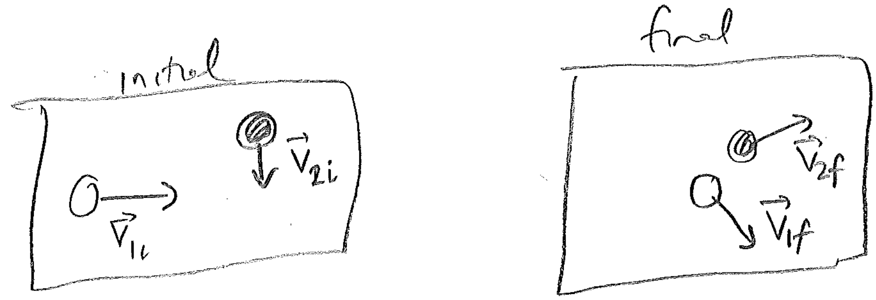
Collisions



Given initial data $\vec{v}_{1i}, \vec{v}_{2i}$ can we predict final $\vec{v}_{1f}, \vec{v}_{2f}$?

- momentarily isolated $\Rightarrow$ momentum conserved

- kinetic energy may or may not be conserved

$$\Delta K = K_f - K_i$$

If kinetic energy conserved ($\Delta K = 0$), collision is "elastic"

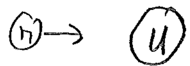
If kinetic energy is lost ($\Delta K < 0$), collision is "inelastic"

(If objects stick together, called "completely" "inelastic")

kinetic energy $\rightarrow$ thermal energy

If kinetic energy gained ($\Delta K > 0$), collision is "superelastic"

chemical energy $\rightarrow$ kinetic (and thermal) energy

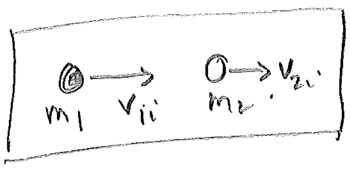


nuclear energy $\rightarrow$ kinetic (+ thermal)

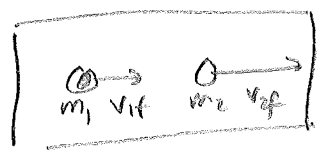
[C12.T4] first question: F_1 2nd question: ambiguous

one-dimensional elastic collision between 2 masses

initial



final



known: m_1 m_2
$\vec{v}_{1ix}$, v_{2ix}

unknown v_{1fx}, v_{2fx}

momentum conservation (x-component)

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 (v_{1ix} - v_{1fx}) = m_2 (v_{2fx} - v_{2ix}) \qquad \underline{\text{eqn A}}$$

Kinetic energy conservation

$$\tfrac{1}{2} m_1 v_{1ix}^2 + \tfrac{1}{2} m_2 v_{2ix}^2 = \tfrac{1}{2} m_1 v_{1fx}^2 + \tfrac{1}{2} m_2 v_{2fx}^2$$

$$m_1 (v_{1ix}^2 - v_{1fx}^2) = m_2 (v_{2fx}^2 - v_{2ix}^2)$$

$$\underline{m_1 (v_{1ix} - v_{1fx})} (v_{1ix} + v_{1fx}) = \underline{m_2 (v_{2fx} - v_{2ix})} (v_{2fx} + v_{2ix}) \qquad \underline{\text{eqn B}}$$

underlined terms are same by eqn A; cancel them out

$$v_{1ix} + v_{1fx} = v_{2fx} + v_{2ix}$$

$$v_{1ix} - v_{2ix} = v_{2fx} - v_{1fx} \qquad \underline{\text{eqn C}}$$

Define relative velocity
$$\vec{v}_{rel} = \vec{v}_1 - \vec{v}_2$$

$$\Rightarrow (v_{rel})_{ix} = -(v_{rel})_{fx}$$

In an elastic collision the magnitude of relative velocity is unchanged

Eqn A + Eqn C = 2 linear eqns in 2 unknowns [solve for HW problem]

[think abt collision in CM frame]

[C12.6] F, A

[C12.5] C

[Q: What is max value of v_{2fx} ?]

Special case: m_2 at rest initially ($v_{2ix}=0$)

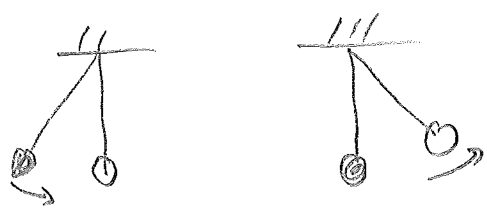
Then solving eqn A + eqn C

$$v_{1fx} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1ix}$$

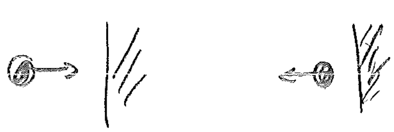
$$v_{2fx} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1ix}$$

Consider various limits:

$m_1 = m_2 \Rightarrow v_{1fx} = 0 \quad v_{2fx} = v_{1ix}$



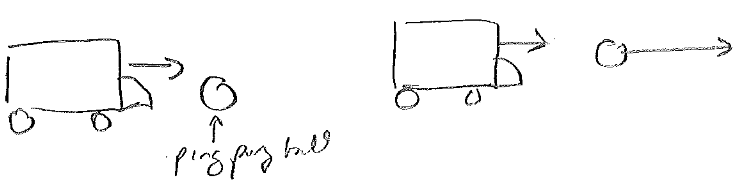
$m_2 \gg m_1$ (brick wall) $\Rightarrow v_{1fx} = -v_{1ix}, \; v_{2fx} \approx 0$



[rel speed same]

[talk about mom + kinetic energy transfer]

$m_1 \ll m_2 \Rightarrow v_{1fx} = v_{1ix}, \; v_{2fx} = 2v_{1ix}$ [rel speed same]



PHYSICS 1130 SOLUTIONS: Problem #31 Due Monday, October 21, 2013

In class, we showed that in a one-dimensional elastic collision between two masses m_1 and m_2, momentum and kinetic energy conservation can be used to demonstrate that $v_{1ix} - v_{2ix} = v_{2fx} - v_{1fx}$. Use this equation together with momentum conservation to solve for each of the unknowns v_{1fx} and v_{2fx}. Your results should have form $Av_{1ix} + Bv_{2ix}$, where A and B are functions of the two masses.

Momentum conservation gives

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \tag{1}$$

and this together with kinetic energy conservation can be used to derive

$$v_{1ix} - v_{2ix} = v_{2fx} - v_{1fx} \tag{2}$$

Equation (2) can be rewritten

$$v_{2fx} = v_{1ix} - v_{2ix} + v_{1fx} \tag{3}$$

Substituting eq. (3) into eq. (1), we obtain

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2(v_{1ix} - v_{2ix} + v_{1fx}) \tag{4}$$

or

$$(m_1 + m_2)v_{1fx} = (m_1 - m_2)v_{1ix} + 2m_2 v_{2ix} \tag{5}$$

or finally

$$v_{1fx} = \frac{m_1 - m_2}{m_1 + m_2} v_{1ix} + \frac{2m_2}{m_1 + m_2} v_{2ix} \tag{6}$$

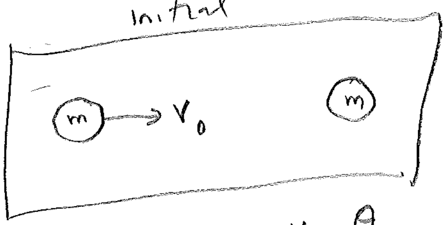
Substituting eq. (6) into eq. (3) and simplifying, we obtain

$$v_{2fx} = \frac{2m_1}{m_1 + m_2} v_{1ix} + \frac{m_2 - m_1}{m_1 + m_2} v_{2ix} \tag{7}$$

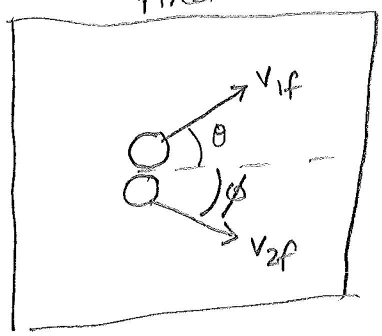
Thus both v_{1fx} and v_{2fx} can be expressed in the form $Av_{1ix} + Bv_{2ix}$, where A and B are ratios of masses.

2-dim'l elastic collision of equal masses

initial



final



knowns: m, v_0, θ

unknowns: v_{1f}, v_{2f}, ϕ

Kinetic energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$

$\Rightarrow v_{2f}^2 = v_0^2 - v_{1f}^2$ (eqn A)

x-momentum: $mv_0 = mv_{1f}\cos\theta + mv_{2f}\cos\phi$

$\Rightarrow v_{2f}\cos\phi = v_0 - v_{1f}\cos\theta$ (eqn B)

y-momentum: $0 = mv_{1f}\sin\theta - mv_{2f}\sin\phi$

$\Rightarrow v_{2f}\sin\phi = v_{1f}\sin\theta$ (eqn C)

- 3 eqns in 3 unknowns v_{1f}, v_{2f}, ϕ.

First eliminate ϕ by using $\sin^2\phi + \cos^2\phi = 1$.

$(\text{eqn B})^2 + (\text{eqn C})^2 \Rightarrow$

$$v_{2f}^2\cos^2\phi = v_0^2 - 2v_0v_{1f}\cos\theta + v_{1f}^2\cos^2\theta$$
$$v_{2f}^2\sin^2\phi = v_{1f}^2\sin^2\theta$$

$$\Rightarrow v_{2f}^2 = v_0^2 - 2v_0v_{1f}\cos\theta + v_{1f}^2 \quad \text{(eqn D)}$$

use eqn A & eqn D & eliminate v_{2f} $\Big\}$ $v_0^2 - 2v_0v_{1f}\cos\theta + v_{1f}^2 = v_0^2 - v_{1f}^2$

$2v_{1f}(v_{1f} - v_0\cos\theta) = 0$

Solve for v_{1f} $\}$ $\Rightarrow$ either $v_{1f} = 0$ or $\boxed{v_{1f} = v_0\cos\theta} \Rightarrow \vec{v}_{1f} = \begin{pmatrix} v_0\cos^2\theta \\ v_0\cos\theta\sin\theta \end{pmatrix}$

Solve for v_{2f} $\}$ (eqn A) $\Rightarrow v_{2f}^2 = v_0^2 - v_0^2\cos^2\theta = v_0^2\sin^2\theta$ $\quad \boxed{v_{2f} = v_0\sin\theta}$ $\Rightarrow \vec{v}_{2f} = \begin{pmatrix} v_0\sin\phi \\ v_0\sin\theta\ldots \end{pmatrix}$

Solve for ϕ $\}$ (eqn B) $\Rightarrow v_0\sin\theta\sin\phi = v_0\cos\theta\sin\theta$

$\sin\phi = \cos\theta \Rightarrow \phi + \theta$ are complementary

$\phi = \frac{\pi}{2} - \theta$

In an elastic collision of equal masses (one initially at rest)
angle between outgoing directions is $90°$

[illegible] ex C12.5 [illegible]

extra

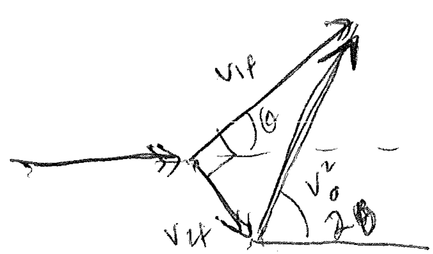
$$\Rightarrow \vec{v}_{1f} = v_0 \cos\theta \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

$$\vec{v}_{2f} = v_0 \sin\theta \begin{pmatrix} \cos\phi \\ -\sin\phi \\ 0 \end{pmatrix} = v_0 \sin\theta \begin{pmatrix} \sin\theta \\ -\cos\theta \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{v}_{1f} - \vec{v}_{2f} = v_0 \begin{pmatrix} c^2\theta - s^2\theta \\ 2 s\theta c\theta \\ 0 \end{pmatrix} = v_0 \begin{pmatrix} c2\theta \\ s2\theta \\ 0 \end{pmatrix}$$

$$|\vec{v}_{1f} - \vec{v}_{2f}| = v_0 \quad \checkmark$$

rel vel same



quick proof of ⊥

$\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2\vec{v}_{1f}\cdot\vec{v}_{2f}$

elastic $\Rightarrow v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

$\Rightarrow \vec{v}_{1f}\cdot\vec{v}_{2f} = 0$

10-14-10

Proving that $|\vec{V}_{Rf}| = |\vec{V}_{Ri}|$ in an elastic collision

Background

mom. cons: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

$\Rightarrow -m_1\Delta\vec{v}_1 = m_2\Delta\vec{v}_2 = \Delta\vec{p}$

kin. en. cons $\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

$\Rightarrow m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$

$m_1(\vec{v}_{1i} + \vec{v}_{1f})\cdot(-\Delta\vec{v}_1) = m_2(\vec{v}_{2f} + \vec{v}_{2i})\cdot(\Delta\vec{v}_2)$

$\Rightarrow (\vec{V}_{1i} + \vec{V}_{1f} - \vec{V}_{2f} - \vec{V}_{2i})\cdot\Delta\vec{p} = 0$

$\vec{V}_{rel} = \vec{V}_1 - \vec{V}_2$

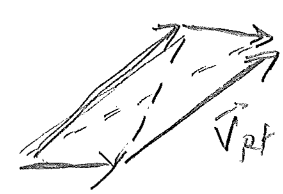
$(\vec{V}_{Ri} + \vec{V}_{Rf})\cdot\Delta\vec{p} = 0$ (1)

If this is one dimensional, then $V_{Rix} = -V_{Rfx}$

In more than one dimension, observe that

$\vec{V}_{1f} = \vec{V}_{1i} - \frac{\Delta\vec{p}}{m_1}$

$\vec{V}_{2f} = \vec{V}_{2i} + \frac{\Delta\vec{p}}{m_2}$



so $\vec{V}_{Rf} = \vec{V}_{Ri} - \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\Delta\vec{p}$

$\Rightarrow \vec{V}_{Ri} - \vec{V}_{Rf} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)\Delta\vec{p}$ (2)

(1) + (2) $\Rightarrow$ diagonals are $\perp$

$\Rightarrow$ rhombus

ie $(\vec{V}_{Ri} + \vec{V}_{Rf})\cdot(\vec{V}_{Ri} - \vec{V}_{Rf}) = 0$

$\Rightarrow V_{Ri}^2 = V_{Rf}^2$

Suppose $\vec{F}_A$ acts on an object or system

$[dK]_A = \vec{F}_A \cdot d\vec{r}$ is the kinetic energy transferred to the object by $\vec{F}_A$

Define the power delivered by the force A as the rate at which energy is ~~delivered~~ transferred to the object

$$P_A = \frac{[dK]_A}{dt} = \vec{F}_A \cdot \frac{d\vec{r}}{dt} = \vec{F}_A \cdot \vec{v}$$

C12

more back to CF-day 2?

Not here for these 18 18

~~Fall 09~~ Spring

C12-3

Power: $P = \frac{d(\text{energy})}{dt}$

Spent a while having them calculate theirs

1 watt = 1 J/s

kW hours

A) >1000 W
B) 1000 W
C) 100 W
D) 1 W
E) 0.1 W
F) 0.01 W
G) 0.01 W
H) << 0.01 W

ask

1 cal = 4.186 J

1 food cal = 1000 cal

(2500 food cal/day $\Rightarrow$ 3×10^6 cal ~ 1.2×10^7 J/day ~ $\frac{1.2\times10^7}{86,400\,s}$ ~ ~~140~~ 120 Watts)

0 → 60 mph in 10 s → 2000 kg

746 W = 1 hp

(1 hp = 746 W)

→ 10^5 W ~ 100 hp

(engine ~ 400 hp)

$[dK] = \vec{F}\cdot d\vec{r}$

$\frac{(dK)}{dt} = \vec{F}\cdot\vec{v}$ = rate at which a force $\vec{F}$ delivers power

[C12.3] → Not more define power as positive!

C12T.1 A trained bicyclist in excellent shape might be able to convert food energy to mechanical energy at a rate of 0.25 hp for a reasonable length of time. Imagine such a person pedaling a stationary bike connected to a perfectly efficient electrical generator. Could such a person generate enough electrical power to run a toaster (T = yes, F = no)? How about a single ordinary lightbulb (T = yes, F = no)?

C12T.2 An object moving with a speed of 5 m/s in the $-x$ direction is acted on by a force with a magnitude of 5 N acting in the $+x$ direction. At what rate does the interaction exerting this force transform this object's kinetic energy to some other form of energy (or vice versa)?

A. +25 W
B. −25 W
C. 5 W
D. −1 W
E. 0
F. Other (specify)

C12T.3 An object moving with a velocity whose components are [4 m/s, −1 m/s, 3 m/s] is acted on by a force whose components are [−5 N, 0, +5 N]. What is the power of the energy transfer involved in this interaction?

A. −35 W
B. −5 W
C. 0
D. +5 W
E. +35 W
F. Other (specify)

C12T.4 Momentum is not conserved in an inelastic collision, true (T) or false (F)? Energy is not conserved in such a collision, T or F?

C12T.5 An object with mass m moving in the $+x$ direction collides head-on with an object of mass $3m$ at rest. After this elastic collision, the first object

A. Moves in the $+x$ direction.

B. Is at rest.

C. Moves in the $-x$ direction.

D. The answer depends on moving object's initial speed.

C12T.6 An object moving with speed v_0 collides head-on with an object at rest that is very much more massive. If the collision is elastic, how does the lighter object's speed v *after* the collision compare with its original speed v_0?

A. v is about equal to v_0.

B. v is noticeably less than v_0.

C. v is one-half of v_0.

D. v is very small.

E. v is exactly zero.

F. $v \approx -v_0$.

C12T.7 If two moving objects collide, we can *always* orient our reference frame so that the collision takes place entirely in the xy plane, T or F?

C12T.8 A object of mass m (object A) moving in the $+x$ direction collides with another object at rest (object B) whose mass is unknown. After the collision, object B is observed moving in the $+y$ direction. This collision *cannot* be elastic, T or F?