How do interactions change an object's $\vec{p}$, K, and $\vec{L}$?

$d\vec{p} = \sum_A [d\vec{p}]_A$

$[d\vec{p}]_A$ = impulse, or momentum transfer by force A

Rate of momentum transfer $= \frac{[d\vec{p}]_A}{dt} = \vec{F}_A$

Rate of change of momentum $\frac{d\vec{p}}{dt} = \sum \vec{F}_A = \vec{F}_{net}$

If (external) net force vanishes, then momentum of the system is conserved

$dK = \sum [dK]_A$

$[dK]_A$ = k-work, or kinetic energy transfer by force A $= \vec{v} \cdot [d\vec{p}]_A$

Rate of kinetic energy transfer $\frac{[dK]_A}{dt} = \vec{v} \cdot \vec{F}_A$ = power delivered by force A

Rate of change of kinetic energy $\frac{dK}{dt} = \sum \vec{v} \cdot \vec{F}_A = \vec{v} \cdot \vec{F}_{net}$ = net power

If no work is done on system, kinetic energy is conserved

$d\vec{L} = \sum [d\vec{L}]_A$

$[d\vec{L}]_A$ = twirl, or angular momentum transfer by force A $= \vec{r} \times [d\vec{p}]_A$

Rate of angular momentum transfer $\frac{[d\vec{L}]_A}{dt}$ = torque $\vec{\tau}_A$ exerted by force A

$\vec{\tau}_A = \vec{r} \times \vec{F}_A$ where $\vec{r}$ = vector from origin to the point where the force is applied

Rate of change of angular momentum $\frac{d\vec{L}}{dt} = \sum \vec{\tau}_A = \vec{\tau}_{net}$

If (external) net torque vanishes, then angular momentum of the system is conserved

No net torque $\Rightarrow$ $\vec{L}^{sys}$ does not change in time

<u>example</u>: stationary rotating object

$$\vec{L}^{rot} = I\vec{\omega}$$

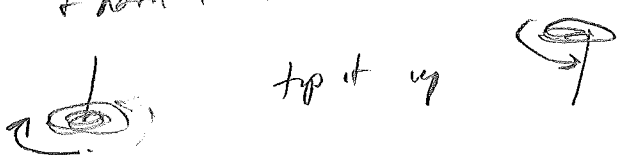
$$I_{init}\,\vec{\omega}_{init} = I_{final}\,\vec{\omega}_{final}$$

[DEMO: squeezatron] [$\vec{F}$ exerted on balls is toward axis $\Rightarrow \vec{\tau} = \vec{r}\times\vec{F} = 0$]

If $I \downarrow$, then $\vec{\omega} \uparrow$

[DEMO: Youtube: "world record skating spin"]

[DEMO: stand on platform
have 2 students spin up the loaded bicycle wheel
& hand it to me



$$\vec{L}^{init}_{wheel} = \vec{L}^{final}_{wheel} + \vec{L}_{me}$$

$$\downarrow \; = \; \uparrow \; + \; \downarrow$$

C 14-3

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net} \Rightarrow d\vec{L} = \vec{\tau}_{net}\, dt$$

$$\vec{L}_f - \vec{L}_i = \vec{\tau}_{net}\, dt$$

$$\vec{L}_f = \vec{L}_i + \vec{\tau}_{net}\, dt$$

[C13.T7]

Ask what direction $\vec{L}$ points & whether it is ↑ or ↓

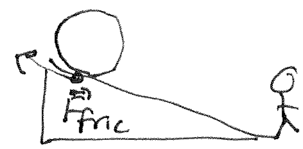
Then discuss how $\vec{\tau}_{grav}$ changes $\vec{L}$

[C13.T8]

[C13.T10]

[C13.T11]

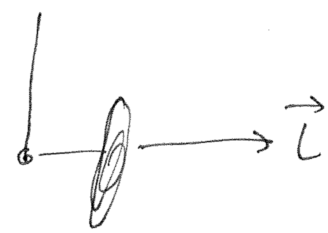
[C13.T9] Draw picture first because otherwise too hard.



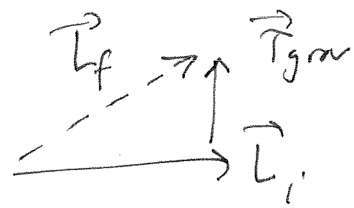
· Ask what direction $\vec{L}$ is in.

· explain how $\vec{\tau}_{fric}$ causes barrel to roll faster

[Demo: gyroscope] [set it in frame,
spin it up,
lift it w/ the string]



From above



[Demo: James Bond briefcase built by Q]

BACKGROUND

Angular momentum of systems

$$\vec{L}^{sys} = \sum_j \vec{r}_j \times \vec{p}_j$$

$$d\vec{L}^{sys} = \sum_j \left(\underbrace{d\vec{r}_j \times \vec{p}_j}_{0 \text{ since } d\vec{r}_j = \vec{v}_j dt} + \vec{r}_j \times d\vec{p}_j \right)$$

$$= \sum_j \sum_A \vec{r}_j \times [d\vec{p}_j]_A$$

$$= \sum_j \sum_{k \neq j} \vec{r}_j \times [d\vec{p}_{j(k)}] + \sum_j \sum_{ext} \vec{r}_j \times [d\vec{p}_j]_{ext}$$

$$= \sum_{j<k} \left(\vec{r}_j \times [d\vec{p}_{j(k)}] + \vec{r}_k \times [d\vec{p}_{k(j)}] \right) + \sum_j \sum_{ext} \vec{r}_j \times \vec{F}_{j,ext}\, dt$$

$$= \sum_{j<k} \underbrace{(\vec{r}_j - \vec{r}_k) \times [d\vec{p}_{j(k)}]}_{0 \text{ because } \vec{F}_{j(k)} \parallel (\vec{r}_j - \vec{r}_k)} + \underbrace{\sum_j \sum_{ext} \vec{\tau}_{j,ext}\, dt}_{\vec{\tau}^{sys}_{ext}}$$

$$\frac{d\vec{L}^{sys}}{dt} = \vec{\tau}^{sys}_{ext}$$

C14T.1 When I was a teenager, I saw a "Candid Camera" sequence in which they filmed a person struggling with a suitcase that (unknown to that person) contained a large and rapidly spinning gyroscope. Imagine that you are carrying a suitcase containing a gyroscope whose angular velocity is horizontal and points directly away from your legs as you carry it with your right hand. Imagine that you make a 90° turn to the left as you walk. What does the suitcase do?

A. It turns to the left along with you.
B. It turns to the right as you try to turn left.
C. Its bottom edge flips up away from your legs as you make the turn.
D. Its bottom edge flips in to tangle with your legs.
E. Its front end dips toward the ground.
F. Its back end dips toward the ground.
T. It attempts to maintain its original orientation.

C14T.2 Experienced players can throw a Frisbee using forehand motion that causes the Frisbee to rotate counterclockwise (when thrown by a right-handed person). If a Frisbee thrown in this way is to skip off the ground, which edge has to hit the ground first?

A. The back edge.
B. The front edge.
C. The left edge.
D. The right edge.
E. One can't make the Frisbee thrown in this way skip.

C14T.3 Which of the following changes in a top's design will cause the largest decrease in the top's precession rate?

A. Increasing its mass by 10%.
B. Decreasing its height by 10%.
C. Increasing its radius by 10%.
D. Increasing its angular speed by 10%.
E. Changes B, C, and D have the same effect.
F. None of these design changes modify the precession rate.

C14T.4 A person is sitting at rest on a stool that is free to rotate about a vertical axis while holding in one hand a bicycle wheel that is rapidly spinning counterclockwise when viewed from above. The person then stops the wheel with the other hand. What happens to the person as a result?

A. The person must rotate counterclockwise.
B. The person must rotate clockwise.
C. The person will rotate in a direction that depends on which hand does the stopping.
D. Nothing; the wheel's angular momentum is carried away by external interactions.
E. Nothing; the wheel's angular momentum is simply dissipated by the friction interaction.

C14T.5 An astronaut floating at rest in space throws a ball to another astronaut, using a side-arm motion that ends up releasing the ball well to the astronaut's right as he or she faces the direction of the ball's forward motion. After throwing the ball, the astronaut (when viewed from above)

A. Remains completely at rest.
B. Rotates clockwise, but his or her center of mass remains at rest.
C. Rotates counterclockwise, but her or his center of mass remains at rest.
D. Rotates clockwise and his or her center of mass moves opposite to the ball's motion.
E. Rotates counterclockwise and her or his center of mass moves opposite to the ball's motion.
F. Other (specify).

C14T.6 If global warming proceeds during the next century as anticipated, it is possible that the polar ice caps will melt, substantially raising sea levels around the world (and flooding coastal cities such as New York). Would this shorten, lengthen, or have strictly no effect on the duration of the day?

A. Lengthen the day slightly.
B. Shorten it slightly.
C. Have strictly *no* effect on the length of the day.