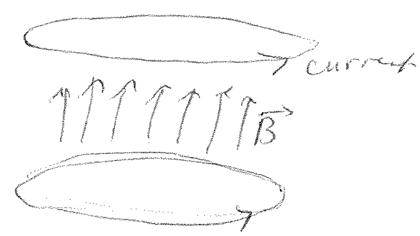
## Magnetic field $\vec{B}$

• Caused by moving charges (currents)
"electromagnet"

Helmholtz coils





unit of magnetic field: 1 Tesla = $\frac{N \cdot s}{C \cdot m}$

Coulomb = unit of charge

(earth's field ~ $10^{-4}$ T)

strong magnets ~ 10 T

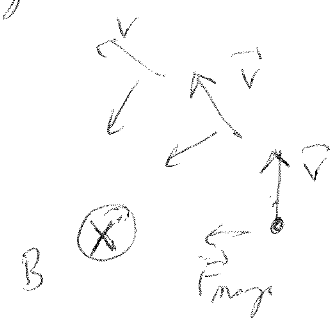
E7-2
2013

Lagrange

$\vec{B}$ exerts a force (Lorentz force) on a moving charge (ion)

$$\vec{F}_{mag} = q\vec{v}\times\vec{B}$$

Causes a charge to move in a circle



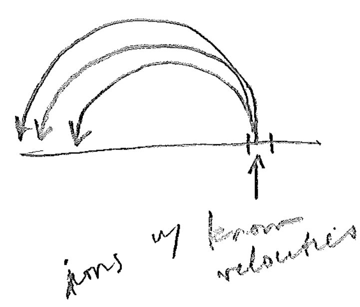

$$\cancel{F_{mag}\ a\ v^2} \qquad F_{mag} = m\left(\frac{v^2}{R}\right)$$

$$qvB = \frac{mv^2}{R}$$

$$\cancel{mv = qBR}$$

$$R = \frac{mv}{qB}$$

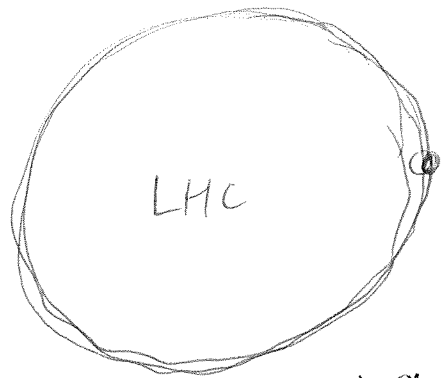
Mass spectrometer...



E7-3

2017

particle accelerator




27 km

$2\pi R = 27{,}000\ \cancel{km}\ m$

$R = 4300\ m$

charge of proton    superconducting

$$p = qBR = \underbrace{(1.6\times10^{-19}\,C)}_{\text{charge of proton}}\underbrace{(8.3\ T)}_{\text{superconducting}}(4300\ m)$$

$\frac{N\cdot s}{C\cdot m}$    $kg\ \frac{m}{s}$

$= 5.7E{-}15\ N\cdot s$

proton

$E = pc = 1.7E{-}6\ J = 1E13\ eV = 10\ TeV$

$\gamma = 10{,}000,\quad \frac{v}{c} = 1 - \frac{1}{2\gamma^2}$

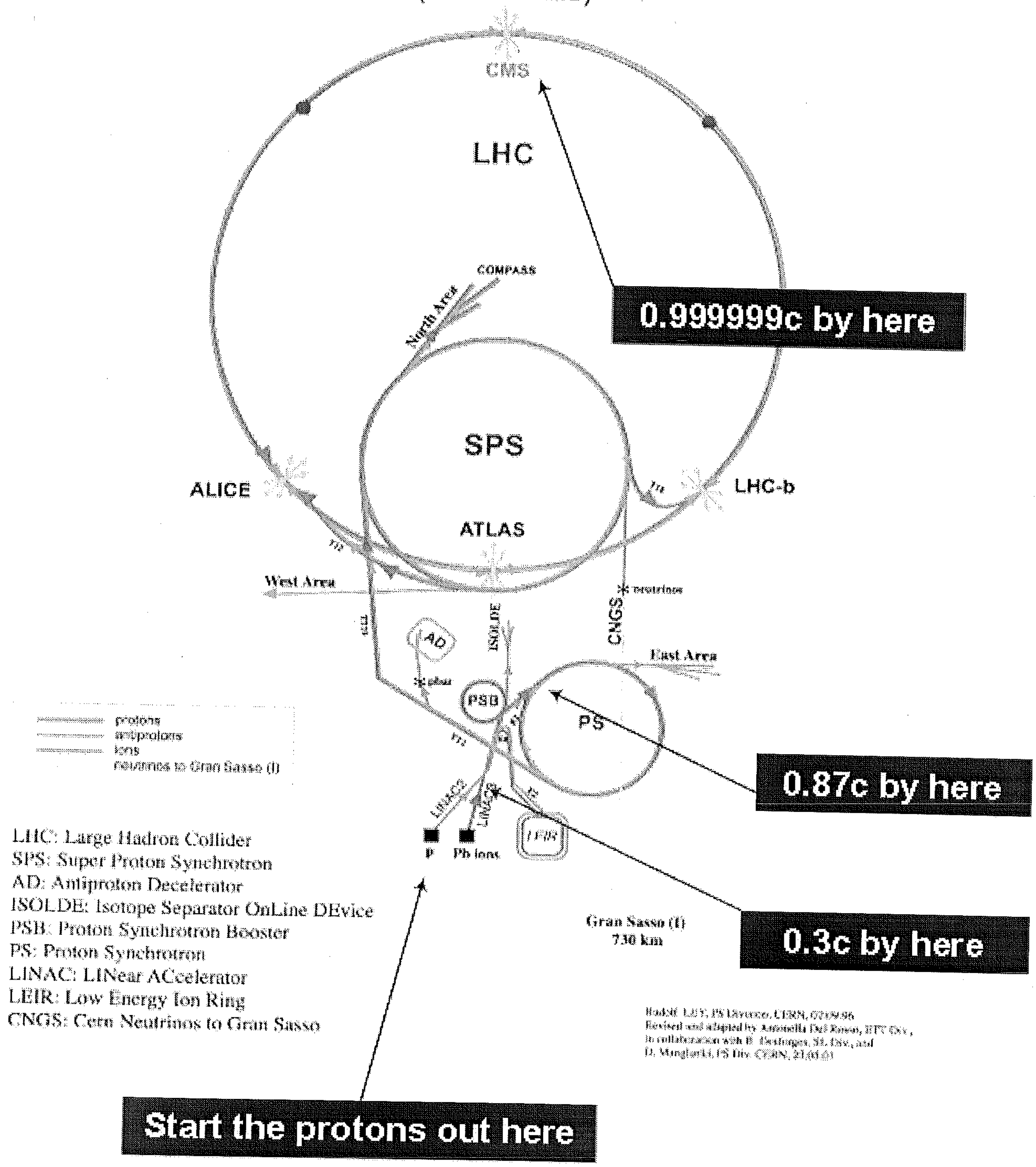
$V = 0.99999999$    $v/c\ 10^{-8}\ c$

~~3~~ (3 m/s less)

300,000,000

299792458 m/s

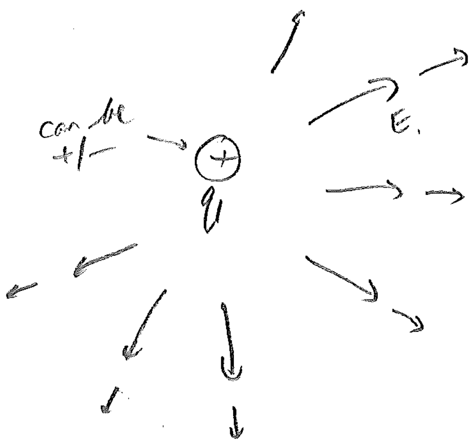
# CERN Accelerators
(not to scale)



ET-1

# Electric & magnetic fields

An electric charge $q_1$ creates an electric field $\vec{E}$ around it



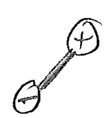

- field fills space, has a direction (& strength)
- how tell it is there?

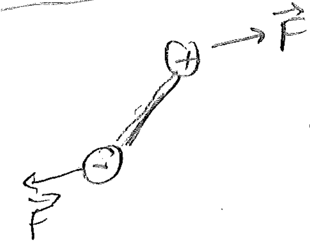
An electric field $\vec{E}$ exerts a force on an electric charge $q_2$

$\vec{F} = q_2 \vec{E}$ : force depends on sign of charge

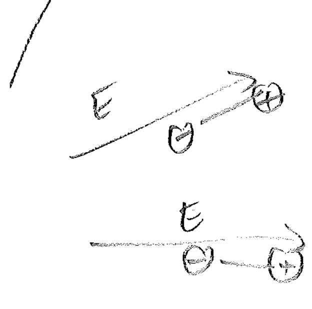

⇒ like charges repel, opposites attract (mediated by electric field)

Electric Dipole : pair of equal & opposite charges





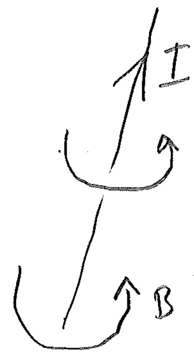

net force is zero but net torque is not.
dipole tends to align w/ field
↓
determines the direction



old notes E7-2

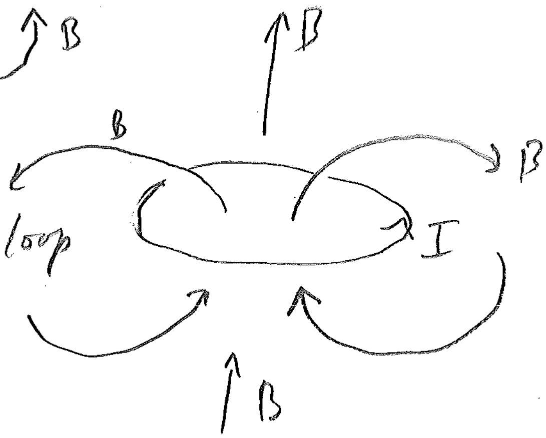
# Magnetic field $\vec{B}$

a moving charge (= current I) creates a magnetic field $\vec{B}$ around it

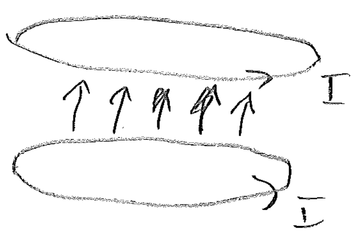



$\vec{B}$ "curls" around I acc. r.h. rule

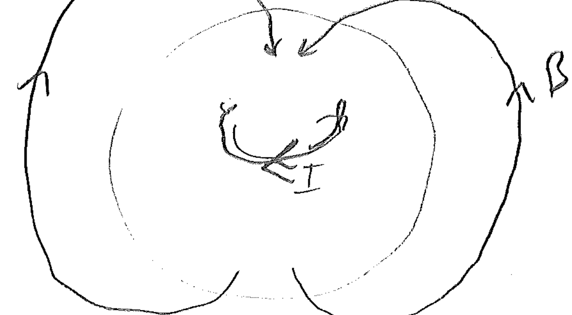
current loop




"Helmholtz" coil creates a fairly uniform B field in region between the current loops




currents within Earth's core presumably responsible for its mag field




units of mag field = Tesla = $\frac{\text{Volt}\cdot\text{s}}{\text{m}^2}$ $\left(= \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)$

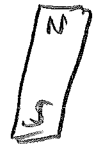
also gauss $\quad$ 1 T = 10,000 gauss

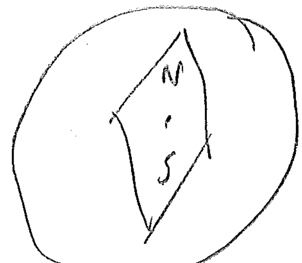
earth's field $\sim \frac{1}{2}$ gauss

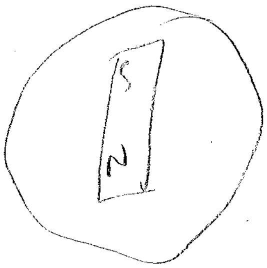
strongest superconducting magnets $\sim$ 20 T old notes E7-3

How detect magnetic field? w/ magnet!

= magnetic dipole





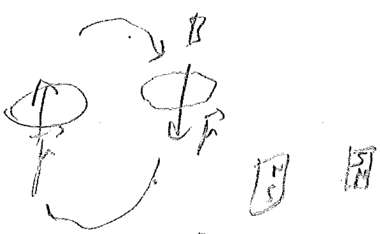


needle lines up w/ magnetic field

But "poles" are really fictions ⇒ can't split it up.

Really: a magnetic field $\vec{B}$ exerts a force on a moving charge ~~(current)~~

$$\boxed{\vec{F} = q\vec{v}\times\vec{B}}$$



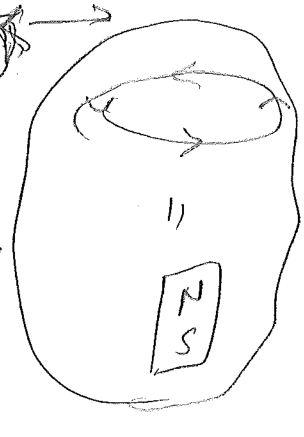
$E = -\vec{\mu}\cdot\vec{B}$

$\vec{F} \parallel \vec{B}$

~~current loop lines up w/ $\vec{B}$ field~~

~~magnets~~

~~(ferromagnets ⇒ microscopic currents in atoms)~~



~~∴~~ a current loop tries to align w/ $\vec{B}$ field

moving ~~charge~~ chg → moves in an arc   $\frac{mv^2}{R} = qvB$

$R = \frac{mv}{qB} = \frac{p}{qB}$

→ mass spectrum

→ measure $p$ of ptcl

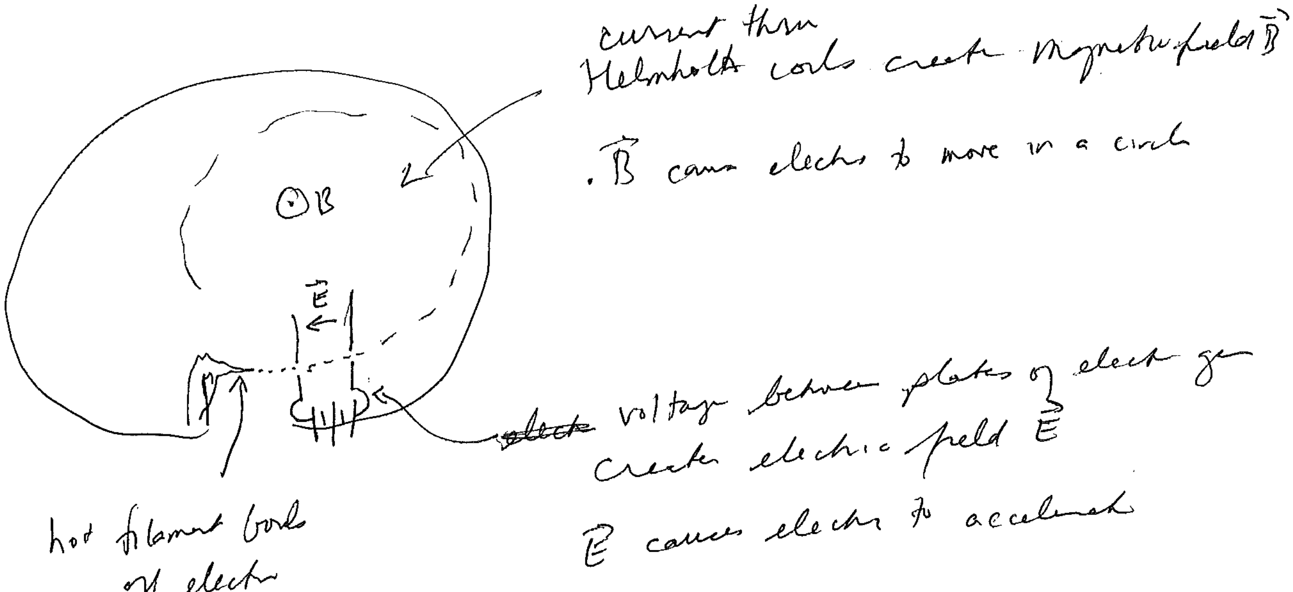
→ LHC ⇒

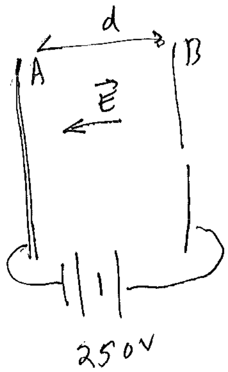
F '2001

old notes

Electrons have negative chg $q = -e$ and mass $m$

Purpose of lab is to determine ratio $\frac{e}{m}$




voltage difference $V$ between plates creates electric field of magnitude $E = \frac{V}{d}$

Recall: potential energy associated w/ grav. interact near earth is

$$U(z) = mgz$$

$$\Rightarrow F_z = -\frac{dV}{dz} = -mg$$

Similarly, pot. en. assoc w/ const electric field $E$ btw plates is

$$U(x) = qEx$$

$$F_x = -\frac{dU}{dx} = -qE$$

generally $\vec{F} = q\vec{E}$ where $\vec{E} = \begin{bmatrix} -E \\ 0 \\ 0 \end{bmatrix}$

F'2001

Consider elec starting at rest at A & ending at B

$$U_A + K_A = U_B + K_B$$

$$qE(0) + 0 = \underbrace{qEd}_{-eEd} + \tfrac{1}{2}mv_f^2$$

$$\tfrac{1}{2}mv_f^2 = eEd$$

$$v_f = \sqrt{\frac{2eEd}{m}}$$

$Ed$ = voltage between ~~plate~~ plates $\Rightarrow$ $\boxed{v_f = \sqrt{\frac{2eV}{m}}}$

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Helmholtz coils ~~are~~ create a magnetic field $\vec{B}$

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Force exerted by magnetic field is $\vec{F} = q\vec{v} \times \vec{B} = -e\,\vec{v} \times \vec{B}$

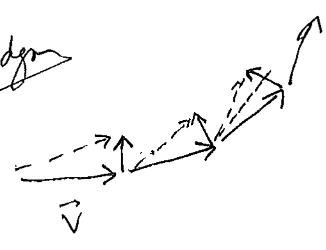
If there is only force acting, then

$B \odot \;\; \longrightarrow \vec{v}$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = -\frac{e}{m}\,\vec{v} \times \vec{B}$$

$\odot B \;\; \longrightarrow \vec{v} \;\; \Rightarrow \uparrow \vec{a}$

trajectory dgm



Since $\vec{a}$ is $\perp$ to $\vec{v}$, elec moves in a circle! $\Rightarrow a = \frac{v^2}{r}$

What is radius of circle? $a = \frac{v^2}{r} = \frac{F_{net}}{m} = \frac{evB}{m}\underbrace{\sin 90°}_{1}$

$$\boxed{r = \frac{vm}{eB}}$$

$$r^2 = \frac{v^2m^2}{e^2B^2} = \frac{2mV}{eB^2} \Rightarrow \frac{e}{m} = \frac{2V}{B^2r^2}$$

$\boxed{E7-1}$

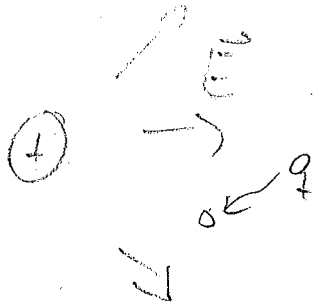
- electric charge creates an electric field
- field fills space, has a direction → represent w/ a sensing vectors
- how tell it's there? electric field exerts force on an electric charge

# Electric field:

force on charge:

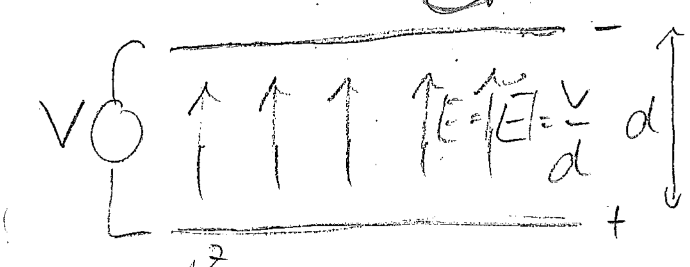
$$\vec{F}_e = q\vec{E}$$

⇒ ~~depends on~~ depends on sign of charge → things like repel, opp attract



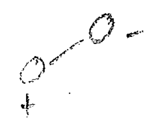

Could find E by measuring force on test charge q

or (skip)

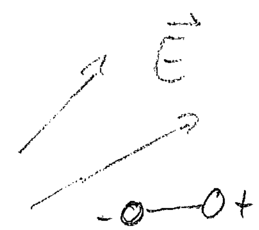



$$\Rightarrow \vec{E} = \begin{pmatrix} 0 \\ 0 \\ V/d \end{pmatrix}$$

⇒ Alternative: build small electric dipoles:




will orient themselves in E field:




What is energy that charge q gains or looses as it travels between two plates?

$$dK = \vec{F}_e \cdot d\vec{r} = q\vec{E} \cdot d\vec{r} = q\, E_z \underbrace{dz}_{d} = q\,\frac{V}{d}\,d = qV$$

do this first

# E7-3

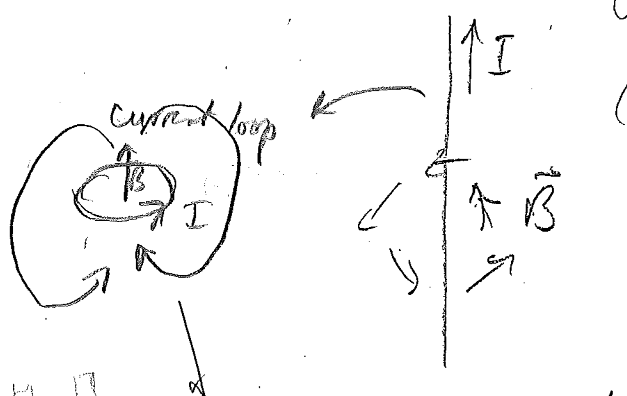
Superficially, magnetic forces seem very different from electric forces, but it turns out that they have same origin:

- $\vec{E}$-fields: ∘ created by charges
  ∘ act on charges
- $\vec{B}$-fields: ∘ created by moving charges ("currents")
  ∘ act on moving charges

<u>Creating $\vec{B}$-fields</u>

⇒ • can create $\vec{B}$-field by currents:

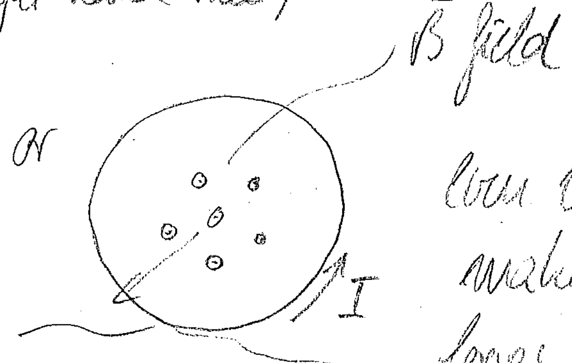
(right hand rule)




earth creates a field due to currents in core ⇒ ~ $\frac{1}{2}$ gauss

(1 Tesla = 10,000 gauss $= \frac{N \cdot s}{C \cdot m} = \frac{Volt \cdot s}{m^2}$)

or




even better: make many loops

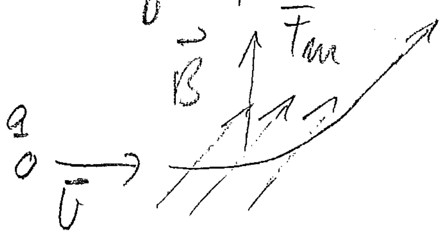
⇒ "Helmholtz coil"

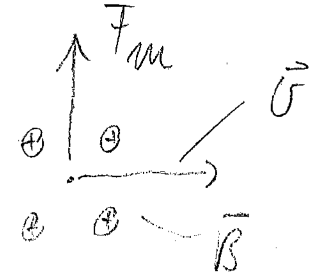
- In ferromagnets: microscopic currents in individual atoms, that all line up and add up

# E7-5

⇒ magnetic forces can't do work
i.e. can't change speed, but <u>can</u> change direction of velocity

Consider pos. charge $q$ moving perpendicularly to $\vec{B}$-field: $\vec{v} \perp \vec{B}$

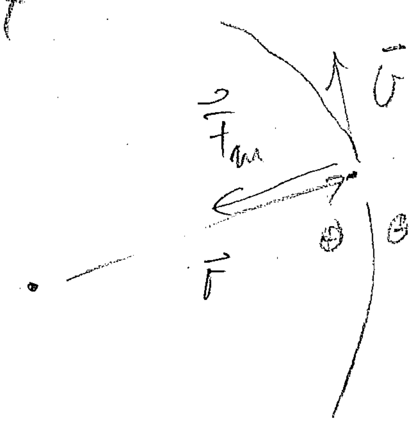



If charge remains in $\vec{B}$-field:

$\vec{F}$ always $\perp$ on $\vec{v}$, $v = \text{const}$

⇒ circular motion




$$m\vec{a} = -\frac{mv^2}{r}\hat{r} = \vec{F}_m$$
$$= -v|q|B\,\hat{r}$$

So $\dfrac{mv^2}{r} = v|q|B$

[neg. charge circles other way around...]

$\boxed{\text{E7-7}}$

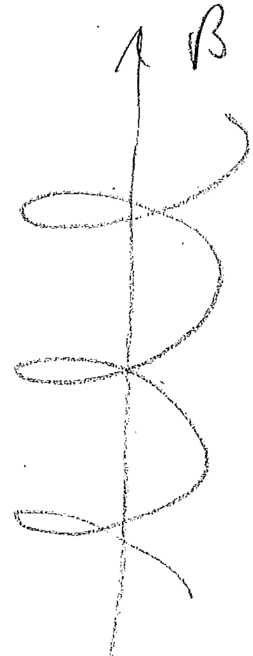
$\Rightarrow$ • motion in $\hat{z}$ direction unaffected:

$$v_z = v\cos\theta$$

• circle in x-z plane with radius

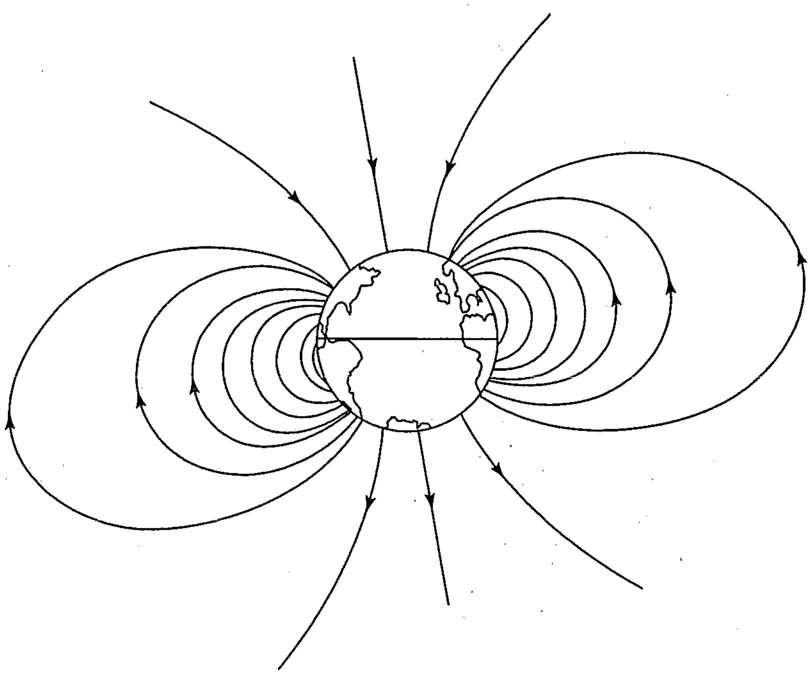
$$R = \frac{mv}{|q|B}\sin\theta$$

$\Rightarrow$ helical motion along $\vec{B}$-field:




($\Rightarrow$ van Allen belts)

Cosmic rays (atomic nuclei stripped bare of their electrons) would continuously bombard Earth's surface if most of them were not deflected by Earth's magnetic field. Given that Earth is, to an excellent approximation, a magnetic dipole, the intensity of cosmic rays bombarding its surface is greatest at the



1. poles.
2. mid-latitudes.
3. equator.