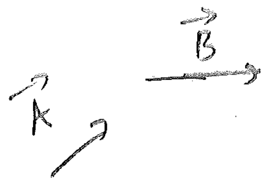
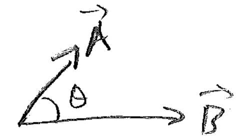
[we defined vector addition,
now multiplication]

Dot product $\vec{A}\cdot\vec{B}$



1) place vectors tail-to-tail



2) define θ as angle between them

3) $\vec{A}\cdot\vec{B} \equiv A\,B\cos\theta$

Observe: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Parallel vectors $\Rightarrow \theta = 0 \Rightarrow \vec{A}\cdot\vec{B} = AB$

Antiparallel " $\Rightarrow \theta = 180° \Rightarrow \vec{A}\cdot\vec{B} = -AB$

Perpendicular vectors $\Rightarrow \theta = 90° \Rightarrow \vec{A}\cdot\vec{B} = 0$

Acute $\Rightarrow \vec{A}\cdot\vec{B} > 0$

Obtuse $\Rightarrow \vec{A}\cdot\vec{B} < 0$

Component rule: $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

special case $\vec{A}\cdot\vec{A} = A^2 = A_x^2 + A_y^2 + A_z^2$

$\boxed{\text{Change in kinetic energy caused by interactions}}$ → not necessarily conservative!

If an obj's momentum changes by $d\vec{p}$
how much does its kinetic energy change?

[Q: Does K change? A. Yes B. No C. Maybe]

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)$$

$$dK = \frac{1}{2}m(2v_x\,dv_x + 2v_y\,dv_y + 2v_z\,dv_z)$$

$$= v_x(m\,dv_x) + v_y(m\,dv_y) + v_z(m\,dv_z)$$

$$= v_x\,dp_x + v_y\,dp_y + v_z\,dp_z$$

$$dK = \vec{v}\cdot d\vec{p}$$

Recall $d\vec{p} = \vec{F}_{net}\,dt$ so

$$dK = \vec{v}\cdot\vec{F}_{net}\,dt$$

[eg. circular orbits $\vec{v} \perp \vec{F} \Rightarrow K = \text{const}$]

[if several forces are acting on an object, change in momentum = vector sum of momentum transfers]

$$d\vec{p} = \sum_{\substack{\text{interactions} \\ A}} [d\vec{p}]_A$$

$[d\vec{p}]_A$ = momentum transfer by interaction $A = \vec{F}_A\, dt$

[change in kinetic energy = sum of kinetic energy transfers]

$$dK = \sum_A [dK]_A$$

$[dK]_A$ = kinetic energy transferred by interaction A

= "K-work" done by interaction A

$\equiv \vec{v} \cdot [d\vec{p}]_A$

$= \vec{v} \cdot \vec{F}_A\, dt$

$= \frac{d\vec{r}}{\cancel{dt}} \cdot \vec{F}_A\, \cancel{dt}$

$= \vec{F}_A \cdot d\vec{r}$

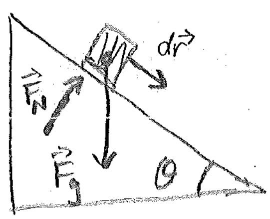
= work done by force $\vec{F}_A$ ("force times displacement")

$$[dK]_A = \vec{v} \cdot [d\vec{p}]_A = \vec{F}_A \cdot d\vec{r}$$

$$dK = \sum_A [dK]_A$$

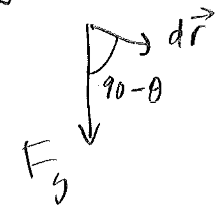
$$\boxed{dK = \sum_A \vec{F}_A \cdot d\vec{r}}$$ work-energy theorem

Example: block on frictionless plane



$$[dK]_N = \vec{F}_N \cdot d\vec{r} = 0$$

$$[dK]_g = \vec{F}_g \cdot d\vec{r} = mg\, dr \cos(90-\theta)$$



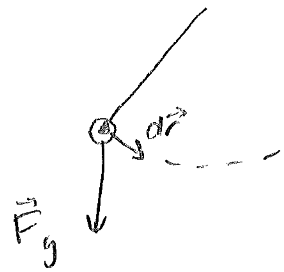
$$= mg \sin\theta \, dr$$

Constant force $\Rightarrow$ $\Delta K = mg \sin\theta \; \Delta r$

where Δr = distance block has moved

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mg\sin\theta\, \Delta r$$

[C8.T4]$_A$



[grav does + work on way down, − work on way up]

[C8.T3]$_C$

[C8.T2]$_C$

$dK = \vec{F} \cdot d\vec{r}$ so same

tricky: [C8.T1]$_D$

$dp = F\,dt \rightarrow$ same p

$K = \frac{(p)^2}{2m}$

~~Rate at wh~~

~~$\frac{[dK]_A}{dt} = \vec{F}_A \cdot \vec{v}$~~

~~Recall~~

Power

done in Ch2 but cd do here? → include 2 min quests for Ch2

Force = rate at which momentum is transferred to a particle

$$\vec{F}_A = \frac{[d\vec{p}]_A}{dt}$$

Power = rate at which kinetic energy is transferred to a particle by an interaction

$$P_A = \frac{[dK]_A}{dt} = \vec{F}_A \cdot \vec{v}$$

$P = \vec{F} \cdot \vec{v}$ = power delivered to a particle by a force $\vec{F}$

units: $N \cdot \frac{m}{s} = \frac{J}{s} =$ Watts

Day II

Day II

C8-11

$$\boxed{\text{change in kinetic energy when only conservative forces act}}$$

(eg. spring, gravity, etc.)

Recall: if only conservative forces act, mechanical energy is conserved

$$E^{mech} = K + V_A = \text{const}$$

→ [for a light object interacting w/ a much heavier one by a single interaction A]

$$dK + dV_A = 0$$

$$dK = -dV_A$$

But we also know that if A is the only interaction

$$dK = [dK]_A = \vec{F}_A \cdot d\vec{r}$$

Hence, for conservative force

$$dV_A = -\vec{F}_A \cdot d\vec{r}$$

(relation between potential energy and force for a conservative interaction A)

$$\boxed{dV = -\vec{F} \cdot d\vec{r} \quad \text{for any conservative interaction}}$$

$$= -(F_x\,dx + F_y\,dy + F_z\,dz)$$

$$\Rightarrow \begin{cases} F_x = -\dfrac{dV}{dx} \\ F_y = -\dfrac{dV}{dy} \\ F_z = -\dfrac{dV}{dz} \end{cases}$$

Could delay this until after C8-14

[Could do example (Moore does) or wait until chapter C11]

since don't actually use force laws yet

gravity near earth

$$V_{grav} = mgz$$

$$F_x = -\frac{\partial V}{\partial x} = 0$$

$$F_y = -\frac{\partial V}{\partial y} = 0$$

$$F_z = -\frac{\partial V}{\partial z} = -mg$$

[could do other forces here or later]

change in kinetic energy when both conservative and nonconservative forces act

$$dK = \sum_{\substack{\text{all}\\ \text{interactions}}} [dK] = \sum_{\text{all}} \vec{F}\cdot d\vec{r}$$

$$= \underbrace{\sum_{\substack{\text{conservative}\\ \text{interaction}}} \vec{F}\cdot d\vec{r}}_{-\sum_{\text{cons.}} dV} + \sum_{\substack{\text{noncons.}\\ \text{int.}}} \vec{F}\cdot d\vec{r}$$

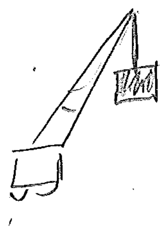
$$\underbrace{dK + \sum_{\text{cons}} dV}_{dE^{mech}} = \sum_{\text{noncons}} \vec{F}\cdot d\vec{r}$$

If only conservative forces act,
or If the nonconservative forces do no work (ie $\vec{F}\cdot d\vec{r} = 0$),
then mechanical energy is conserved

$$dE^{mech} = 0 \Rightarrow E^{mech} = \text{const}$$

$$E_i^{mech} = E_f^{mech}$$

Q: A crane is lifting a box at const speed.



Is mechanical energy of the system conserved?

[C8.T7] bead on wire: F_N does no work

[C8.T9]$_A$ (most say B)

cons of energy valid, but v_f is not zero

[C8-T8] fun, but not necessary question, good for discussion

Background

Mechanical energy of isolated systems

$$dK^{sys} = \sum_j dK_j = \sum_j \sum_{k \neq j} [dK_{j(k)}]$$ only internal interactions

$$= \sum_j \sum_{k \neq j} \vec{F}_{j(k)} \cdot d\vec{r}_j$$

$$= \sum_{j<k} \left(\vec{F}_{j(k)} \cdot d\vec{r}_j + \vec{F}_{k(j)} \cdot d\vec{r}_k \right)$$

$$= \sum_{j<k} \vec{F}_{j(k)} \cdot (d\vec{r}_j - d\vec{r}_k)$$ Newton's 3rd

$$= - \sum_{j<k} dV_{jk}$$ if only conservative forces do work

$$\Rightarrow \boxed{E^{sys}_{mech} = \sum_j K_j + \sum_{j<k} V_{jk} = \text{const}}$$

C8T.1 Two hockey pucks are initially at rest on a horizontal plane of frictionless ice. Puck A has twice the mass of puck B. Imagine that we apply the same constant force to each puck for the same interval of time dt. How do the pucks' kinetic energies compare at the end of this interval?

A. $K_A = 4K_B$
B. $K_A = 2K_B$
C. $K_A = K_B$
D. $K_B = 2K_A$
E. $K_B = 4K_A$
F. Other (specify)

C8T.2 Two hockey pucks are initially at rest on a horizontal plane of frictionless ice. Puck A has twice the mass of puck B. Imagine that we apply the same constant force to each puck until each puck crosses a finish line 1 m from its starting point. How do the pucks' kinetic energies compare when each crosses this finish line?

A. $K_A = 4K_B$
B. $K_A = 2K_B$
C. $K_A = K_B$
D. $K_B = 2K_A$
E. $K_B = 4K_A$
F. Other (specify)

C8T.3 A crate is being lifted vertically upward at a constant speed. Which interaction contributes *negative* k-work to the crate as it rises? (Ignore friction.)

A. The crate's contact interaction with the cable lifting it.

B. The crate's gravitational interaction with the earth.

C. Other (specify).

D. Neither A nor B; the crate's kinetic energy is constant!

C8T.4 Imagine that we suspend an object from the ceiling by a string and then set it swinging back and forth. Which of the following is/are responsible for significant changes in the object's kinetic energy as it swings?

A. The object's gravitational interaction with the earth.

B. The object's contact interaction with the string.

C. The centrifugal interaction pulling the object outward.

D. A and B only.

E. All the above.

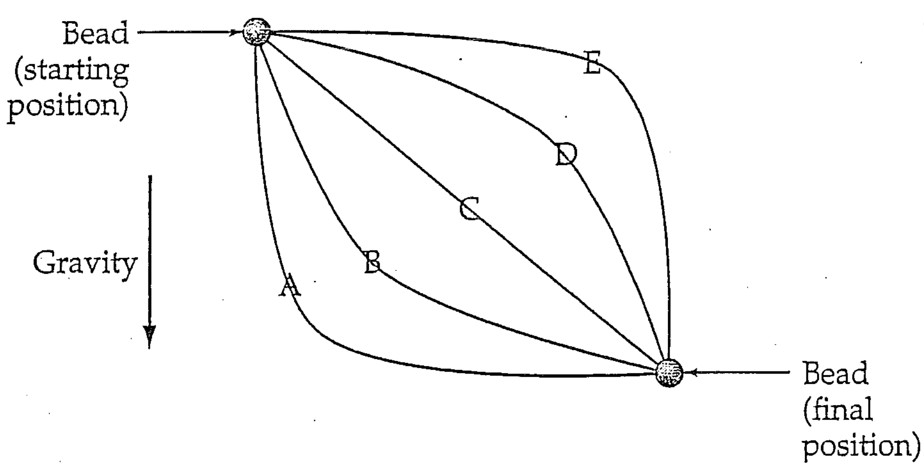
F. None of the above.

T. Other (specify).

C8T.5 When an object slides down a frictionless incline, the contact interaction between the object and the incline does not contribute to the object's kinetic energy, true (T) or false (F)? This means that it also does not transfer any momentum to the object, T or F?

C8T.6 Imagine that a very heavy ball is suspended by a chain. Imagine that I pull the ball away from its equilibrium position, hold it against my nose (with the chain taut), and release the ball from rest. The ball swings away from me and then back toward me. I can be confident that it will not smash my nose (as long as I don't move), T or F?

C8T.7 A bead slides from rest down a frictionless wire in the earth's gravitational field. The diagram shows a set of possible shapes that the wire might have. At the bottom of which will the bead have the highest speed? (If the final speed is the same for all shapes, answer F.)



C8T.8 In the situation described in problem C8T.7, along the wire of which shape will the bead take the *shortest time* to get from the top to the bottom? (If the time is the same for all shapes, answer F.)

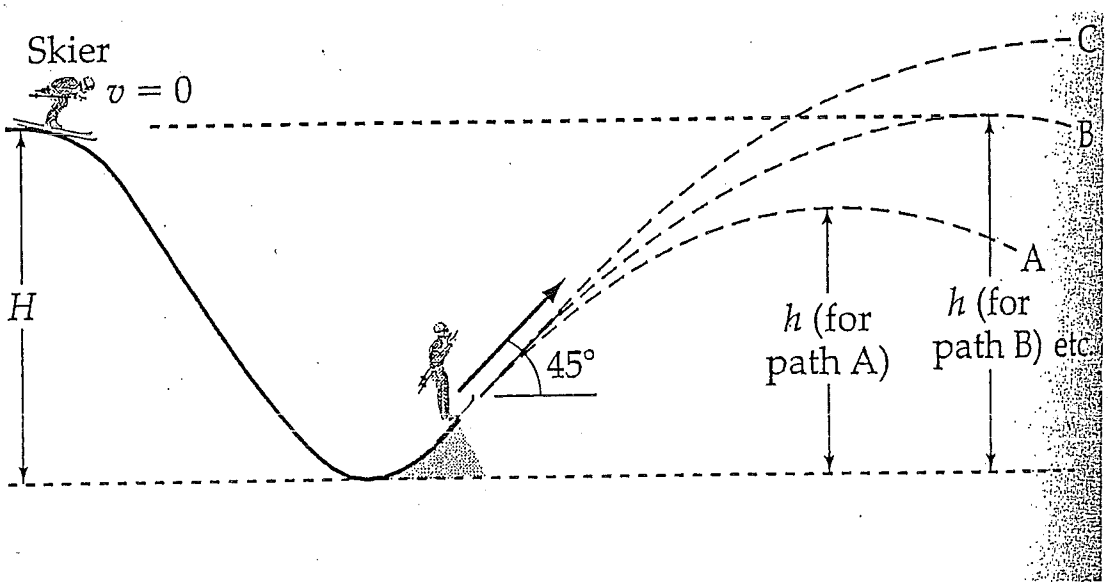
C8T.9 A skier starts from rest, slides down a frictionless slope, and slides off a ski jump angled upward at 45° with respect to the vertical. How does the skier's height h at the peak of the jump compare to his or her original height H?

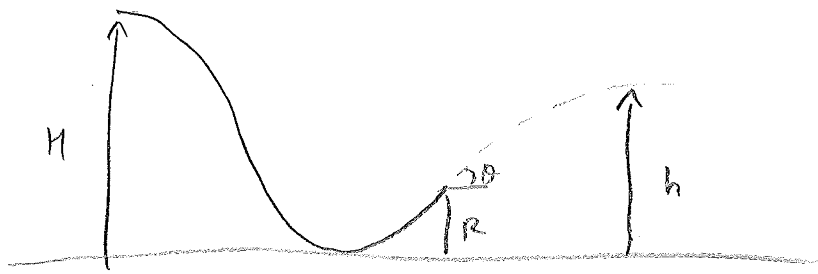
A. $h < H$

B. $h \approx H$

C. $h > H$

D. There is not enough information provided to tell.





$$\frac{1}{2}mv_0^2 + mgH = \frac{1}{2}mv_f^2 + mgR = \frac{1}{2}mv_f^2\cos^2\theta + mgh$$

$$v_f^2 = v_0^2 + 2g(H-R)$$

$$mg(H-h) = \frac{1}{2}m\left(v_0^2 + 2g[H-R]\right)\cos^2\theta - \frac{1}{2}mv_0^2$$

$$= mg(H-R)\cos^2\theta - \frac{1}{2}mv_0^2\sin^2\theta$$

$$H-h = (H-R)\cos^2\theta - \frac{v_0^2}{2g}\sin^2\theta$$

$$h = H - (H-R)\cos^2\theta + \frac{v_0^2}{2g}\sin^2\theta$$

if $\theta = 45° \Rightarrow h = \frac{H+R}{2} + \frac{v_0^2}{4g}$

$h \geq H$ if $\frac{v_0^2}{2g}\sin^2\theta > (H-R)\cos^2\theta$

$$v_0^2 > \frac{2g(H-R)}{\tan^2\theta}$$