

Resistive forces: friction & drag

friction: (contact)
 drag: (more than a medium fluid medium)

Friction

① static: when there is no relative motion between surfaces in contact (eg rolling % slipping)

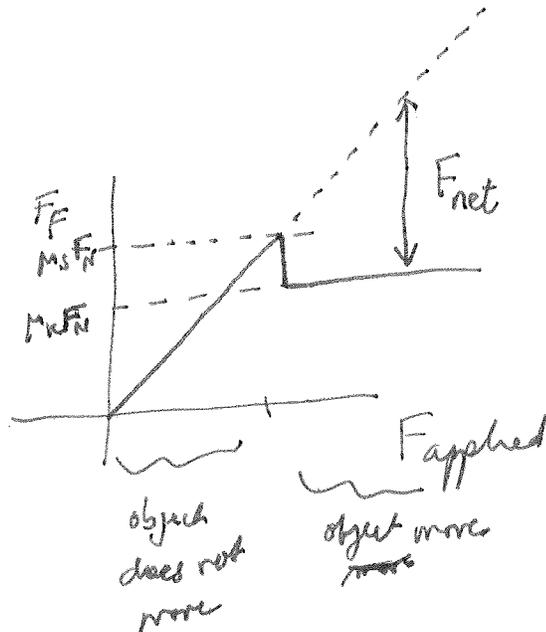
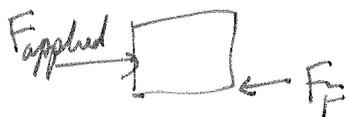
$$F_{SF} \leq F_{SF, \max} = \mu_s F_N$$

② kinetic: when surfaces are in relative motion (eg sliding)

$$F_{KF} = \mu_k F_N$$

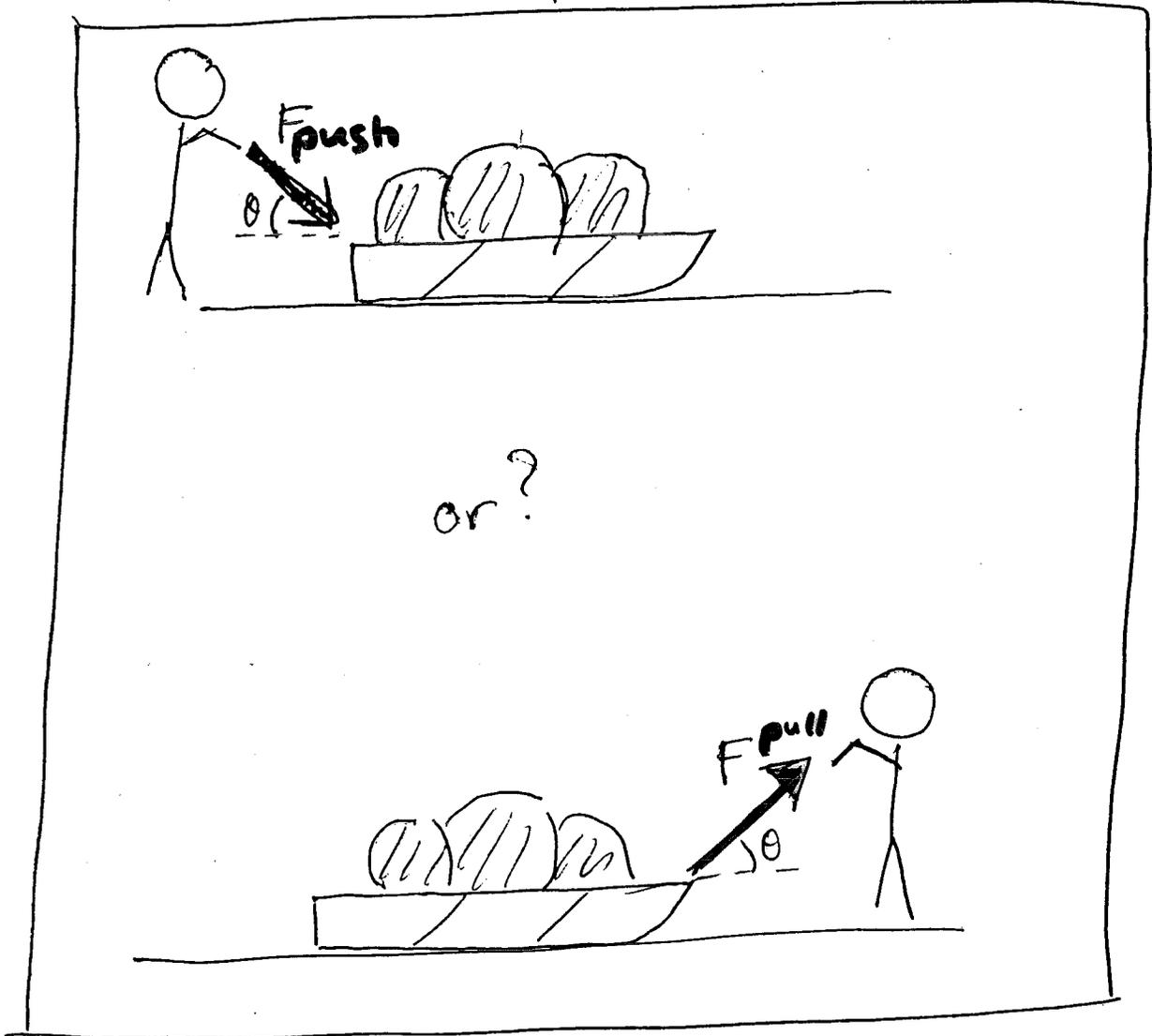
μ_k = coefficient of kinetic friction

Usually $\mu_k < \mu_s$



[explain why should have anti-lock brakes] ← tires on asphalt $\mu_s = 0.6$
 $\mu_k = 0.4$

Is it easier to
push or pull a sled?



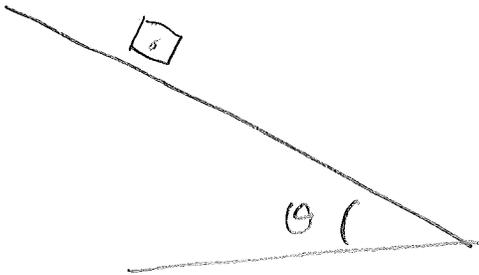
Does it make any difference?

Book sliding down a plank:

What is its acceleration?

How fast is it moving?

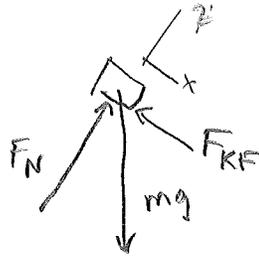
How far does it go?



• motion along plank \Rightarrow orient axes (label clearly)



• FBD



• Analyze force into components



$$F_z^{net} = F_N - mg \cos \theta$$

$$F_x^{net} = mg \sin \theta - F_{KF} = mg \sin \theta - \mu_k F_N$$

$$F^{net} = ma$$

$$a_z = 0 \Rightarrow F_z^{net} = 0 \Rightarrow F_N = mg \cos \theta$$

$$F_x^{net} = mg \sin \theta - \mu_k mg \cos \theta = ma_x$$

$$a_x = g(\sin \theta - \mu_k \cos \theta)$$

Note: if $\tan \theta = \mu_k$ then $a_x = 0$
(const velocity)

If $\tan \theta > \mu_k$, $a_x > 0$

If $\tan \theta < \mu_k$, $a_x < 0$ i.e. slowing down

$$V_x(t) = \int_0^t a_x(t) dt + v_{0x}$$

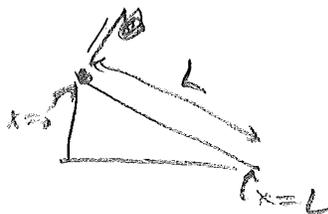
a_x is constant so

$$V_x(t) = g(\sin\theta - \mu_k \cos\theta) t + v_{0x}$$

$$\begin{aligned} x(t) &= \int_0^t v_x(t) dt + x_0 \\ &= \int_0^t [g(\sin\theta - \mu_k \cos\theta) t + v_{0x}] dt + x_0 \\ &= \frac{1}{2} g(\sin\theta - \mu_k \cos\theta) t^2 + v_{0x} t + x_0 \end{aligned}$$

[N6.T10]

If block starts at rest
at top of plane



how long to get to bottom?

How fast is it moving at bottom?

$$a = g(\sin\theta - \mu_k \cos\theta) = \text{const}$$

$$x(t) = \frac{1}{2}at^2 + \underbrace{v_{0x}}_0 t + \underbrace{x_0}_0 = L$$

$$t = \sqrt{\frac{2L}{a}}$$

$$v(t) = at + \frac{v_{0x}}{0}$$

$$= a \sqrt{\frac{2L}{a}}$$

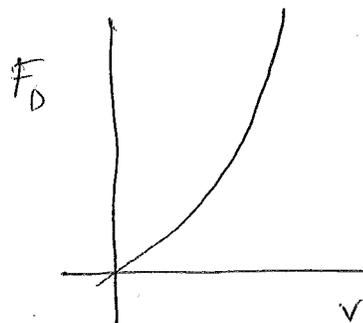
$$= \underline{\underline{\sqrt{2aL}}}$$

Drag force (eg in air or water)

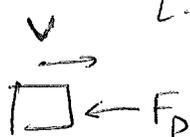
$$F_D \approx \underbrace{c_1 v}_{\text{low speeds}} + \underbrace{c_2 v^2}_{\text{high speeds}}$$

applies at low speeds
on small particles (dust)
or viscous liquids

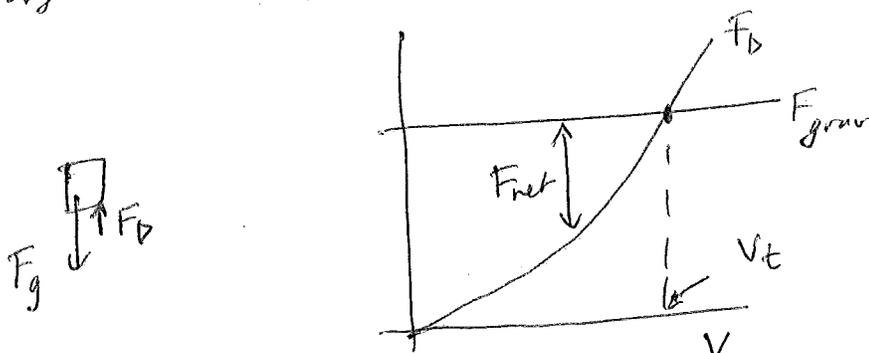
applies at
higher speeds
or for larger objects



If no other forces, object will come to rest
[force causes deceleration]



If there is a const applied force (eg gravity)
object will approach terminal velocity



→ revisit this in ch N11

As $v \uparrow$, $F_D \uparrow$, $F_{net} \downarrow$ until $F_{net} = 0$ at $v = v_t$

~~$F_{net} = F_g - F_D$~~
 ~~$= mg$~~

Terminal velocity in air

assume $F_D \approx C_2 v^2$

$$C_2 = \frac{1}{2} C_p A$$

C = drag coefficient ($= 0.5$ for sphere)

ρ = fluid density ($1.2 \frac{\text{kg}}{\text{m}^3}$ for air)

A = cross-sectional area

$$F_{\text{net}} = 0 \Rightarrow F_{\text{grav}} = C_2 v_T^2 \Rightarrow v_T = \sqrt{\frac{mg}{C_2}} = \sqrt{\frac{2mg}{C_p A}}$$

Consider a sphere of water of radius R

$$C = \frac{1}{2}$$

$$m = \frac{4}{3} \pi R^3 \rho_w$$

$$\rho_w = 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$A = \pi R^2$$

$$\Rightarrow v_T = \sqrt{\frac{16}{3} g R \frac{\rho_w}{\rho A}} = (210 \frac{\text{m}}{\text{s}}) \sqrt{\frac{R}{(1\text{m})}} \approx 500 \text{ mph} \sqrt{\frac{R}{(1\text{m})}}$$

(Imp $= 0.415 \text{ m/s}$)

person $\sim \text{~~1.7m~~ } 60 \text{ m/s} = 140 \text{ mph}$

large raindrop $\sim \text{~~1cm~~ } 20 \text{ m/s}$
 $r \sim 3 \text{ mm}, v_T \approx 10 \frac{\text{m}}{\text{s}}$

Speed of a Falling Raindrop

The Physics Factbook™

Edited by Glenn Elert -- Written by his students

An educational, [Fair Use](#) website

[topic index](#) | [author index](#) | [special index](#)

Bibliographic Entry	Result (w/surrounding text)	Standardized Result
Corbert, John H. <i>Physical Geography Manual</i> . 1974. 5th ed. N.p.: Kendall/Hunt, 2003. 127.	"A large drop of about 5 mm (3/16 in.) diameter reaches a maximum speed of about 9 m/sec."	9 m/s
"Climate." <i>Encyclopedia Britannica</i> . 2007, Encyclopedia Britannica Online. 25 May 2007.	"Large raindrops, up to six millimeters in diameter, have terminal velocities of about 10 metres per second and so may cause considerable compaction and erosion of the soil by their force of impact."	10 m/s
Beard, K.V. <u>Terminal Velocity and Shape of Cloud and Precipitation Drops</u> [pdf]. <i>Journal of the Atmospheric Sciences</i> (May 1976): 851-864.	[see graph 1 below]	9 m/s - 13 m/s
Spilhaus, A.F. <u>Raindrop Size, Shape, and Falling Speed</u> [pdf]. <i>Journal of Meteorology</i> . 5 (June 1948): 108-110.	[see graph 2 below]	9.3 m/s
Holladay, April. <u>Falling raindrops hit 5 to 20 mph speeds</u> . Wonderquest. Albuquerque: 19 Dec 2001.	"At sea level, a large raindrop about 5 millimeters across (house-fly size) falls at the rate of 9 meters per second (20 miles per hour). Drizzle drops (less than 0.5 mm across, i.e., salt-grain size) fall at 2 meters per second (4.5 mph)."	9 m/s

From 1st edition

Drag $F_D = \frac{1}{2} C_D A \rho v^2$

Stokes $F_f = 6\pi\eta r v$

Reynolds $R = \frac{\rho v d}{\eta}$

$$\begin{cases} A = \text{cross sectional area} \\ \rho = \text{fluid density} \\ C_D = \text{drag coeff} \\ \eta = \text{viscosity} \end{cases}$$

$d = \text{diameter}$

If $F_D = F_f$ then

for sphere
 $C_D = \frac{1}{2}, A = \pi r^2$

$$\frac{F_D}{F_f} = \frac{\frac{1}{2}(\frac{1}{2})\pi r^2 \rho v^2}{6\pi\eta r v} = \frac{R}{48}$$

$R : 0 \rightarrow 10 \Rightarrow \text{Stokes}$

$R : 300 \rightarrow 3 \times 10^5 \Rightarrow \text{Drag}$

$R > 3 \times 10^5 : C_D \text{ not constant}$

- N6T.1 The magnitude of the normal force on a box sitting on an incline is equal to that of its weight, true (T) or false (F)?
- N6T.2 A certain crate sits on a rough floor. You find that you have to apply a horizontal force of 200 N to get the crate moving. If you put some massive objects in the crate so that its mass is doubled, how much force does it take to get the crate moving now?
- A. Still 200 N
 - B. 400 N
 - C. 800 N
 - D. It depends (specify)
- N6T.3 Two boxes of the same mass sit on a rough floor. These boxes are made of the same kind of cardboard and are identical except that one is twice as large as the other. If it takes 200 N to start moving the smaller box, how much force does it take to start moving the larger one?
- A. Still 200 N
 - B. 400 N
 - C. 800 N
 - D. It depends (specify)
- N6T.4 The coefficient of static friction between Teflon and scrambled eggs is about 0.1. What is the smallest tilt angle from the horizontal that will cause the eggs to slide across the surface of a tilted Teflon-coated pan?
- A. 0.002°
 - B. 5.7°
 - C. 15°
 - D. 33°
 - E. Other (specify)
- X N6T.5 If you want to stop a car as quickly as possible on an icy road, you should
- A. Jam on the brakes as hard as you can.
 - B. Push on the brakes as hard as you can without locking the car's wheels and thus making the car skid.
 - C. Pump the brakes.
 - D. Do something else (specify).

- ✓ N6T.6 Putting wider tires on your car will clearly give you more traction, T or F?
- N6T.7 Assume that the coefficient of static friction between your car's tires and a certain road surface is about 0.75. Your car can climb a 45° slope, T or F?
- ✓ N6T.8 Imagine that an external force of 100 N must be applied to keep a bicycle and rider moving at a constant speed of 12 mi/h against opposing air drag. To double the bike's speed to 24 mi/h, we must increase the magnitude of the force exerted on the bike to
- A. 141 N
 B. 200 N
 C. 400 N
 D. It depends on the bike's shape and area
 E. Other (specify)
- ✓ N6T.9 Imagine that a certain engine can cause the road to exert a certain maximum forward force F_{SF} on a certain car. If we change the car's design to reduce its drag coefficient by a factor of 2, by what factor will the car's maximum speed increase (other things being equal)?
- A. No increase
 B. 1.41
 C. 2
 D. 4
 E. Depends (specify)
 F. Other (specify)
- N6T.10 A truck is traveling down a steady slope such that for each 1 m the truck goes forward along the slope it goes down 0.04 m (we call this a 4% grade). Imagine that the truck's brakes fail. What is the approximate increase in the truck's speed after 30 s, assuming the engine is not used and there is little drag or other friction? [$g = 22$ (mi/h)/s.]
- A. 0.9 mi/h
 B. 11 mi/h
 C. 26 mi/h
 D. 53 mi/h
 E. 660 mi/h
 F. Other (specify)

$\tan \theta = 0.04$
 $a = 0.4 \text{ m/s}^2$
 $\Rightarrow v \approx 0.9 \frac{\text{mi}}{\text{h}}$
 $\Rightarrow v \approx 26 \frac{\text{mi}}{\text{h}}$

A157 problem
utilizes ~~the~~
cost acct.

Review for midterm

$$|\vec{u} \times \vec{w}| = uw \sin \theta = u_{\perp} w$$

r. h. rules
compass for

$$\vec{L} = \vec{r} \times \vec{p} \quad \left\{ \begin{array}{l} \text{choose origin!} \\ \vec{r} = \text{from } o \text{ to object} \end{array} \right.$$

$$\vec{L} = \vec{L}^{cm} + \vec{L}^{rot} = \vec{r} \times M \vec{v}_{cm} + I \vec{\omega}$$

extended object

$$\frac{d\vec{L}}{dt} = \sum_{\text{external}} \vec{\tau}$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

\vec{r} = fr. o. t. point where force is applied

$$\text{Isolated} \Rightarrow \sum \vec{\tau} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{conserved}$$

forces \rightarrow $\vec{F}_{net} = m \vec{a}_{cm}$

Static $\vec{F}_{net} = 0, \vec{\tau}_{net} = 0$

use FBD to analyze indiv. force

Non static $\vec{F}_{net} = m \vec{a}$

use \vec{a} to find \vec{F}_{net} \rightarrow $\vec{F}_{net} \neq$ find \vec{a}

$$\vec{r} \rightarrow \vec{v} = \frac{d\vec{r}}{dt} \rightarrow \vec{a} = \frac{d\vec{v}}{dt}$$

motion dynamics

eg $u_{cm} \rightarrow$ \vec{a} toward ctr = $\frac{v^2}{r}$ toward center of circle.

$$\vec{s} \rightarrow \vec{v} = \int \vec{a} dt, \vec{r} = \int \vec{v} dt$$

N, F_{sf}, T are reactive

$$F_{sf} \leq \mu_s F_N$$

$$F_{kf} = \mu_k F_N$$