[Return to system of particles]

$$\vec{p}^{sys} = \sum m_j \vec{v}_j$$

$$K^{sys} = \sum \frac{1}{2} m_j v_j^2$$

[Q: which of the following are true?] surprisingly I got all 4 answers

A: $\vec{p}^{sys} = M \vec{v}_{cm}$

B: $K^{sys} = \frac{1}{2} M v_{cm}^2$

C: both

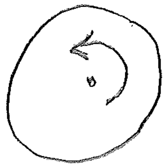
D: neither

Talk about examples of falseness of B. eg
upturned bicycle wheel spinning : $\vec{v}_{cm} = 0$

$\vec{p}^{sys} = 0$

$K^{sys} \neq 0$

[How characterize rotation?]



r.p.m = revolutions per minute

<u>frequency f</u> $= \frac{\# \text{ cycles}}{\text{sec}}$

1 cycle = 1 revolution

[if 10 $\frac{\text{cyc}}{\text{sec}}$, how long for one cycle?]

<u>period T</u> = time for 1 cycle $= \frac{1}{f}$

1 cycle = $360° = 2\pi$ radians

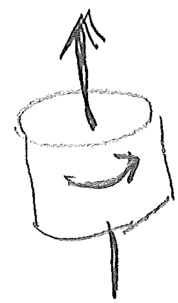
<u>angular frequency ω</u> $= \frac{\# \text{ radians}}{\text{sec}} = 2\pi f = \frac{2\pi}{T}$

(omega)

a.k.a <u>angular speed</u>

[Q: What is angular speed of earth's rotation?]

$$\frac{2\pi}{86400\,s} = 7.3 \times 10^{-5} \ \frac{\text{rad}}{\text{sec}}$$



axis of rotation

use right hand rule to define direction.

[we used rh-rule for coord systems]

Angular velocity $\vec{\omega}$ = vector w/ magnitude ω (angular speed) and direction given by axis of rotation

[C9.T3]$_B$

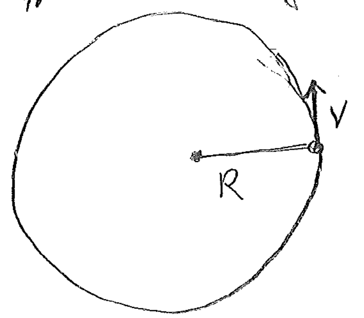
[C9.T4]$_D$

This may be hard for them; we'll use this to motivate the following

[C9.T1]$_B$

$v = 50$ cm/s
$R = 10$ cm
what is ω?

[Ask them how to figure this out.]

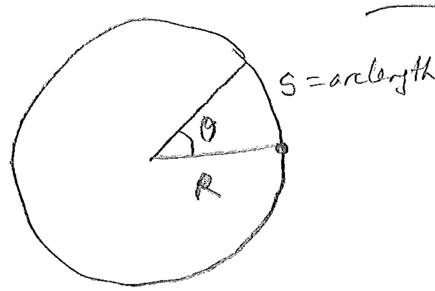


here's one way →

$$T = \frac{2\pi R}{V}$$

$$\omega = \frac{2\pi}{T} = \frac{V}{R}$$

here's another →



$$s = R\theta \quad \leftarrow \text{radians}$$

$$v = \frac{|ds|}{dt}, \quad \omega = \frac{|d\theta|}{dt}$$

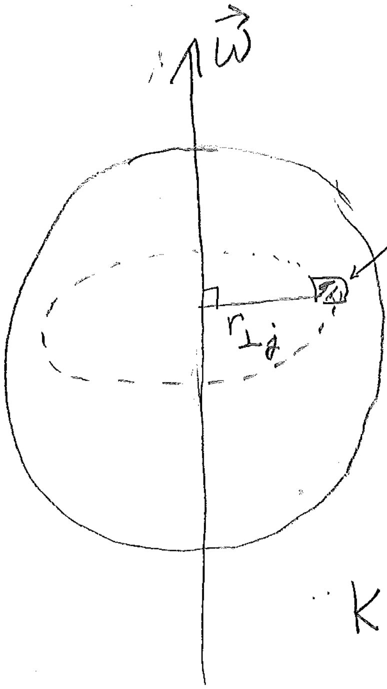
$$\Rightarrow v = R\omega$$

$$\boxed{v = R\omega}$$

[C9.T2]$_D$

$v = \omega R = \frac{2\pi}{T} R =$

[Now we're ready to compute K for rotating object]



Small element of mass m_j

$r_{\perp j}$ = distance from m_j to axis of rotation

v_j = speed of element $= \omega r_{\perp j}$

$$K^{rot} = \sum \frac{1}{2} m_j v_j^2$$

$$= \sum \frac{1}{2} m_j r_{\perp j}^2 \omega^2$$

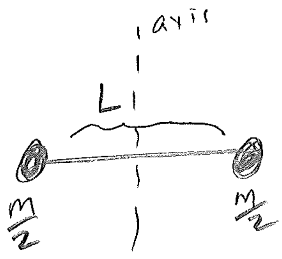
$$= \frac{1}{2} \underbrace{\left(\sum m_j r_{\perp j}^2 \right)}_{\text{call this } I \text{, moment of inertia}} \omega^2$$

$$I = \sum_j m_j r_{\perp j}^2$$

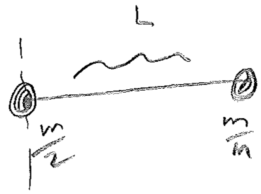
If continuous

$$I = \int r_\perp^2 \, dm$$

Dumb-bell

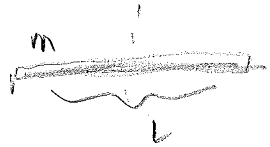


$$I = \frac{m}{2}\left(\frac{L}{2}\right)^2 + \frac{m}{2}\left(\frac{L}{2}\right)^2 = \frac{mL^2}{4}$$



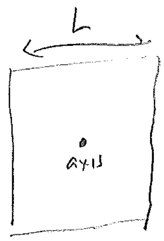
$$I = \frac{m}{2}L^2 + 0 = \frac{mL^2}{2}$$

[depends on axis]



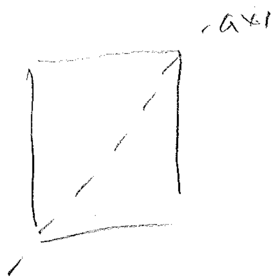
$I \stackrel{?}{=}$ $\left[\frac{1}{12}mL^2\right]$

[C9.T5]



$$4\left(\frac{m}{4}\right)\left(\frac{L}{\sqrt{2}}\right)^2 = \frac{1}{2}mL^2$$

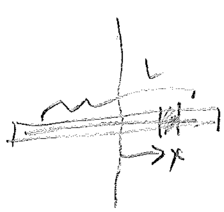
[C9.T6]



$$2\left(\frac{m}{4}\right)\left(\frac{L}{\sqrt{2}}\right)^2 = \frac{1}{4}mL^2$$

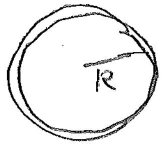
[C9.T7]

rod

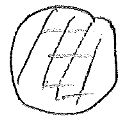


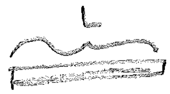
$$I = \int r_\perp^2\, dm$$
$$= \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 \left(\frac{m}{L}\right) dx = \frac{m}{3L} x^3 \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{mL^2}{12}$$

Moments of inertia about the cm.

ring / hoop 

$I = mR^2$

disk  $I = \frac{1}{2} mR^2$

rod 

$I = \frac{1}{12} mL^2$

solid sphere  $I = \frac{2}{5} mR^2$

hollow sphere  $I = \frac{2}{3} mR^2$

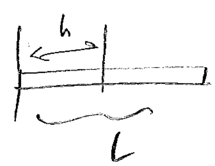
(the disk, rod, solid sphere and hollow sphere:) requires doing an integral

$$\left[\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{m}{L} dx\, x^2 = \frac{m}{L} \frac{x^3}{3} \Big|_{-\frac{L}{2}}^{\frac{L}{2}} = \frac{mL^2}{12} \right]$$

[C9.T9]

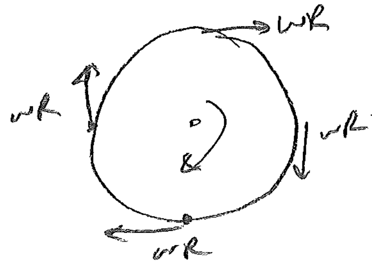
[optional] Parallel axis thm: moment of inertia about an axis that is a distance h from the cm

$$I_{\text{about axis}} = I_{\text{about cm}} + Mh^2$$

eg rod rotating about one end: 

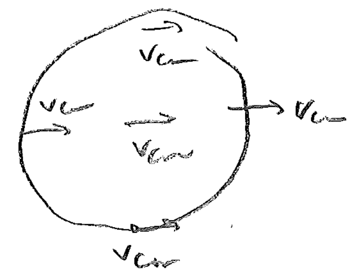
$$\frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3} mL^2$$

Rotating not translating



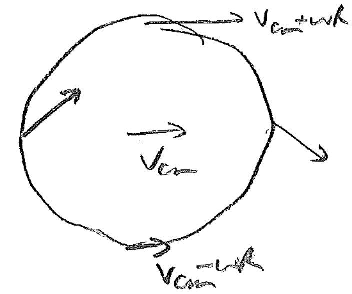
$$K^{rot} = \frac{1}{2} I \omega^2$$

Translating not rotating



$$K = \frac{1}{2} M v_{cm}^2$$

Both rotating & translating



$$K = ?$$

Kinetic energy of a rotating & translating object

$$K = \sum \tfrac{1}{2} m_j v_j^2 \qquad \vec{v}_j = \vec{v}_{cm} + \vec{v}_j'$$

$$= \sum \tfrac{1}{2} m_j \vec{v}_j \cdot \vec{v}_j$$

$$= \sum \tfrac{1}{2} m_j (\vec{v}_{cm} + \vec{v}_j') \cdot (\vec{v}_{cm} + \vec{v}_j')$$

$$= \tfrac{1}{2} \underbrace{\left(\sum m_j\right)}_{M} v_{cm}^2 + \underbrace{\sum m_j \vec{v}_j'}_{=0} \cdot \vec{v}_{cm} + \sum \tfrac{1}{2} m_j v_j'^2$$

Recall $\vec{r}_{cm} = \dfrac{\sum m_j \vec{r}_j}{M}$

$\vec{v}_{cm} = \dfrac{\sum m_j \vec{v}_j}{M}$

$$\sum m_j \vec{v}_j' = \sum m_j \vec{v}_j - \underbrace{\left(\sum m_j\right)}_{M} \vec{v}_{cm} = 0$$

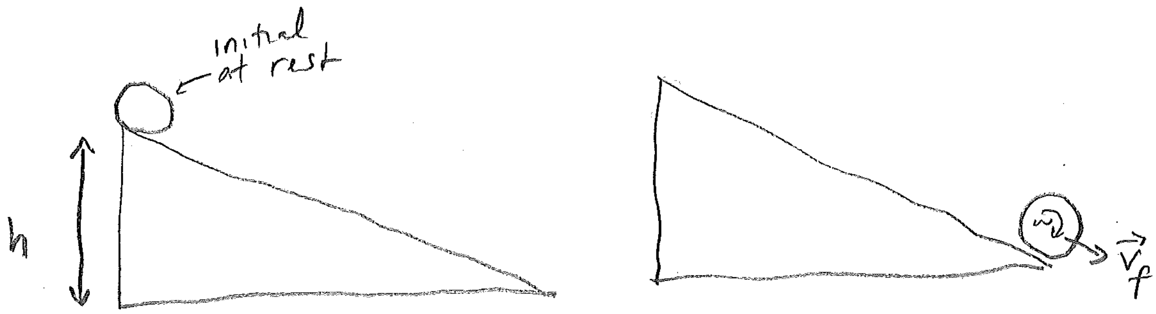
$$K = K^{cm} + K^{rot}$$

$$= \tfrac{1}{2} M v_{cm}^2 + \tfrac{1}{2} I \omega^2$$

If object is rolling w/o slipping,
point of contact is stationary $\Rightarrow$ $v_{cm} - \omega R = 0$

$$\Rightarrow \omega = \frac{v_{cm}}{R}$$

Can rolling down incline w/o slipping



$$\omega_f = \frac{v_f}{R}$$

$$\Delta K = -\Delta V^{grav}$$

$$K_f - K_i = K_f^{cm} + K_f^{rot} = -(0 - mgh)$$

$$\frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = mgh$$

$$\frac{1}{2}\left(m + \frac{I}{R^2}\right) v_f^2 = mgh$$

$$\Rightarrow \quad v_f^2 = \frac{2mgh}{m + \frac{I}{R^2}}$$

$$v_f = \sqrt{\frac{2gh}{1 + \left(\frac{I}{mR^2}\right)}}$$

[C9.T8]

v_f reduced by a factor of $\dfrac{1}{\sqrt{1 + \left(\frac{I}{mR^2}\right)}}$ [DEMO: solid vs hollow cyl]

[DEMO: roll cans of broth & pumpkin down incline
use meter stick to hold them.]

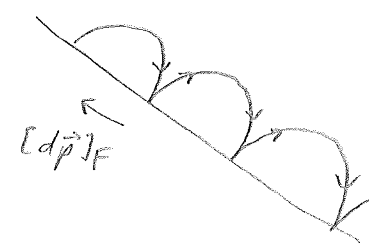
Rolling w/o slipping requires <u>friction</u>

but friction does no work

because $\vec{v}_{rel} = 0$ at point of contact

$$[dK]_{fric} = \vec{v} \cdot [d\vec{p}]_{fric}$$

$$= 0$$



Background

Kinetic energy of a moving, rotating object

Moore does this in problem C9.S10

$\vec{u}_j$ = velocity of m_j in cm frame

$$\vec{v}_j = \vec{v}_{cm} + \vec{u}_j$$

$$K^{sys} = \sum K_j$$

$$= \sum \tfrac{1}{2} m_j v_j^2$$

$$= \sum \tfrac{1}{2} m_j \left(v_{cm}^2 + 2\vec{v}_{cm} \cdot \vec{u}_j + u_j^2 \right)$$

$$= \tfrac{1}{2} \underbrace{\left(\sum m_j \right)}_{M} v_{cm}^2 + \vec{v}_{cm} \cdot \underbrace{\left(\sum m_j \vec{u}_j \right)}_{\substack{0 \\ \text{by def of cm}}} + \tfrac{1}{2} \sum m_j u_j^2$$

$$= K^{cm} + K^{rot}$$

C9T.1 A particle moving along a circular path 10 cm in radius moves at a speed of 50 cm/s. Its angular speed is

A. 50 rad/s
B. 5 rad/s
C. 0.2 rad/s
D. 1.26 rad/s
E. 900° s^{-1}
F. Other (specify)

C9T.2 A merry-go-round makes a complete revolution once every 6.28 s. What is the speed (in meters per second) of a horse 5 m from the merry-go-round's center?

A. 0.80 m/s
B. 1.0 m/s
C. 1.26 m/s
D. 5.0 m/s
E. 31.4 m/s
F. Other (specify)

C9T.3 A cylindrical barrel is rolling on the ground toward you. Its angular velocity points

A. To your right.
B. To your left.
C. Toward you.
D. Away from you.
E. Toward you at the top, away from you on the bottom.
F. Other (specify).

C9T.4 A bicyclist passes in front of you, moving perpendicular to your line of sight from your left to your right. The angular velocity of its wheels points roughly

A. To your right.
B. To your left.
C. Toward you.
D. Away from you.
E. To your right at the top, to your left on the bottom.
F. Other (specify).

C9T.5 Four point particles, each with mass $\frac{1}{4}M$, are connected by massless rods so that they form a square whose sides have length L. What is the moment of inertia I of this object if it is spun around an axis going through the center of the square perpendicular to the plane of the square?

A. $\frac{1}{16}ML^2$
B. $\frac{1}{8}ML^2$
C. $\frac{1}{4}ML^2$
D. $\frac{1}{2}ML^2$
E. ML^2
F. Other (specify)

C9T.6 What would be the moment of inertia I of the square described in problem C9T.5 if it were spun around an axis going through one particle and its diagonal opposite?

A. $\frac{1}{16}ML^2$
B. $\frac{1}{8}ML^2$
C. $\frac{1}{4}ML^2$
D. $\frac{1}{2}ML^2$
E. ML^2
F. Other (specify)

C9T.7 Two wheels have the same total mass M and radius R. One wheel is a uniform disk. The other is like a bicycle wheel, with lightweight spokes connecting the rim to the hub. Which has the larger moment of inertia?
A. The solid disk.
B. The wheel with spokes.
C. There is insufficient information for a meaningful answer.

C9T.8 A solid ball and a hollow ball are released from rest at the top of an incline and roll without slipping to the bottom. Which reaches the bottom first?
A. The solid ball.
B. The hollow ball.
C. Both balls arrive at the same time.
D. It depends on the masses of the balls.

C9T.9 A 10-kg sphere with a radius of 10 cm spinning at 10 rotations per second has a rotational kinetic energy of
A. 5 J
B. 10 J
C. 31 J
D. 63 J
E. 500 J
F. Other (specify)