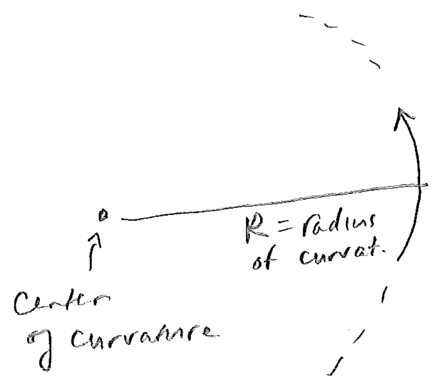
uniform circular motion

constant speed (not velocity)

acceleration is toward center of circle: $a = \frac{v^2}{R} = \omega^2 R$

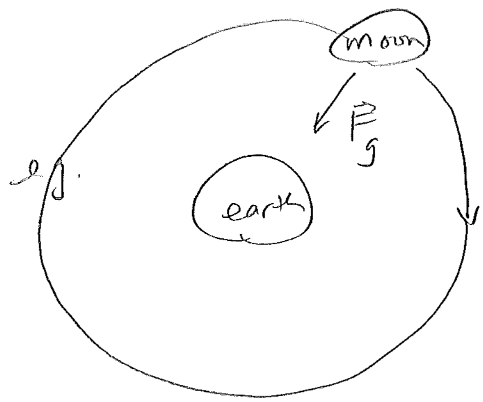
i.e. acceleration is centripetal
(center seeking)



acceleration requires a net force $F^{net} = \frac{mv^2}{R}$

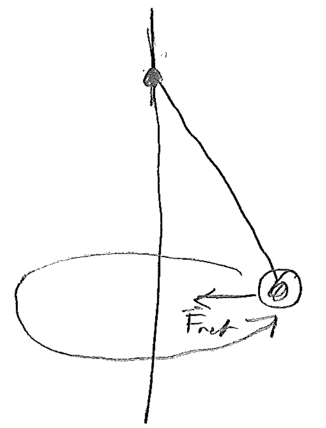
centripetal force, due to some physical force

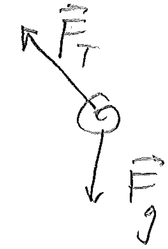
characterizes
what it does
not what causes it



gravity supplies the
centripetal force

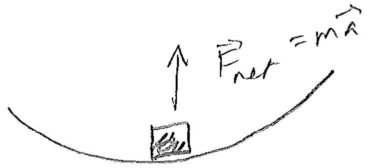
tether ball





(x-component of) tension supplies centripetal force

[N8. T1]$_A$

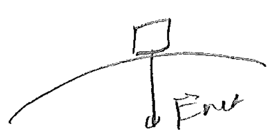


$$F_z^{net} = F_N - mg = \frac{mv^2}{R}$$

$$F_N = mg + \frac{mv^2}{R}$$

normal force supplies centripetal force

car going over hill



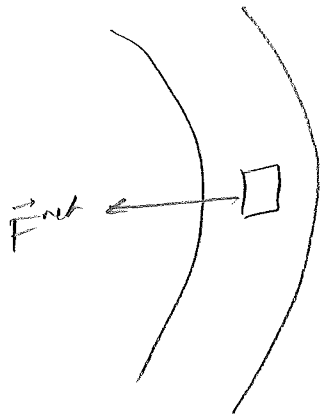
$$F_z^{net} = F_N - mg = -\frac{mv^2}{R}$$

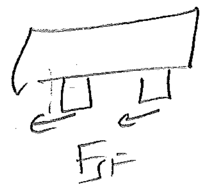
$$F_N = mg - \frac{mv^2}{R}$$

If $v = \sqrt{gR}$, then $F_N = 0$

[loop track]

Car going around a curve





$$F_{SF} = \frac{mv^2}{R}$$

static friction supplies force

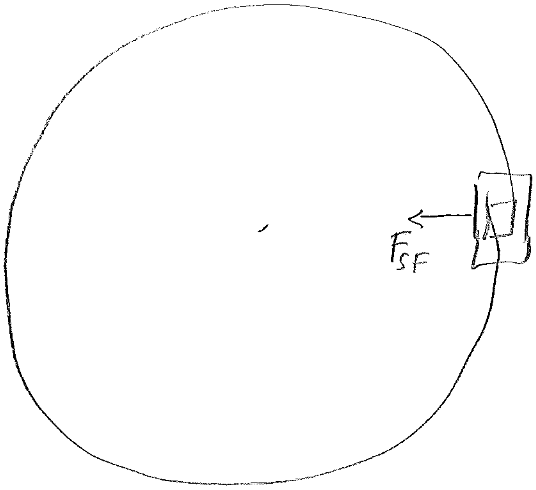
provided $F_{SF} \leq \mu F_N$

[optional discussion of centrifugal force →]

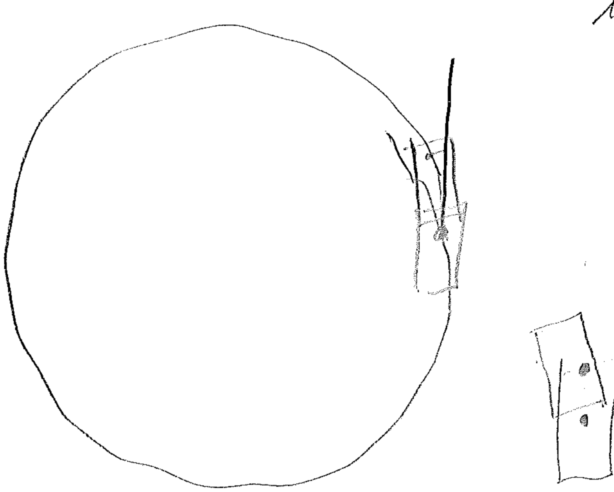
[what if road icy]

Banked curve $\Longrightarrow$

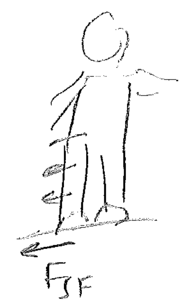
optional discussion of centrifugal force



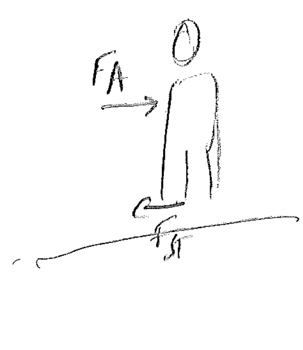
object on disk slides to the right not because of a "centrifugal" force but because of a lack of a centripetal (friction) force causing it to follow the circle by accelerating it inward



standing on a merry go round



Muscles transfer that force up your body (as if you were responding to someone trying to push you over



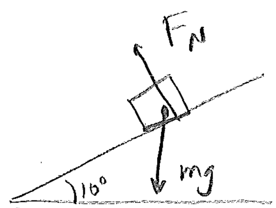
*N8S.9 As you are riding in a 1650-kg car, you approach a hairpin curve in the road whose radius is 50 m. The roadbed is banked inward at an angle of 10°.

(a) Suppose the road is very icy, so that the coefficient of static friction is essentially zero. What is the maximum speed at which you can go around the curve?

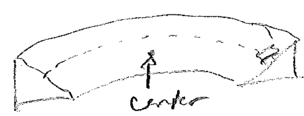
(b) Now suppose that the road is dry and that the static friction coefficient between the tires and the asphalt road is 0.6. What is the maximum speed at which you can safely go around the curve?

$m = 1650$ kg

$R = 50$ m



→ Q: direction of acceleration?



$F^z_{net} = F_N \cos\theta - mg = 0 \quad \Rightarrow F_N = \frac{mg}{\cos\theta}$.

$F^x_{net} = -F_N \sin\theta = -\frac{mv^2}{R} \quad \Rightarrow v^2 = gR \tan\theta$

$v = \sqrt{gR(\tan\theta)} = \sqrt{(9.8)(50)(0.176)} = 9.3\,\frac{m}{s} = 21$ m.p.h.

$F_N \cos\theta - F_{sf} \sin\theta = mg$

$-F_N \sin\theta - F_{sf} \cos\theta = -\frac{mv^2}{R}$

max speed

$F_N(\cos\theta - \mu \sin\theta) = mg$

$F_N(\sin\theta + \mu \cos\theta) = \frac{mv^2}{R}$

$\frac{v^2}{gR} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta} = 0.87$

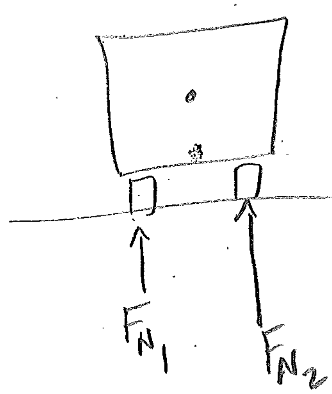
$v = \sqrt{(9.8)(50)(.87)} \approx 20.6\,\frac{m}{s} = 45$ mph.

Optional

N8-6

*N8S.11 Consider a car turning a corner to the left. Using the methods of chapter N5, show that in order to balance the torques on the car that seek to rotate the car around an axis going through its center of mass along the direction of its motion, the roadbed has to exert a greater normal force on the right wheels of the car than on the left wheels. Since this normal force will be transmitted to the car's body through the springs of the car's suspension, this means that the suspension springs on the right will have to compress more than those on the left, and thus the car will lean to the right, as asserted in section N8.4.

A car leans *away* from a curve (see problem N8S.11).



Ask? why?

N8T.1 When a speeding roller-coaster car is at the bottom of a loop, the magnitude of the normal force exerted on the car's wheels due to its interaction with the track is

A. Greater than the weight of the car and its passengers.
B. Equal to the weight of the car and its passengers.
C. Less than the weight of the car and its passengers.

N8T.2 A child grips tightly the outer edge of a playground merry-go-round as other kids push on it to give it a dizzying rotational velocity. When the other kids let go, the horizontal component of the net force on the child points most nearly

A. Inward toward the center of the merry-go-round.
B. Outward away from the center of the merry-go-round.
C. In the direction of rotation.
D. Nowhere: the horizontal component of the net force is zero.
E. In some other direction (specify).

N8T.3 A car is traveling counterclockwise along a circular bend in the road whose effective radius is 100 m. At a certain instant of time, the car is traveling due *north*, has a speed of 10 m/s, and is in the process of *increasing* that speed at a rate of 1 m/s^2. The direction of the car's acceleration at that instant is most nearly

A. North
B. Northeast
C. East
D. Northwest
E. West
F. Southwest
T. Zero

N8T.4 A plane in a certain circular holding pattern banks at an angle of 8° when flying at a speed of 150 mi/h. If a second plane flies in the same circle at 300 mi/h, what is its banking angle?

A. A bit less than 16°.
B. Exactly 16°.
C. A bit more than 16°.
D. A bit less than 32°.
E. Exactly 32°.
F. A bit more than 32°.
T. Answer depends on the planes' masses.

N8T.5 Car 1 with mass m rounds a curve of radius R traveling at a constant speed v. Car 2 with mass $2m$ rounds a curve of radius $2R$ traveling at a constant speed $2v$. How does the magnitude F_2 of the sideward static friction force acting on car 2 compare with the magnitude F_1 of the sideward static friction force acting on car 1?

A. $F_1 = 4F_2$
B. $F_1 = 2F_2$
C. $F_1 = F_2$
D. $F_2 = 2F_1$

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B. $F_1 = 2F_2$
C. $F_1 = F_2$
D. $F_2 = 2F_1$
E. $F_2 = 4F_1$
F. Other (specify)

Non-uniform circular motion

The acceleration $\vec{a}$ has both

a component toward the center, $a_r = -\frac{v^2}{R}$

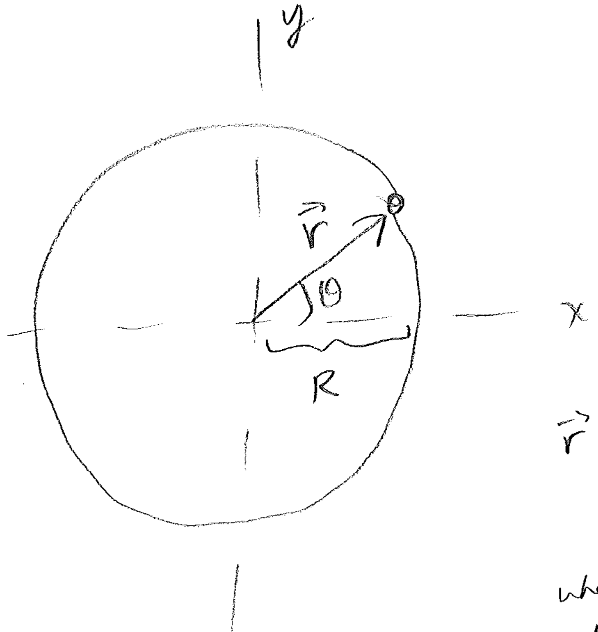
caused by the centripetal component of $\vec{F}^{net}$

and

a component parallel (or antiparallel) to the direction of motion

a_θ, caused by the ~~tangential~~ component of $\vec{F}^{net}$.

not necessarily uniform
circular motion

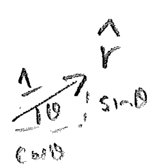


The position of a point on a circle of radius R is

$$\vec{r} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix} = R \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

where R is constant but θ may be a function of time $\theta(t)$

Define $\hat{r}$ = unit vector parallel to $\vec{r}$



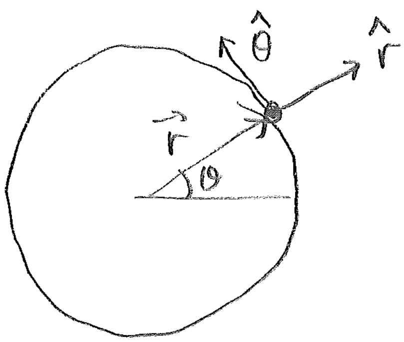
$$\hat{r} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \qquad |\hat{r}| = \sqrt{\cos^2 + \sin^2} = 1$$

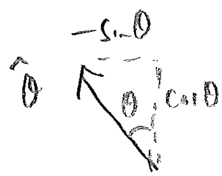
Then $\quad \vec{r} = R\hat{r} \qquad \hat{r} = \dfrac{\vec{r}}{R}$

Instantaneous velocity of the point is

$$\vec{v} = \frac{d\vec{r}}{dt} = R\begin{pmatrix} -\sin\theta \frac{d\theta}{dt} \\ \cos\theta \frac{d\theta}{dt} \\ 0 \end{pmatrix} = R\frac{d\theta}{dt}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

Define $\hat{\theta}$ = unit vector $\perp$ to $\hat{r}$ in direction increasing θ





$$\hat{\theta} = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

check $\hat{r}\cdot\hat{\theta} = 0$

Then $\vec{v} = R\frac{d\theta}{dt}\hat{\theta}$

Speed $v = |\vec{v}| = R\left|\frac{d\theta}{dt}\right| = R\omega$

ω = angular speed

[NB: Moore uses $\hat{v}$ instead of $\hat{\theta}$ in 2nd ed., but $\hat{v}$ not defined if $v=0$, so $\vec{a}$ at top of pendulum swing is ill-defined]

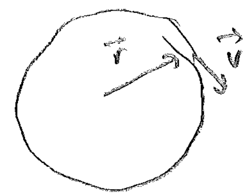
N8-2-4

Define v_θ = component of $\vec{v}$ in $\hat{\theta}$ direction

$$\vec{v} = v_\theta \, \hat{\theta} \qquad \text{so} \qquad v_\theta = R \frac{d\theta}{dt}$$

$v_\theta > 0$ if θ increasing

$v_\theta < 0$ if θ is decreasing



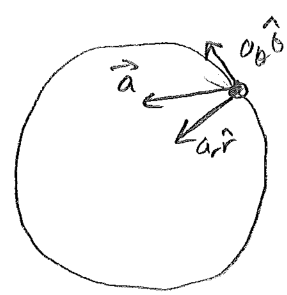
$$\vec{V} = V_\theta \hat{\theta} = V_\theta \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{dv_\theta}{dt} \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} + V_\theta \begin{pmatrix} -\cos\theta \frac{d\theta}{dt} \\ -\sin\theta \frac{d\theta}{dt} \\ 0 \end{pmatrix}$$

$$= \frac{dv_\theta}{dt} \hat{\theta} - V_\theta \frac{d\theta}{dt} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \qquad \frac{d\theta}{dt} = \frac{V_\theta}{R}$$

$$= \frac{dv_\theta}{dt} \hat{\theta} - \frac{V_\theta^2}{R} \hat{r}$$

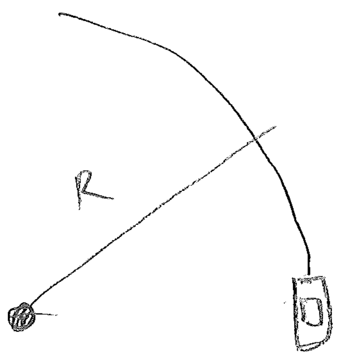


$$= a_\theta \hat{\theta} + a_r \hat{r}$$

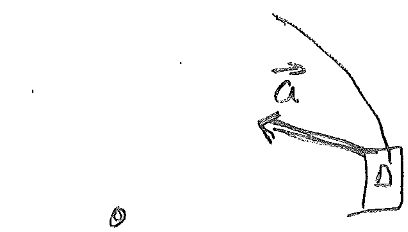
$$a_\theta = \frac{dv_\theta}{dt}, \quad a_r = -\frac{V_\theta^2}{R}$$

$$a = |\vec{a}| = \sqrt{\left(\frac{dv_\theta}{dt}\right)^2 + \left(\frac{V_\theta^2}{R}\right)^2}$$

Car going around a curve



[N8.T3]



Q: what forces are acting? F_{SF}

[Can talk about how road exerts the force on tires]

[N8.T5]

If car rounds corner at const speed, $a = \frac{v^2}{R}$

$F_{SF} = F_{net} = ma = \frac{mv^2}{R}$

But $F_{SF} \le \mu_s F_N = \mu_s mg$

so

$\frac{mv^2}{R} \le \mu_s mg$

$v < \sqrt{\mu_s g R}$

this is very close to prob. N8.S4

assign a diff problem or refrain from doing this

N8-2-7 ~~N8-2-9~~

~~Nonuniform circular motion: acceleration has both a component toward the center, given by $\frac{v^2}{R}$, and a component along the direction of motion~~ proportional to the net force in that direction

[N8.T6]$_D$

- use motion diagram to see that $\vec{a}$ is along direction of motion.
- also, no component toward center since $V = 0$.
- ~~Treating~~
- component along direction of motion $\sim g\sin\theta$ by calc'g net force in that direction
- Finally $T = mg\cos\theta$

[N8.T8]$_F$

- motion diagram to see that $\vec{a}$ is toward center
- also, no net force component along direction of motion
- $T - mg = \frac{mv^2}{R}$
- Determine V using energy conservation

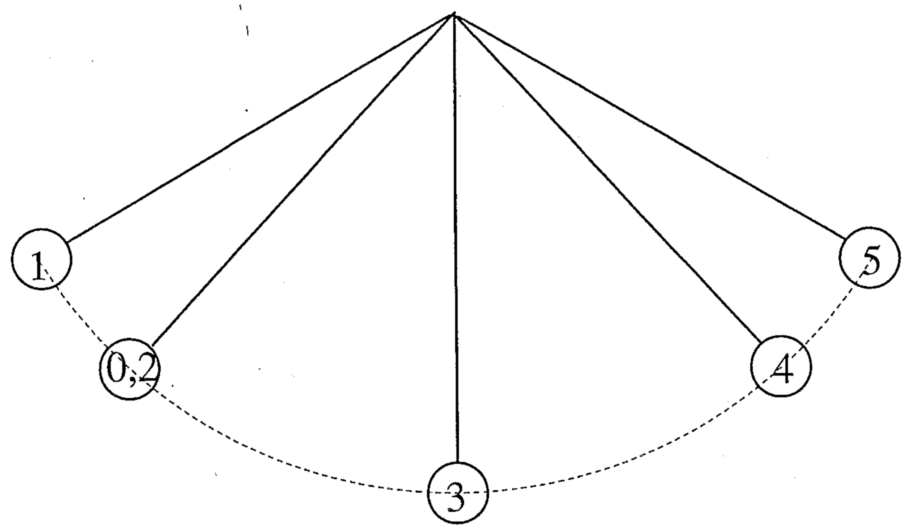
[N8.T7]$_E$

- motion diagram
- net force along motion direction $\Rightarrow a \sim g \neq 0$
- accel toward center $\sim \frac{v^2}{R}$

use these to compute tension

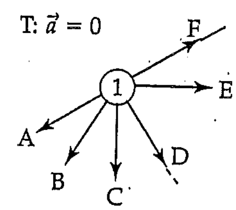
~~$T = mg\cos\theta + \frac{mv^2}{R}$~~

Should have been assigned as a problem in N02

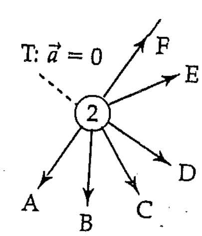


A mass (bob) swings from the end of a string. At point 1, the bob is at the extreme point of the swing and thus is instantaneously at rest. At point 3, the bob is directly below its suspension point and has its maximum speed.

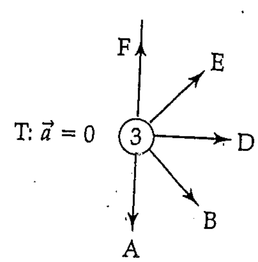
N8T.6 Which one of the arrows to the right most closely indicates the direction of the bob's acceleration when it is at point 1?



N8T.7 Which one of the arrows to the right most closely indicates the direction of the bob's acceleration when it is at point 2?



N8T.8 Which one of the arrows to the right most closely indicates the direction of the bob's acceleration when it is at point 3?



$$\dot{\phi} = 2\sqrt{\frac{g}{l}\left[\sin^2\left(\frac{\phi_0}{2}\right) - \sin^2\left(\frac{\phi}{2}\right)\right]}$$

Small angle approx: $T_0 = 2\pi\sqrt{\frac{l}{g}}$

$$t = \frac{T_0}{4\pi}\int \frac{d\phi}{\sqrt{\sin^2\left(\frac{\phi_0}{2}\right) - \sin^2\left(\frac{\phi}{2}\right)}}$$

$$\frac{T}{4} = \frac{T_0}{4\pi}\int_0^{\phi_0} \frac{d\phi}{\sqrt{\sin^2\left(\frac{\phi_0}{2}\right) - \sin^2\left(\frac{\phi}{2}\right)}}$$

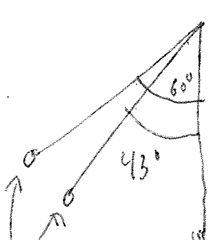
$$\frac{T}{T_0} = \frac{1}{\pi}\int_0^{\phi_0} \frac{d\phi}{\sqrt{\sin^2\left(\frac{\phi_0}{2}\right) - \sin^2\left(\frac{\phi}{2}\right)}}$$

$\phi_0 = 60^\circ \Rightarrow \frac{T}{T_0} = 1.07318$

$$\frac{T}{8} = \frac{T_0}{4\pi}\int_0^{\phi_1} \frac{d\phi}{\sqrt{\sin^2\left(\frac{\phi_0}{2}\right) - \sin^2\left(\frac{\phi}{2}\right)}} \underset{\text{numerically}}{\Longrightarrow} \phi_1 = 0.7495 \pm 0.0001$$

$$\Downarrow$$

$$\underline{\underline{42.94^\circ}}$$



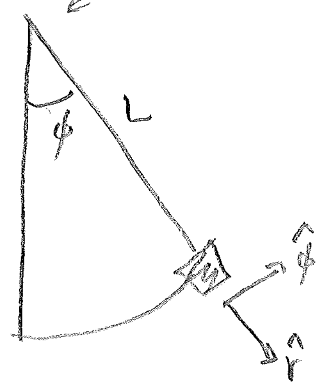
equal times

see 1130/17/Figures/pendulum.fig

[can do this later w/ oscillation]

<u>Pendulum</u>

(let $\phi = 0$ be equilibrium)

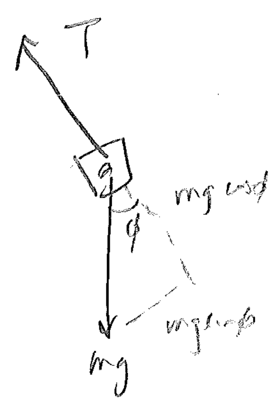


can skip

$$\left[\begin{aligned} \vec r &= L\hat r \\ \vec v &= \underbrace{L\frac{d\phi}{dt}}_{v_\phi}\hat\phi \end{aligned}\right]$$

$$\vec a = \underbrace{\frac{dv_\phi}{dt}}_{a_\phi}\hat\phi - \underbrace{\frac{v_\phi^2}{L}}_{a_r}\hat r$$

Forces acting



$$F_r^{net} = mg\cos\phi - T = ma_r = -\frac{mv_\phi^2}{L}$$

solve for T

$$T = mg\cos\phi + \frac{m}{L}v_\phi^2$$

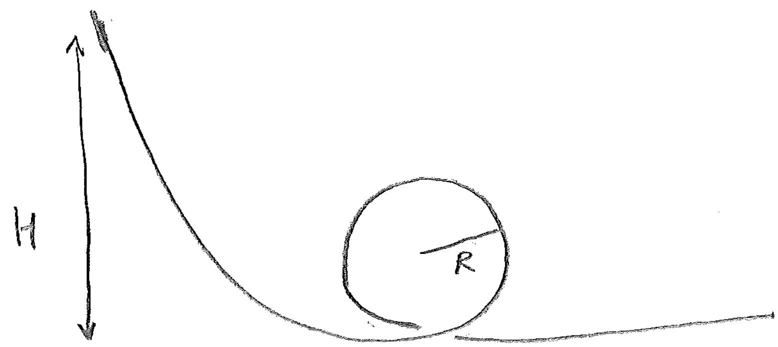
$$F_\phi^{net} = -mg\sin\phi = ma_\phi = m\frac{dv_\phi}{dt} = mL\frac{d^2\phi}{dt}$$

$$\Rightarrow \frac{d^2\phi}{dt} + \frac{g}{L}\sin\phi = 0$$

differential equation describing how ϕ changes in time [solve later]

[Can do this after problem done] N 8-2-9
[assigned as a problem]

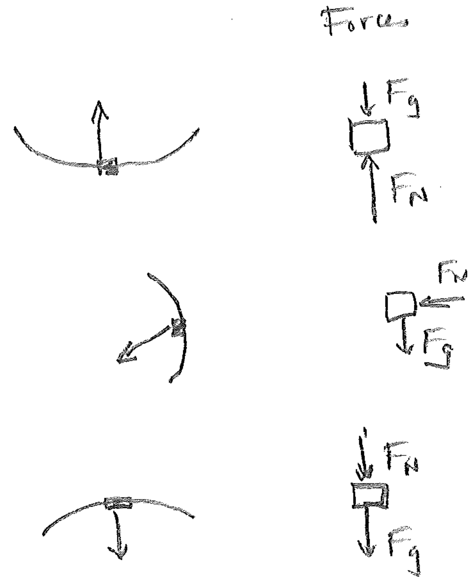
DEMO: Loop-the-loop



Ask what is direction of acceleration here

here

here



At top $F_N + F_g = \frac{mv^2}{R}$

What is minimum speed at top of loop? $v > \sqrt{gR}$

(perhaps devise a series of 2MQ for them)