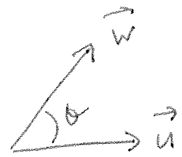
Multiplication of vectors



Dot Product = scalar

$\vec{u}\cdot\vec{w} = uw\cos\theta$

Cross product = vector

magnitude $|\vec{u}\times\vec{w}| = uw\sin\theta$

direction is $\perp$ to both $\vec{u}$ and $\vec{w}$

direction given by right hand rule

push $\vec{u}$ into $\vec{w}$, thumb gives direction

e.g. $\hat{x}\times\hat{y} = \hat{z}$

$\hat{y}\times\hat{z} = \hat{x}$

$\hat{z}\times\hat{x} = \hat{y}$

$\vec{u}\cdot\vec{u} = u^2$ $\qquad$ $\vec{u}\times\vec{u} = 0$

$\vec{u}\cdot\vec{w} = \vec{w}\cdot\vec{u}$ (commutative) $\qquad$ $\vec{u}\times\vec{w} = -\vec{w}\times\vec{u}$ (anti commutative)

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = u_x w_x + u_y w_y + u_z w_z$$

$$\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \times \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} u_y w_z - u_z w_y \\ u_z w_x - u_x w_z \\ u_x w_y - u_y w_x \end{pmatrix}$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}$$

$\vec{L}$ = angular momentum of a particle with respect to origin O

$= \vec{r} \times \vec{p}$

$\vec{p}$ = momentum of the particle ($= m\vec{v}$)

$\vec{r}$ = position of particle relative to O

ie vector from O to the particle

[if I drop a piece of chalk, does it have momentum? yes
Just before it hits the floor

Does it have angular momentum?

[yes / no / maybe]

What is the direction of $\vec{L}$

[up / down / north / south / east / west / zero]

Depends on choice of O. [like potential energy depends on ~~ref~~ ref. pos

[if origin is me ⇒ east
if origin is you ⇒ west, or southwest or northwest
if origin is pt from which dropped ⇒ zero

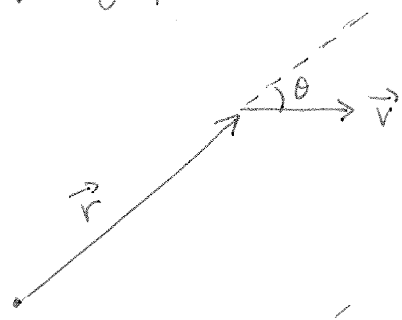
[C13.T2]

[C13.T3]

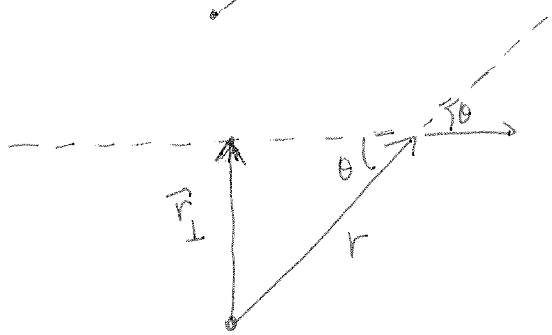
Airplane flying overhead [west to east] at const height + velocity.

Direction of $\vec{L}$?

[Q: Does $\vec{L}$ of plane change?]



$\vec{L} = rmv\sin\theta$ ⊗

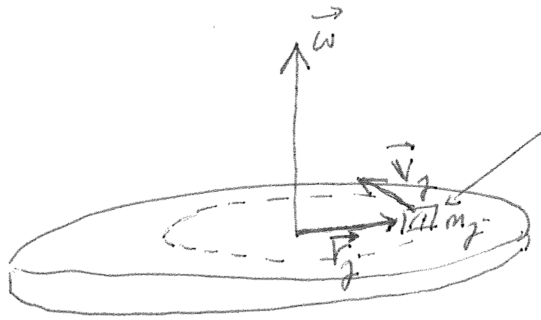


$r_\perp = r\sin\theta$ = distance of closest approach

$|L| = r_\perp mv$

[doesn't change]

Angular momentum of a rotating object <u>with respect</u> to its axis of rotation



small element of mass m_j

$r_{\perp j}$ = distance from axis of rotation

v_j = speed of element = $\omega r_{\perp j}$

Each element contributes $\vec{r}_j \times m_j \vec{v}_j$ to angular momentum

$$|\vec{r}_j \times m_j \vec{v}_j| = r_{\perp j}\, m_j (\omega r_{\perp j}) \sin(90^\circ)$$

$$= m_j r_{\perp j}^2 \,\omega$$

$\vec{r}_j \times m_j \vec{v}_j$ is parallel to $\vec{\omega}$ so $\vec{r}_j \times m_j \vec{v}_j = m_j r_{\perp j}^2 \vec{\omega}$

Add up all the ~~pieces~~ elements: $\vec{L} = \left(\sum_j m_j r_{\perp j}^2\right)\vec{\omega}$

$$= I\vec{\omega}$$

(C13.T6)

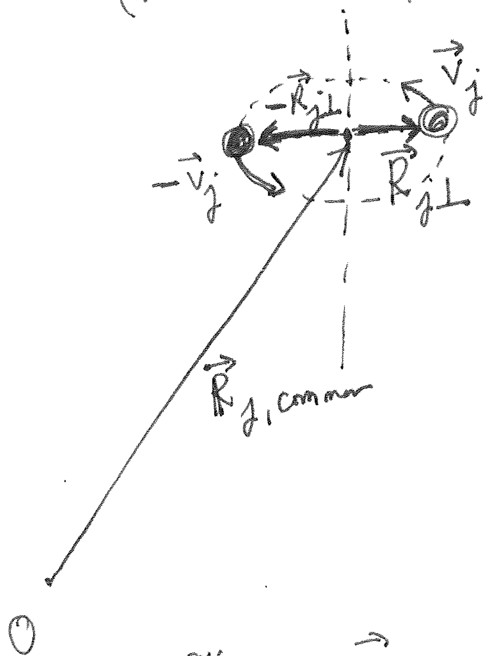
(C13)

BACKGROUND

w.r.t axis of rotation
(Thus, cm is on axis of rotation!)

Angular momentum of a symmetric, stationary rotating object

(result is indep. of origin).

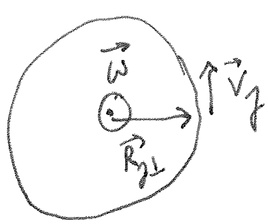


$$\vec{L}_j = \vec{r}_j \times m_j \vec{v}_j$$

$$= (\vec{R}_{j,\text{cm}} + \vec{R}_{j\perp}) \times m_j \vec{v}_j$$

when add the 2 paired masses only $\vec{R}_{j\perp}$ contributes since velocities are opposite

$$\vec{L}^{\text{sys}} = \sum \vec{L}_j = \sum \vec{R}_{j\perp} \times m_j \vec{v}_j$$



$$= \sum m_j R_{j\perp} v_j \hat{\omega}$$

$$= \sum m_j R_{j\perp}^2 \omega \hat{\omega}$$

$$= \underbrace{\left(\sum m_j R_{j\perp}^2\right)}_{I} \underbrace{(\omega \hat{\omega})}_{\vec{\omega}}$$

$$\vec{L}^{\text{rot}} = I \vec{\omega} \quad \text{for a } \underline{\text{symmetric}} \text{ object}$$

[otherwise I is a tensor]

C13-4

~~Rotating stationary object~~

~~$\vec{L}^{rot} = I\vec{\omega}$~~

~~$I = \sum m_j R_{\perp j}^2$~~

~~(explain qualitatively how direction of $\vec{L} \parallel \vec{\omega}$)~~

~~[C13T6?]~~

Moving and rotating object

Angular momentum about origin O is

$$\vec{L}_{about\ O} = \underbrace{\vec{L}^{cm}_{about\ O}}_{\text{treat object as a point mass } M} + \vec{L}^{rot}_{about\ axis\ of\ rotation}$$

$$= \vec{r}_{cm} \times M\vec{v}_{cm} + I\vec{\omega}$$

position of cm relative to O

[NB don't call this "translational angular momentum" bc try confuse it w/ $\vec{p}$; call it "ang mom of cm"]

(analogous to $K = K^{cm} + K^{rot}$
$= \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2$)

BACKGROUND

Angular momentum of a symmetric moving, rotating object

$\vec{v}_j = \vec{v}_{cm} + \vec{u}_j$ $\qquad$ $\vec{u}_j$ = velocity of m_j in cm frame

$$\vec{L}^{sys} = \sum_j L_j$$

$$= \sum_j \vec{r}_j \times m_j \vec{v}_j$$

$$= \sum_j \vec{r}_j \times m_j (\vec{v}_{cm} + \vec{u}_j)$$

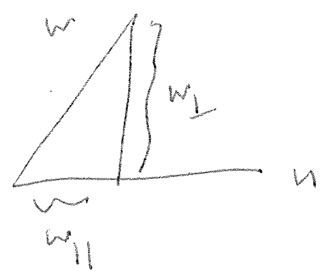
indep of origin for a stationary object

$$= \underbrace{\left(\sum m_j \vec{r}_j\right)}_{M\vec{r}_{cm}} \times \vec{v}_{cm} + \underbrace{\sum \vec{r}_j \times m_j \vec{u}_j}_{I\vec{\omega}}$$

$$= \underbrace{\vec{L}^{cm}}_{\text{depends on origin}} + \vec{L}^{rot}$$

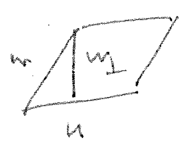
[omitted]

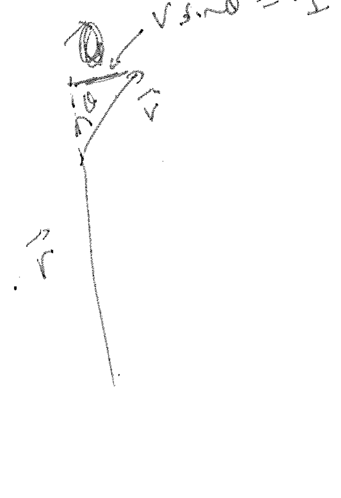
$\vec{u} \cdot \vec{w} = u w_{\parallel} = u_{\parallel} w$



$|\vec{u} \times \vec{w}| = u w_\perp = u_\perp w$

= area of parallelogram





$|L| = m r v_\perp$

Also: $m r_\perp v$

C13T.1 A child swings a 0.1-kg ball on a string in a horizontal circle of radius 2.0 m once every 0.63 s. The ball's angular momentum about the child has a magnitude of

A. 0.63 $kg\cdot m^2/s$
B. 2.0 $kg\cdot m^2/s$
C. 4.0 $kg\cdot m^2/s$
D. 10 $kg\cdot m^2/s$
E. 20 $kg\cdot m^2/s$
F. Other (specify)

C13T.2 If you are standing 30 m due east of a car traveling at 25 m/s southwest, what is the direction of the car's angular momentum relative to you?

A. Southwest
B. Northeast
C. East
D. West
E. Up
F. Down

C13T.3 You are standing 30 m due east of a 50-kg person who is running at a speed of 2.0 m/s due west. What is the magnitude of that person's angular momentum about you?

A. 3000 $kg\cdot m^2/s$
B. 1000 $kg\cdot m^2/s$
C. 300 $kg\cdot m^2/s$
D. 100 $kg\cdot m^2/s$
E. 0
F. Other (specify)

C13T.4 The lengths of the hour and minute hands of a clock are 4 cm and 6 cm, respectively. If the vectors $\vec{u}$ and $\vec{w}$ represent the hour and minute hands, respectively, then $\vec{u} \times \vec{w}$ at 5 o'clock is

A. 24 cm^2 up
B. 24 cm^2 down
C. 21 cm^2 up
D. 21 cm^2 down
E. 12 cm^2 up
F. 12 cm^2 down

C13T.5 The letter N is symmetric (in the sense discussed in section C13.4) for rotations about an axis pointing in which of the directions listed below?

A. →
B. ↑
C. Perpendicular to the plane of the letter
D. A and B
E. A and C
F. A, B, and C

C13T.6 A disk with a mass of 10 kg and a radius of 0.1 m rotates at a rate of 10 turns per second. The magnitude of the disk's total angular momentum is

A. 63 $kg \cdot m^2/s$
B. 31 $kg \cdot m^2/s$
C. 10 $kg \cdot m^2/s$
D. 6.3 $kg \cdot m^2/s$
E. 3.1 $kg \cdot m^2/s$
F. 1.0 $kg \cdot m^2/s$
T. Other (specify)

C13T.7 Imagine that you are the pitcher in a baseball game. The batter hits a foul ball vertically in the air. If the ball has a weight of 2 N and an initial upward velocity of about 30 m/s, and you are 40 m from where the ball is hit, what is the gravitational torque (magnitude and direction) on the ball about you just after it is hit?

A. 2400 N·m upward
B. 2400 N·m to your left
C. 2400 N·m to your right
D. 80 N·m upward
E. 80 N·m to your left
F. 80 N·m to your right
T. Other (specify)

C13T.8 As the ball described in problem C13T.7 continues to rise, the magnitude of the torque on the ball about you due to the ball's weight

A. Increases.
B. Essentially remains the same.
C. Decreases.

C13T.9 A cylinder rolls without slipping down an incline directly toward you. The contact interaction between the cylinder and the incline exerts a friction torque on the cylinder about the cylinder's center of mass. What is the direction of this torque?

A. Toward you.
B. Away from you.
C. To your right.
D. To your left.
E. Upward.
F. Downward.

C13T.10 Imagine that you are looking down on a turntable that is spinning counterclockwise. If an upward torque is applied to the turntable, its angular speed

A. Increases.
B. Decreases.
C. Remains the same.

C13T.11 A wheel of radius 50 cm rotates freely on an axle of radius 0.5 cm. If you want to slow the wheel to rest with your hand, you can either exert a friction force with your hand on the wheel's rim (call the magnitude of this force F_{rim}) or exert a friction force on the wheel's axle (call the magnitude of this force F_{axle}). If you had to bring the wheel to rest in 2.0 s either way, how would the force that you'd have to exert on the rim compare to the force that you'd have to exert on the axle?

A. $F_{\text{rim}} = 100F_{\text{axle}}$
B. $F_{\text{rim}} = 10F_{\text{axle}}$
C. $F_{\text{rim}} = F_{\text{axle}}$
D. $10F_{\text{rim}} = F_{\text{axle}}$
E. $100F_{\text{rim}} = F_{\text{axle}}$