axis to point upward)?

A. Its speed.
B. Its velocity.
C. Its x-velocity and y-velocity.
D. Its z-velocity.
E. Its acceleration.
F. Its x-velocity, y-velocity, and acceleration.
T. Some other combination of the given quantities.

N10T.5 Imagine that we throw a baseball with an initial speed of 12 m/s in a direction 60° upward from the horizontal. What is the baseball's speed at the peak of its trajectory? (*Hint:* You do not need to do a *lot* of calculating here.)

A. 12 m/s
B. 10.4 m/s
C. 6 m/s
D. 3 m/s
E. 0 m/s
F. Other (specify)

N10T.6 Imagine that you serve a tennis ball with an initial speed of 10 m/s in a direction 10° below the horizontal. What is its speed at the peak of its trajectory?

A. 10 m/s
B. 9.8 m/s
C. 1.7 m/s
D. 0 m/s
E. There is no "peak" to this tennis ball's trajectory
F. Other (specify)

N10T.7 Imagine that you throw a tennis ball vertically into the air. At the exact top of its trajectory it is at rest. What is the magnitude of its acceleration at this point?

A. $9.8\ \mathrm{m/s^2}$
B. $-9.8\ \mathrm{m/s^2}$
C. $0 < a < 9.8\ \mathrm{m/s^2}$
D. 0
E. Other (specify)

N10T.8 Two balls have the same size and surface texture, but one is twice as heavy as the other. How many times larger is the terminal speed of the more massive ball falling through air than that of the lighter ball?

A. The balls fall with the *same* speed in air.
B. The massive ball's terminal speed is $[2]^{1/2}$ times larger than the other's.
C. The massive ball's terminal speed is 2 times larger than the other's.
D. The massive ball's terminal speed is 4 times larger than the other's.
E. The massive ball's terminal speed is some other multiple of the other's (specify).

N10T.9 Two balls have the same weight and surface texture, but one has twice the diameter of the other. How many times larger is the terminal speed of the smaller ball falling through air than that of the bigger ball?

A. The balls fall with the *same* speed in air.
B. The smaller ball's terminal speed is $[2]^{1/2}$ times larger than other's.
C. The smaller ball's terminal speed is 2 times larger than the other's.
D. The smaller ball's terminal speed is 4 times larger than the other's.
E. The smaller ball's terminal speed is some other multiple of the other's (specify).