[think board management!]

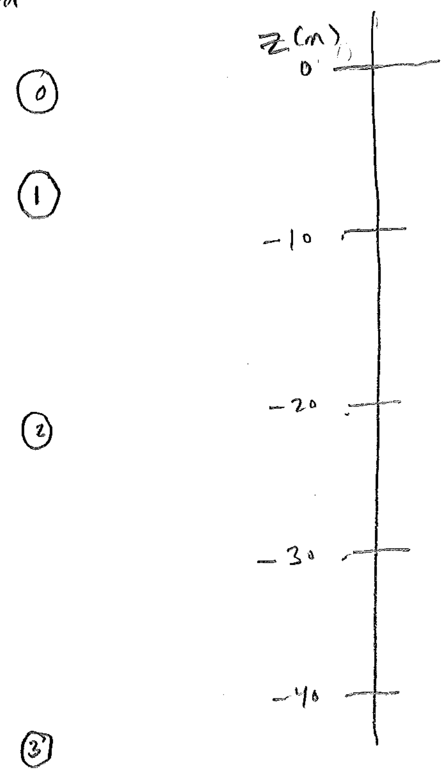
[upper left]

Motion: position, velocity, acceleration

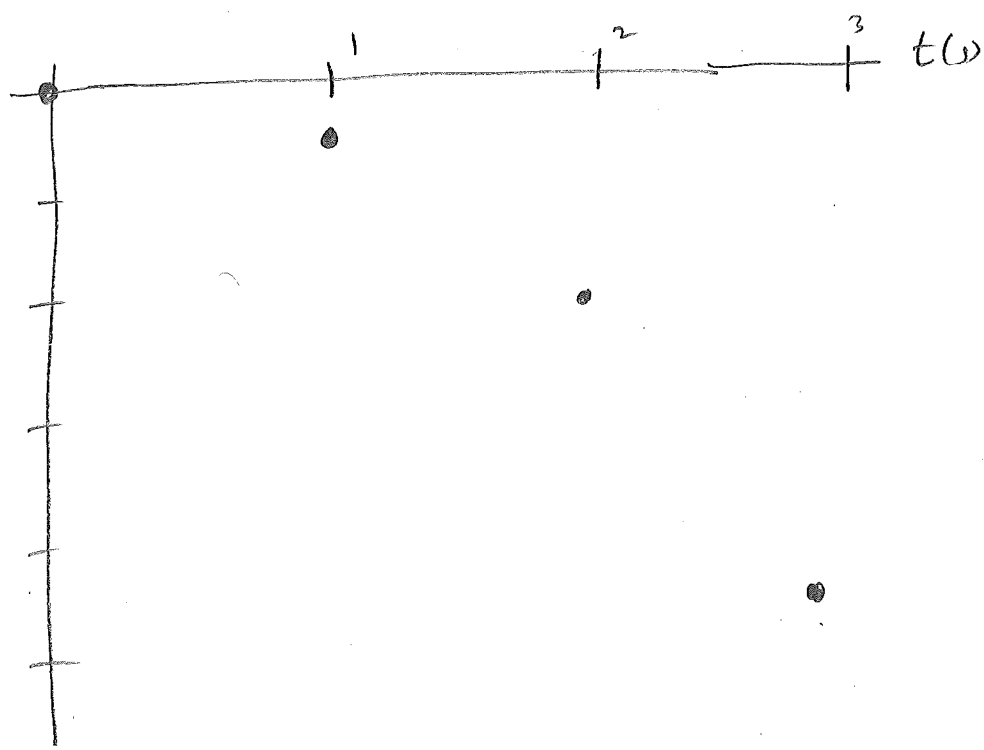
Discretize motion of a falling object [as in making movies in lab]

[Lower board]

motion diagram



[Now add horizontal axis] and data pts

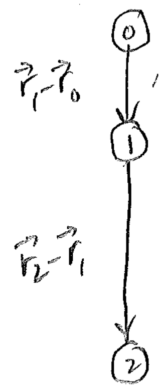


[upper board]

position $\vec{r}_n$

displacement $\Delta\vec{r} = \vec{r}_{n+1} - \vec{r}_n$

[lower board: add displacement vectors]

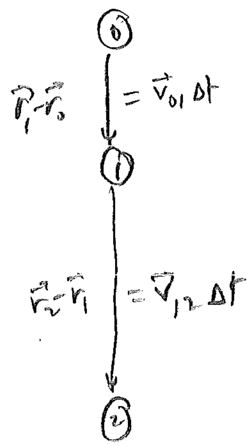


[upper board]

$\vec{V}_{01}$ = average velocity between 0 and 1 $= \dfrac{\vec{r}_1 - \vec{r}_0}{\Delta t}$

$\vec{V}_{01}\,\Delta t = \vec{r}_1 - \vec{r}_2$

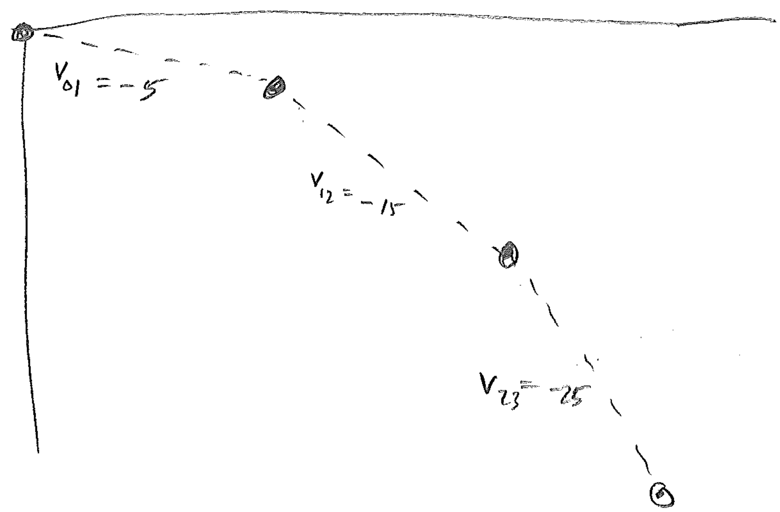
[lower board]



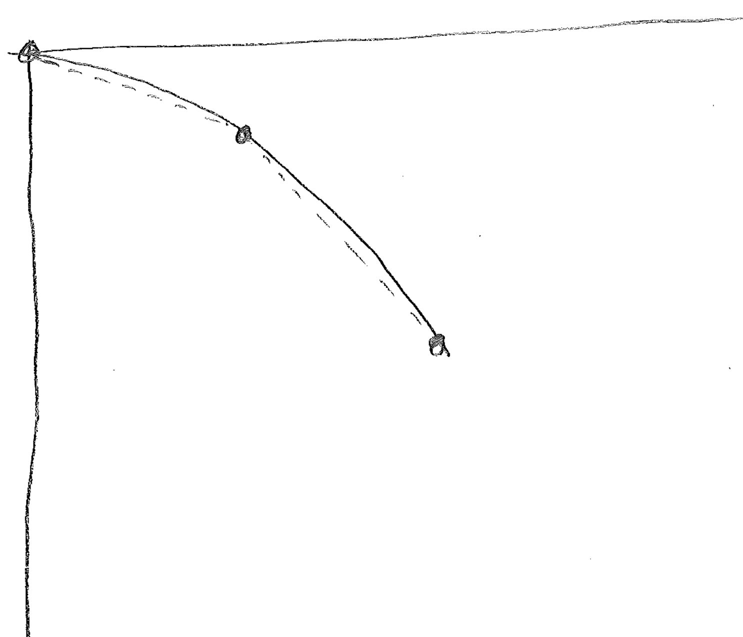
[upper board]

z-component of average velocity $\frac{\Delta z}{\Delta t}$ = slope

[lower board: connect dots w/ dashed lines]



[Now add curve]



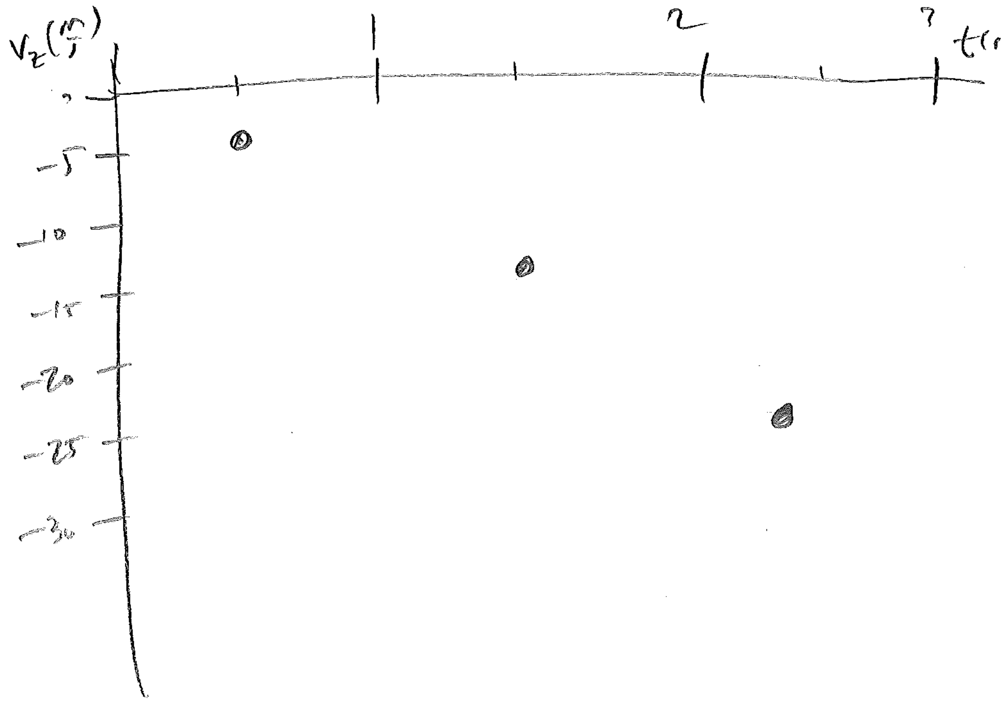
[upper board]

instantaneous velocity $v_z = \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t} = \frac{dz}{dt}$ = slope of line tangent to curve

* average velocity V_{01} is equal to the instantaneous velocity v_z at some instant between 0 and 1 (about halfway)

[NB only true for components!]

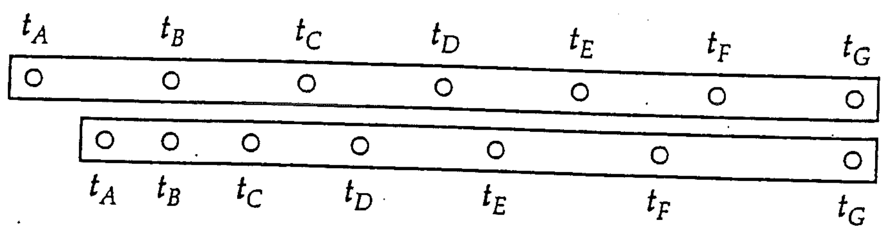
[Now switch boards, & on the board beneath the z vs' t plot]



[N2.T1]$_E$

[N2.T2]$_F$

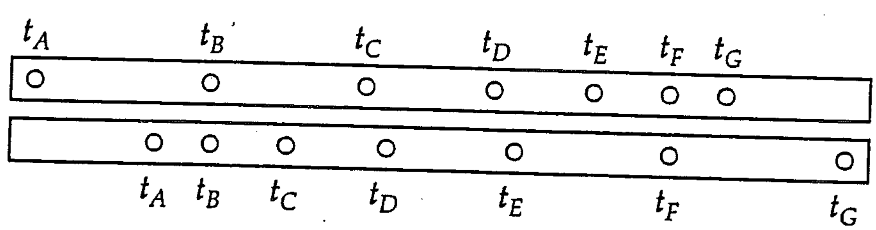
N2T.1 Two marbles are rolling along parallel tracks (which may or may not be inclined). A stroboscopic photograph showing a top view of the positions of the marbles at equally spaced instants of time looks like this:



At what instant(s) of time do the marbles have the same instantaneous velocity?

A. At time t_B.
B. At time t_G.
C. At both t_B and t_G.
D. Some instant between t_C and t_D.
E. Some instant between t_D and t_E.
F. Roughly time t_D.

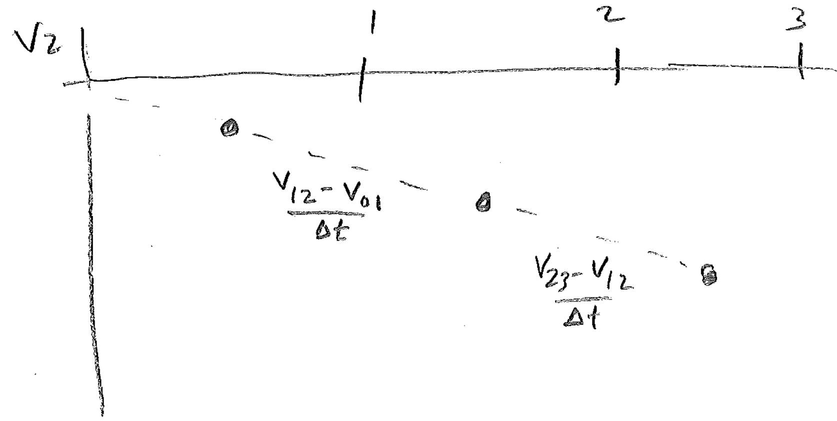
N2T.2 Two marbles are rolling along parallel tracks (which may or may not be inclined). A stroboscopic photograph showing a top view of the positions of the marbles at equally spaced instants of time looks like this:



At what instant(s) of time do the marbles have the same instantaneous velocity?

A. At time t_B.
B. At time t_F.
C. At both t_B and t_F.
D. Some instant between t_C and t_D.
E. Some instant between t_D and t_E.
F. Roughly time t_D.

avg acceleration $\frac{\Delta v_z}{\Delta t}$ = slope



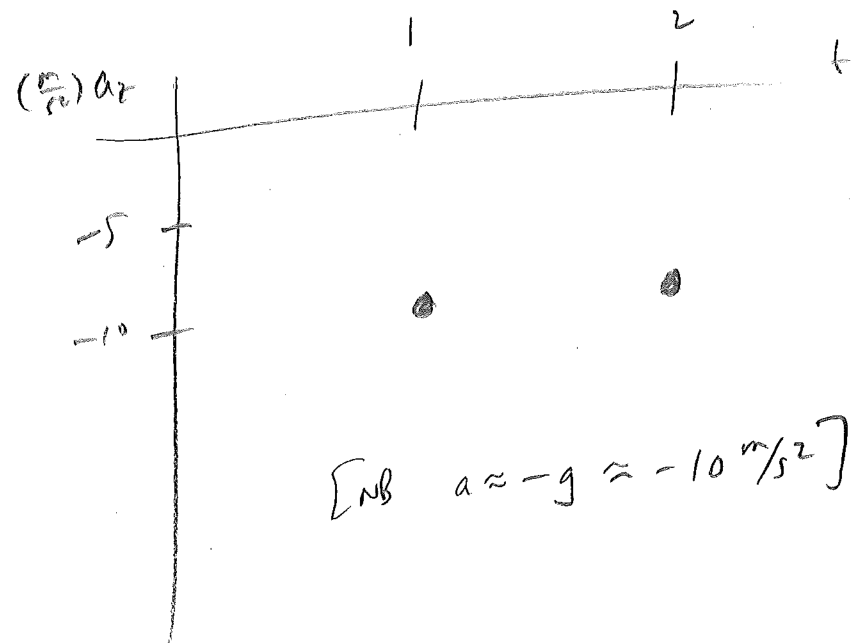
instantaneous acceleration

$a_z = \lim \frac{\Delta v_z}{\Delta t} = \frac{dv_z}{dt}$

= slope of line tangent to graph of $v_z(t)$

* average acceleration $\frac{v_{12} - v_{01}}{\Delta t}$ is equal to instantaneous acceleration at some instant between $\frac{1}{2}$ and $\frac{3}{2}$ (about halfway → 1)

[perhaps on under board] →

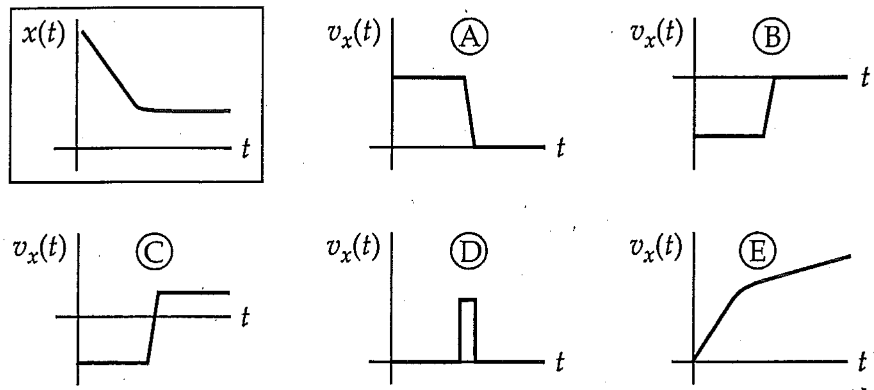


[NB $a \approx -g \approx -10\ ^m/s^2$]

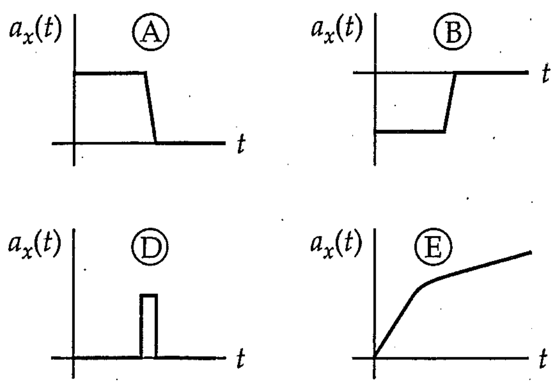
[N3. T9]

[N3. T10]

N3T.9 An object's x-position $x(t)$ is shown in the boxed graph of the following set of graphs. Which of the other graphs in the set most correctly describes its x-velocity?



N3T.10 Which graph best describes the x-acceleration of the object described in problem N3T.9?

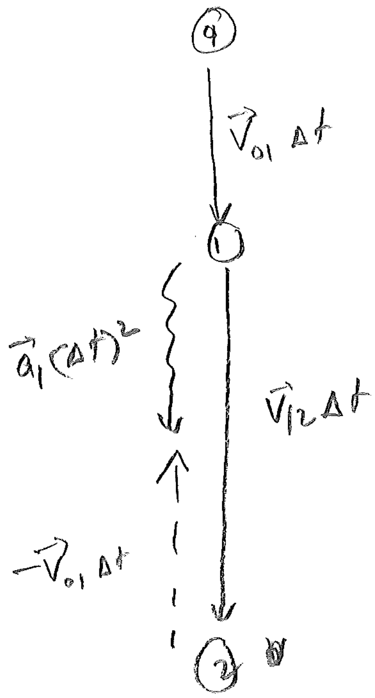


Define $\vec{a}_1$ = (approx) instantaneous acceleration at time 1 $= \frac{\vec{V}_{12} - \vec{V}_{01}}{\Delta t}$

$$\vec{a}_1 \Delta t = \vec{V}_{12} - \vec{V}_{01}$$

then $\vec{a}_1 (\Delta t)^2 = \vec{V}_{12}\,\Delta t - \vec{V}_{01}(\Delta t)$

difference of displacement vectors

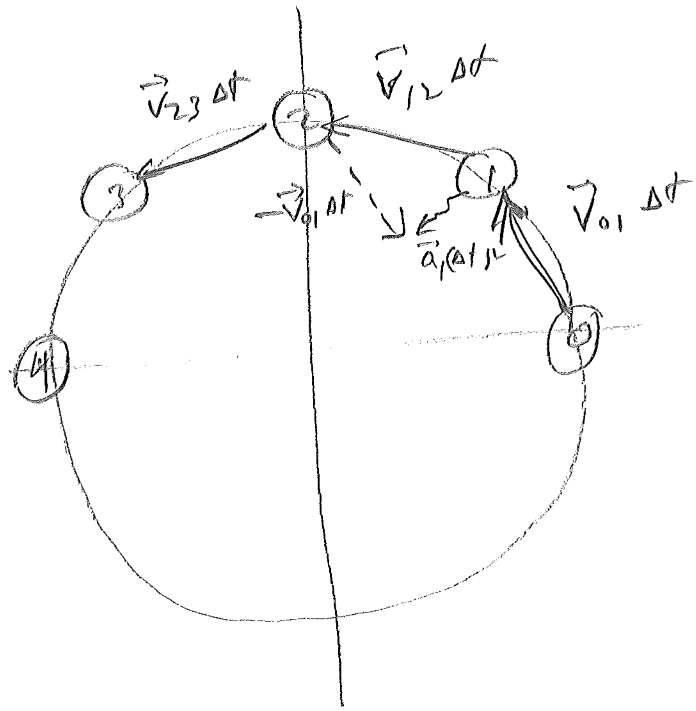


[N2-T8]

uniform circular motion (constant speed) [N2.T4] ← 50-50!
[N2.T5]

$$\vec{v}_{01}\Delta t = \Delta\vec{r}$$

$$\vec{a}_1(\Delta t)^2 = \vec{v}_{12}\Delta t - \vec{v}_{01}\Delta t$$



UCM ⇒ acceleration is always toward center of circle

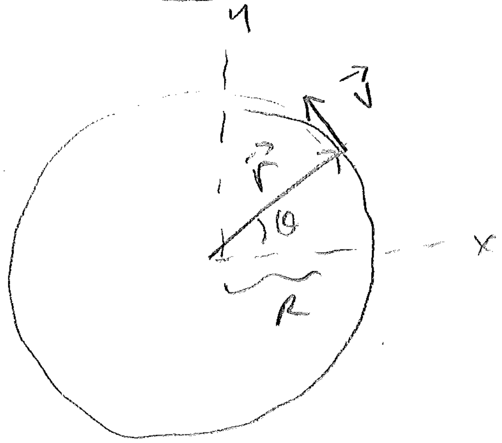
[N2.T11]

[Did this at beginning of N3]

$v = \omega R$ N 2-8

Uniform circular motion ⇒ const speed

⇒ θ is changing linearly w/ time



$$\vec{r} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \begin{pmatrix} -R\sin\theta \frac{d\theta}{dt} \\ R\cos\theta \frac{d\theta}{dt} \\ 0 \end{pmatrix} = R\frac{d\theta}{dt}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$

$$v = |\vec{v}| = R\left|\frac{d\theta}{dt}\right| = R\omega = \text{const}$$

$$\vec{v} = R\frac{d\theta}{dt}\begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix}$$ ← (ω = $\frac{d\theta}{dt}$ = const so only take $\frac{d}{dt}$ of $\sin$/$\cos$)

$$\vec{a} = \frac{d\vec{v}}{dt} = R\frac{d\theta}{dt}\begin{pmatrix} -\cos\theta \frac{d\theta}{dt} \\ -\sin\theta \frac{d\theta}{dt} \\ 0 \end{pmatrix} = -R\left(\frac{d\theta}{dt}\right)^2\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$= -\omega^2\begin{pmatrix} R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix} = -\omega^2\vec{r}$$

acceleration points toward center

$$|a| = \omega^2 R = \left(\frac{v}{R}\right)^2 R = \frac{v^2}{R}$$ for UCM

N2T.3 An object travels halfway around a circle at a constant instantaneous speed v. What is the magnitude of its average velocity during this time interval?

A. v
B. $1.41v$
C. $1.57v$
D. $v/1.41$
E. $v/1.57$
F. Some other multiple of v (specify).
T. We do not have enough information to answer.

[already did a problem on this: C03 – toy train]

N2T.4 An object can have a constant speed and still be accelerating, true (T) or false (F)?

N2T.5 An object's acceleration vector always points in the direction that it is moving, T or F?

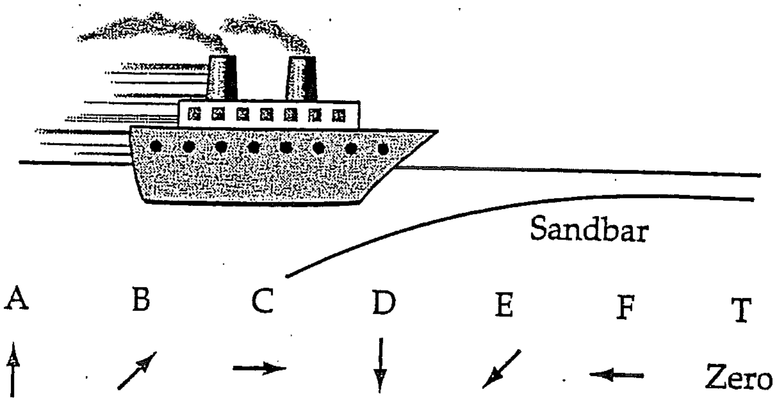
N2T.6 An object can have zero velocity (at an instant of time) and still be accelerating, T or F?

N2T.7 The x-velocity of an object can be positive at the same time that its x-acceleration is negative, T or F?

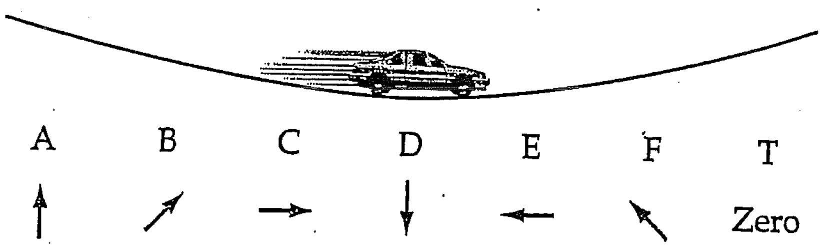
N2T.8 An object falling vertically at a speed of 20 m/s lands in a snowbank and comes to rest 0.5 s later. What is the object's average acceleration during this interval?

A. 10 m/s^2 up
B. 10 m/s^2 down
C. 40 m/s^2 up
D. 40 m/s^2 down
E. 5 m/s^2 up
F. 80 m/s^2 down
T. Other (specify)

N2T.9 A boat hits a sandbar and slides some distance before coming to rest. Which of the arrows shown below best represents the direction of the boat's acceleration as it is sliding? (*Hint:* Draw a motion diagram.)



N2T.10 A car moving at a constant speed travels past a valley in the road, as shown. Which of the arrows shown most closely approximates the direction of the car's acceleration at the instant that it is at the position shown? (*Hint:* Draw a motion diagram.)



N2T.11 A bike (shown in a top view in the diagram) travels around a curve with its brakes on, so that it is constantly slowing down. Which of the arrows shown below most closely approximates the direction of its acceleration at the instant that it is at the position shown? (*Hint:* Draw a motion diagram.)