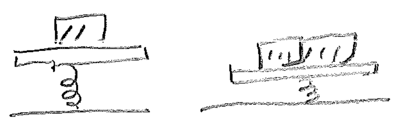
[• We usually associate potential energy w/ long-range forces (e.g. gravity) but sometimes we can use potential energy for contact forces (any such interaction is called "conservative")]

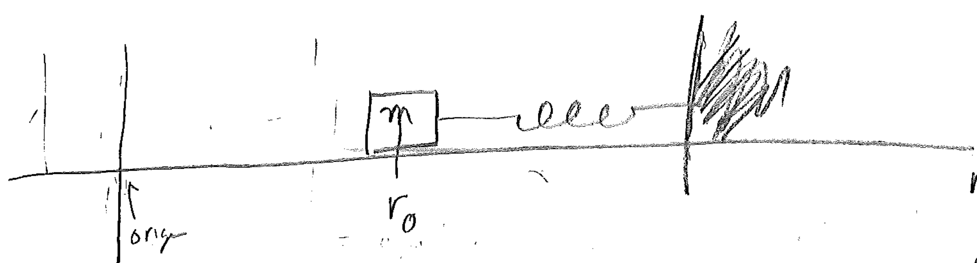
Potential energy is associated w/ conservative forces

1) gravity
2) electrostatic
3) spring

[e.g. how does normal force work?
It is responsive; magnitude depends on other forces; e.g. the weight of an object.
How does it do this? Slight compression of the surface, which you don't really notice.
More noticeable w/ a spring scale]



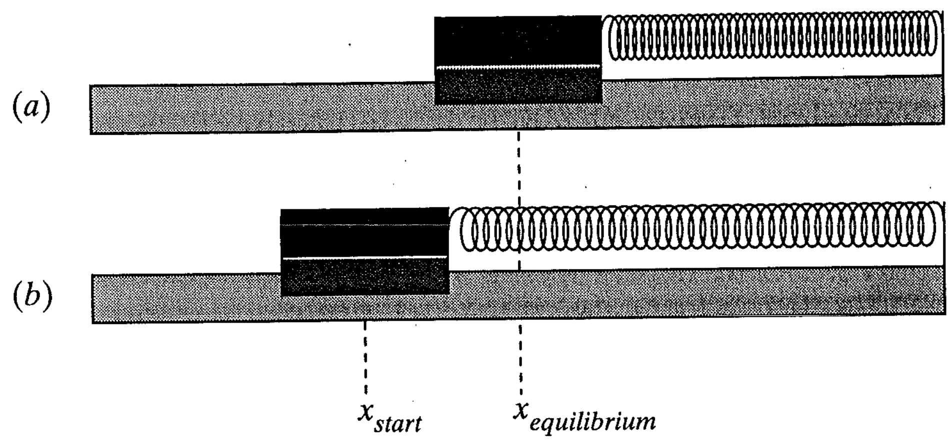
One can store energy in a spring

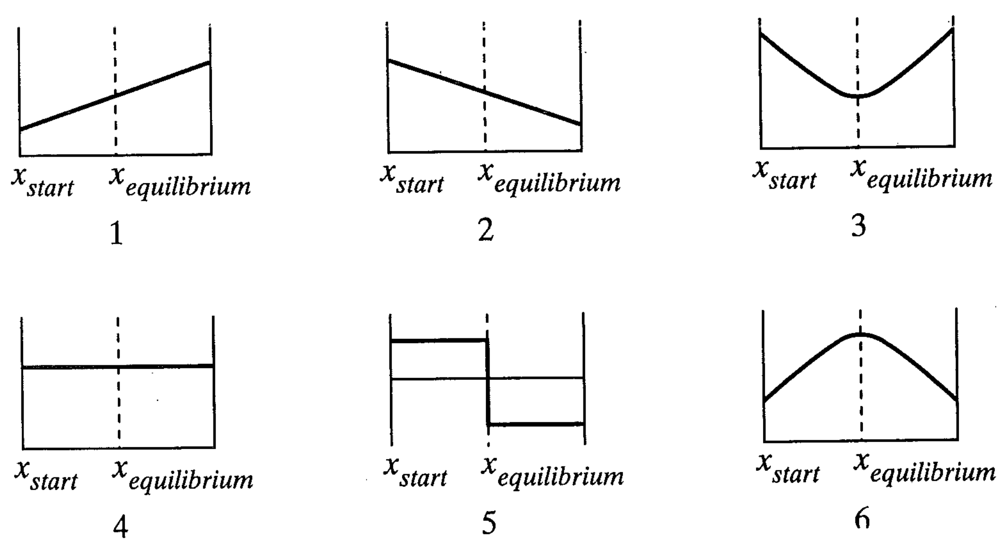


Let r_0 = position of mass at which spring is relaxed (exerts no force)

[Q: In part (a) of figure, an air track ...] Answer 3

In part (*a*) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (*b*), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?





[To either compress or stretch string requires energy so V increases in both directions.]

Spring potential energy $V_{spr} = \frac{1}{2} k_s (r - r_0)^2$

k_s = spring constant

[Note: r_0 is position at which spring is relaxed.
It is also the reference position for V_{spr} because $V_{spr} = 0$ there.]

[Q: "A spring w/ spring const..."] (c)

~~[Q: C7.T8 (graph reading)]~~ later

c7.85 modified

A spring with spring constant $k_s = 10\ \mathrm{J/cm^2}$ has a relaxed length of 6.0 cm. If the spring is compressed to one-third its length, what is the change in potential energy?

(a) 20 J

(b) 60 J

(c) 80 J ←

(d) 180 J

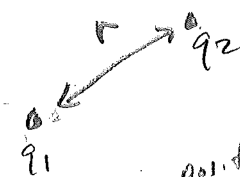
(e) other

Electrostatic interaction

Long range interaction between charged objects

Electrostatic potential energy

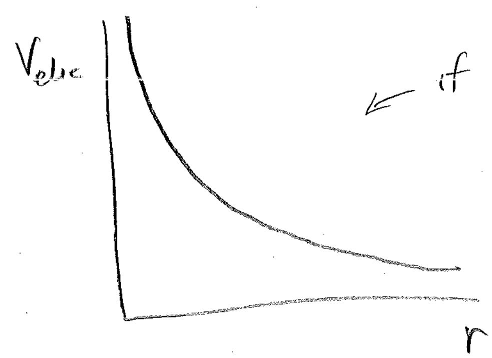
$$V_{elec} = \frac{K q_1 q_2}{r}$$



q = charge in Coulombs (C) → proton has positive charge $e = 1.6 \times 10^{-19}$ C

r = distance between charges

$K = 8.99 \times 10^{9} \frac{J \cdot m}{C^2}$

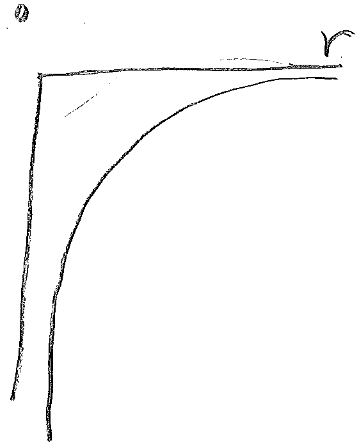


← if $q_1 q_2 > 0$ (same sign)

Reference position is at $r = \infty \Rightarrow V_{elec} = 0$ when charges are far apart (no energy is stored)

[C7.T1]

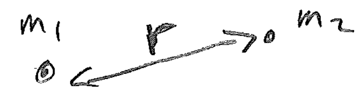
If $q_1 q_2 < 0$

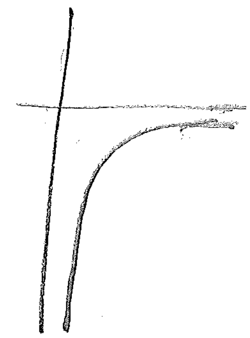


[attractive force]

Universal law of gravitation

$$V_{grav} = -\frac{G m_1 m_2}{r}$$



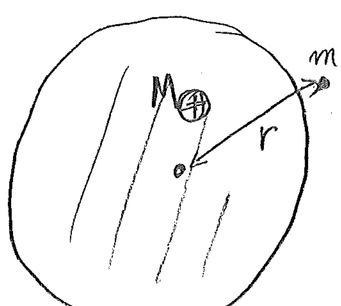


m = mass of point-like objects

G = Newton's const = $6.67 \times 10^{-11} \frac{J \cdot m}{kg^2}$

Reference separation is at $r = \infty$

Newton showed that a sphere of uniformly distributed mass M exerts the same force as if it were all concentrated at the center of the sphere [difficult integral]

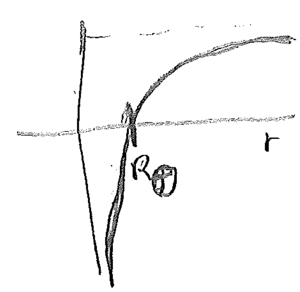


$$V_{grav} = -\frac{G M_\oplus m}{r}$$

r = distance to center of earth

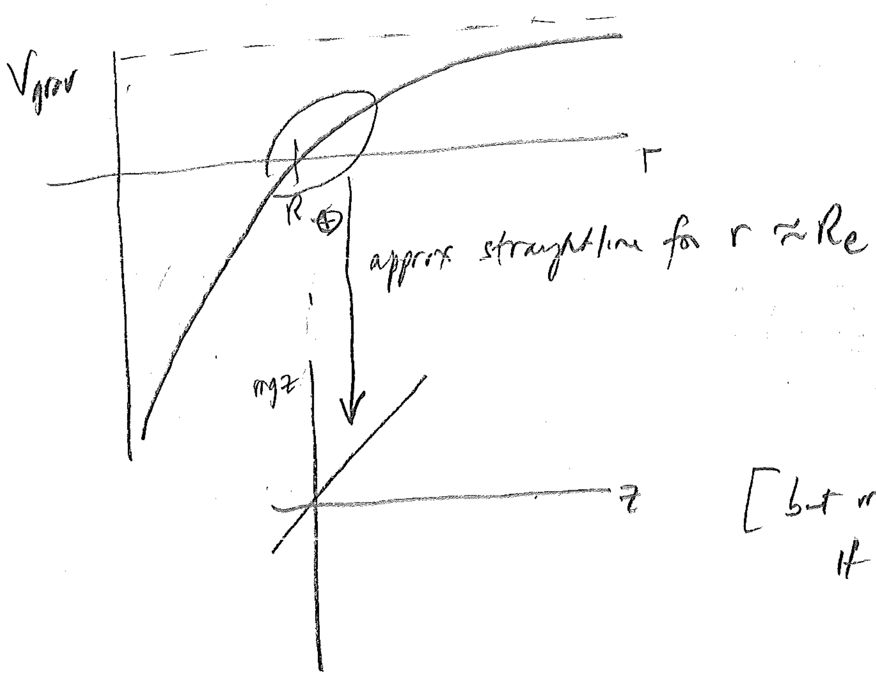
If instead we want reference separation to be at earth's surface, so $V_{grav}(R_\oplus) = 0$ then

$$V_{grav} = -\frac{G M_\oplus m}{r} + \frac{G M_\oplus m}{R_\oplus}$$



How does $V_{grav} = -\frac{GM_\oplus m}{r} + \frac{GM_\oplus m}{R_\oplus}$ valid near earth's surface?

Compare w/ $V_{grav} = mgz$ where $z = r - R_E$



[but mgz gives an overestimate if z is too large]

Slopes of the two potential energies must be equal at $r = R_\oplus$

$$\frac{dV_{grav}}{dr} = +\frac{GM_\oplus m}{r^2} \qquad \frac{d}{dz}(mgz) = mg$$

$$\frac{GM_\oplus}{R_\oplus^2} m = mg$$

$$\Rightarrow g = \frac{GM_\oplus}{R_\oplus^2} = \frac{(6.67E-11)(5.98E24)}{(6.38E6)^2} = 9.8 \ m/s^2$$

grav: [C7.T6]

Spring: [C7.T8]
scale reading

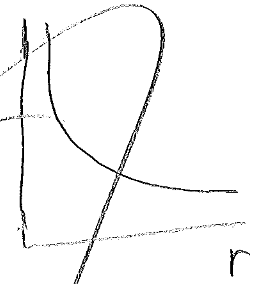
[C7.T4]

others:

errs in mg2? [C7.T5]

[didn't get to in S'10]

electrostatic interaction



$$V_e = \frac{k q_1 q_2}{r}$$

q_1 q_2

$k = 8.99 \times 10^9 \; J \frac{m}{C^2}$

q = charge in coulombs (electron has 1.6×10^{-19} C)

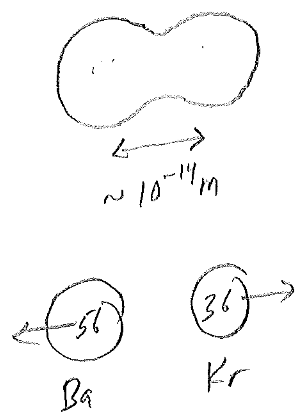
reference point is at ∞.

Liquid drop model of fission

uranium Z = 92

repulsion almost outweighs attraction

U



$0 = \Delta K + \Delta V_e$

$= (K_f - K_i) + \left(\frac{k q_1 q_2}{r_f} - \frac{k q_1 q_2}{r_i}\right)$

$K_i \to 0$, $\frac{k q_1 q_2}{r_f} \to 0$

$$K_f = \frac{k q_1 q_2}{r_i} = \frac{(9E9)(56)(36)(1.6E{-19})^2}{(10^{-14} m)}$$

$\approx 5 \times 10^{-11}$ J

≈ 300 MeV

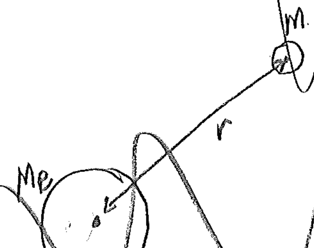
↓

~ energy released in nuclear fission

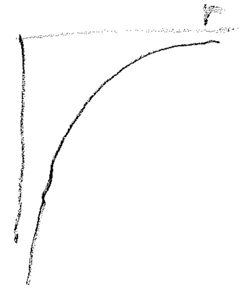
[C17.4]

r, d m₂ (pt masses) → Newton showed it (sphere)

Gravitational potential not near the surface of the earth



$$V_g = -\frac{G M_e m}{r}$$



G = Newton's const = $6.67 \times 10^{-11} \frac{J \cdot m}{kg^2}$

(Consider 1.0 kg object in a plane at 35 km ⇒ is this near earth's surface?] calculate:

Q: Let's compare w/ formula $V = mgz$

What does Newton's formula give for object at earth's surface?

$$V = -\left(\frac{G M_e}{R_e}\right) m \quad \text{vs.} \quad V = 0$$

Problem? No, ~~[illegible]~~ only differences matter ...

~~only changes matter~~

$$\Delta V_g = (1.00)(35{,}000)(9.8) \approx 35{,}000$$ (9.80)

$= 343{,}000$ J

⇒ [C7.B.4] compare Newton w/ approx

$m = 1.00$ kg

$z = 35.0$ km

$R_\oplus = 6380$ km

$M = 5.98 \times 10^{24}$ kg

625 181 80 J
12,770 80 J
341,100 J

$$\Delta V = \left(-\frac{G M_e}{R_e + z} m\right) - \left(-\frac{G M_e}{R_e}\right) m = G M_e m \left(-\frac{1}{R_e + z} + \frac{1}{R_e}\right)$$

$$= \frac{G M_e m(-R_e + R_e + z)}{R_e(R_e + z)} = \frac{G M_e \, m z}{R_e (R_e + z)}$$

← estimate error

$$\approx \left(\frac{G M_e}{R_e^2}\right) m z$$

$\sim 9.8 \frac{m}{s^2}$ so $g = \frac{G M_e}{R_e^2}$ & $V \approx mgz$

about ½%

[which is more accurate?]

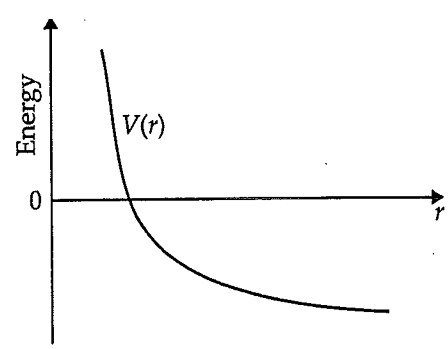
Stop in slides [C7.T6] [C7.T5] F10

C7T.1 The electrostatic potential energy between the proton and electron in a normal hydrogen atom (with the conventional choice of reference separation) is
A. negative
B. zero
C. positive
D. any of the above

C7T.2 Water molecules weakly attract one another (this is why water molecules coalesce into a liquid and then a solid at decreasing temperatures). What do you think is the interaction responsible for this attraction?
A. Incompletely canceled strong interactions between the molecule's quarks.
B. Incompletely canceled electromagnetic interactions between the molecule's charged parts.
C. The weak interaction.
D. The gravitational interaction.

C7T.3 The general gravitational potential energy formula $V(r) = -Gm_1m_2/r$ is always negative for $r > 0$, but the empirical formula $V(r) = mgz$ is always positive for $z > 0$. Why are the signs in these expressions different?
A. The empirical formula is wrong.
B. The equations refer to different kinds of interactions.
C. The first equation does not apply to objects that are not point particles.
D. The equations assume different reference separations.
E. Other (specify).

C7T.4 The following graph shows the potential energy function of a certain interaction. This interaction is
A. always attractive.
B. always repulsive.
C. attractive for small r, repulsive for large r.
D. repulsive for small r, attractive for large r.
E. there is not enough information for a meaningful answer.



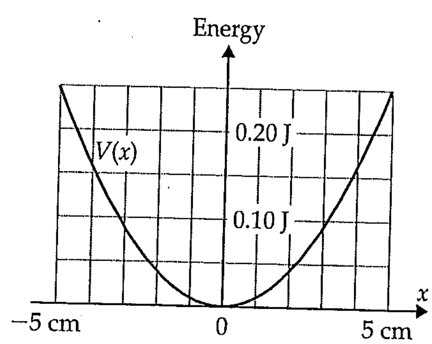
C7T.5 Assuming that $z = 0$ at the earth's surface, at roughly what value of z is the potential energy equation $V(r) = mgz$ (with $g = 9.80$ m/s) wrong by about 1%?
A. $z = 1$ km
B. $z = 15$ km
C. $z = 60$ km
D. $z = 250$ km
E. The equation is never in error: it is exact.

C7T.6 On a planet with twice the mass and twice the diameter of the earth, the value of g at its surface would be what factor times g at the surface of the earth?
A. 2 times larger
B. 4 times larger
C. 8 times larger
D. 2 times smaller
E. 4 times smaller
F. 8 times smaller
T. Other (specify)

C7T.7 When an atom in a crystal is moved 0.02 nm to the right or left, its potential energy of interaction with the crystal increases by 4×10^{-18} J. If the spring potential energy function is a good model for the potential energy of this interaction, what is the effective value of k_s?
A. 10^{-14} J/m^2
B. 2×10^{-14} J/m^2
C. 10,000 J/m^2
D. 20,000 J/m^2
E. Other (specify)

C7T.8 The potential energy function for a spring is shown in the graph to the right. What is the approximate spring constant for this spring?
A. 0.05 J/m^2
B. 5 J/m^2
C. 50 J/m^2
D. 100 J/m^2
E. 200 J/m^2
F. Other (specify)

$\frac{0.25\,J}{\frac{1}{2}(.05)^2} = 2\left(\frac{.25}{.0025}\right) = 200$



In the following figure, a 10-kg weight is suspended from the ceiling by a spring. The weight-spring system is at equilibrium with the bottom of the weight about 1 m above the floor. The spring is then stretched until the weight is just above the eggs. When the spring is released, the weight is pulled up by the contracting spring and then falls back down under the influence of gravity. On the way down, it

1. reverses its direction of travel well above the eggs.
2. reverses its direction of travel precisely as it reaches the eggs.
3. makes a mess as it crashes into the eggs.