Statics (part 2)

object not moving $\Rightarrow \vec{p} = 0 \Rightarrow \frac{d\vec{p}}{dt} = \vec{F}_{net} = 0$

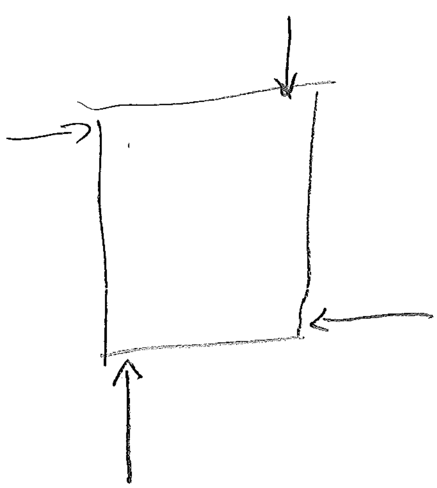
object not rotating $\Rightarrow \vec{L} = 0 \Rightarrow \frac{d\vec{L}}{dt} = \vec{\tau}_{net} = 0$

Net torque on a static object is zero.

Recall torque exerted by a force A w.r.t. origin O

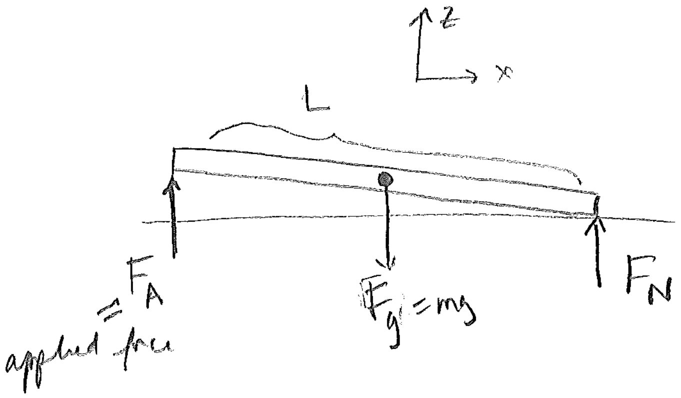
$\vec{\tau}_A = \vec{r} \times \vec{F}_A$

$\vec{r}$ = vector from O to the point where force is applied



[$\vec{F}_{net} = 0$ but will not remain static]

[N5.T5]$_D$ [use meterstick to demonstrate]



N.B. Draw forces where they act in FBD!

F_g acts through cm (effectively)

(2 unknowns: F_A & F_N)

$$F_{net,z} = F_A + F_N - mg \Rightarrow F_A = mg - F_N$$

[symmetry suggests $F_N = F_A$] (need another eqn: $\vec{\tau}_{net}$)

<u>Calculate torques</u>

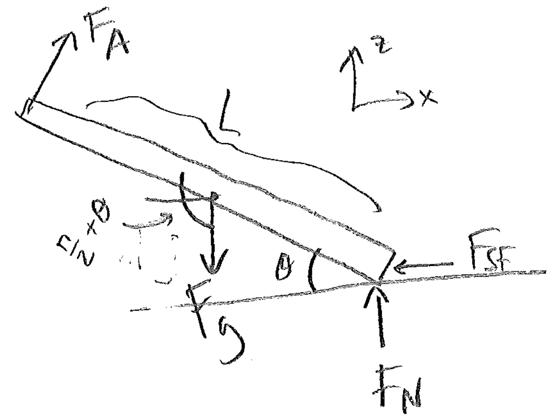
- choose center of board as origin

$$\vec{\tau}_g = 0$$
$$\vec{\tau}_N = \frac{L}{2} F_N (-\hat{y})$$
$$\vec{\tau}_A = \frac{L}{2} F_A (+\hat{y})$$
$$\vec{\tau}_{net} = 0 = \frac{L}{2}(F_A - F_N)\hat{y} = 0 \Rightarrow F_A = F_N = \frac{1}{2} mg$$

- choose point of contact w/ gnd as origin

$$\vec{\tau}_N = 0$$
$$\vec{\tau}_g = \frac{L}{2} mg (-\hat{y})$$
$$\vec{\tau}_A = L F_A (+\hat{y})$$
$$\vec{\tau}_{net} = L\left(F_A - \frac{mg}{2}\right)\hat{y} = 0 \Rightarrow F_A = \frac{1}{2} mg$$

$[N5.T6]_B$ [take poll, but instead of discussing, just do the following calc. ⇒ prep for ladder problem]



3 unknowns: F_A, F_N, F_{SF}

3 eqns $\begin{cases} \tau_{net} = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$

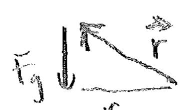
• choose origin at pt of contact:

$\vec{\tau}_N = 0$

$\vec{\tau}_{SF} = 0$

$\vec{\tau}_g = \frac{L}{2} mg \sin\left(\frac{\pi}{2}+\theta\right)(-\hat{y}) = -\frac{L}{2} mg \cos\theta\, \hat{y}$

Alternatively $\tau_g = r_\perp F = \left(\frac{L}{2}\cos\theta\right) mg\, (-\hat{y})$



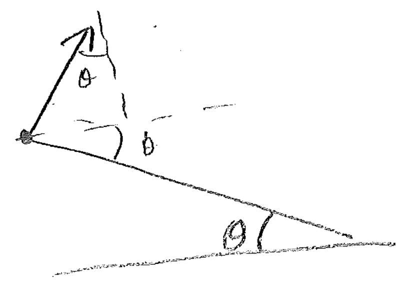
$\vec{\tau}_A = L F_A \hat{y}$

$\vec{\tau}_{net} = 0 = \left(L F_A - \frac{1}{2} L mg \cos\theta\right)\hat{y} \Rightarrow F_A = \frac{1}{2} mg \cos\theta$

[as force decreases]

[Q: Does board slip? At what angle? → do demo]

Board will not slip provided $F_{SF} \leq \mu F_N$



$(F_{net})_x = F_A \sin\theta - F_{SF} = 0$

$F_{SF} = \frac{1}{2} mg \cos\theta \sin\theta$

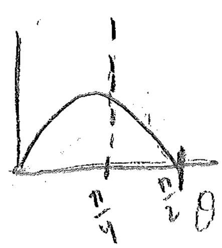
$(F_{net})_z = F_N - mg + F_A \cos\theta$

$F_N = mg - \frac{1}{2} mg \cos^2\theta$

$F_{SF} \leq \mu F_N$

$\frac{1}{2} mg \sin\theta \cos\theta \leq \mu \left(mg - \frac{1}{2} mg \cos^2\theta\right)$

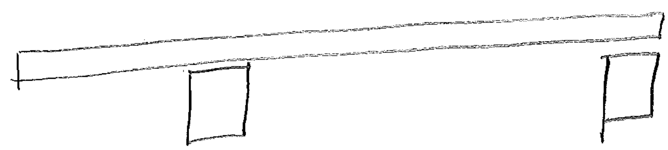
$\frac{\sin\theta \cos\theta}{2 - \cos^2\theta} \leq \mu$



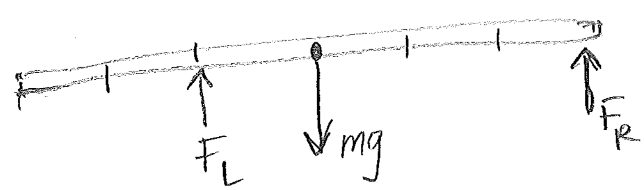
- maximum $\frac{1}{2\sqrt{2}} = 0.354$
- maximized at $35°$

N5-5

[DEMO: board on scales]



Scales register the normal force



$\vec{F}_{net} = 0 \Rightarrow F_R + F_L = mg$ [How much does each scale carry?]

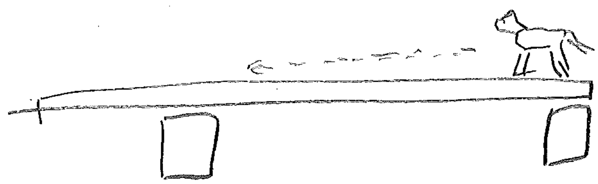
$\vec{\tau}_{net} = 0$ [Ask class to compute either F_L or F_R working in pairs. Can choose your origin.]

[F_R as origin: $4F_L = 3mg \Rightarrow F_L = \frac{3}{4}mg$]

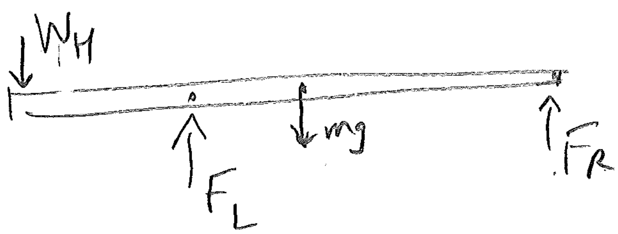
[F_L as origin: $4F_R = mg \Rightarrow F_R = \frac{1}{4}mg$]

[mg as origin: $F_L = 3F_R$]

[Now add Barbie horse: check batteries!]



[How far can she walk & be safe?
Can she walk to the end?]



[What happens to $F_L + F_R$? When will it begin to tip?]

Board will tip if $F_R = 0$

$\vec{\tau}_{net}$ (about F_L): $2W_H = mg$

If $W_H \geq \frac{1}{2} mg$ board will tip!

Net torque due to gravity on a system of masses

$$\vec{\tau} = \sum \vec{r}_i \times m_i\vec{g}$$

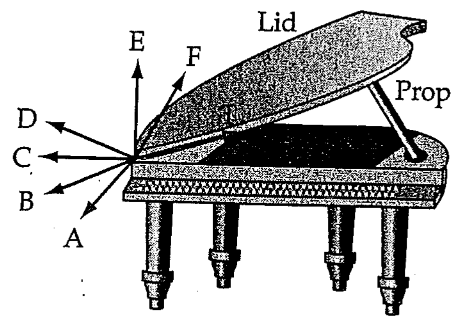
$$= \sum_i m_i\vec{r}_i \times \vec{g}$$

$$= \sum M\vec{R}_{cm} \times \vec{g}$$

$$= \sum \vec{R}_{cm} \times M\vec{g}$$

as if all weight acting at cm.

N5T.3 The lid of a grand piano is propped open as shown. Which arrow most closely approximates the direction of the force that the hinge exerts on the lid?



N5T.4 Imagine that a helicopter's rotor spins clockwise. The helicopter engine must continually exert a torque on the rotor to keep it spinning against the drag that the air exerts on the rotor. Note that a helicopter is usually designed so that its center of mass is directly under the rotor. In order for the helicopter to hover motionless in the air, a small rotor at the helicopter's tail is necessary. As viewed by someone looking at the tail from the helicopter's front, the small rotor must blow air

A. To the left.
B. To the right.
C. Vertically upward.
D. Vertically downward.
E. In some combination of these directions.

N5T.5 A board of mass m lies on the ground. What is the magnitude of the force that you would have to exert to lift *one end* of the board barely off the ground (assuming that the other end still touches the ground)?

A. $2mg$
B. mg
C. The answer depends on the length of the board.
D. $\frac{1}{2}mg$
E. Other (explain).

N5T.6 Imagine that you continue to lift the board described in problem N5T.5. Assume that the force you exert is always perpendicular to the board, and that one end of the board always remains on the ground. What happens to the magnitude of the force you exert on the end as the angle between the board and the ground increases? It

A. Increases
B. Decreases
C. Remains the same