Energy conservation

Energy of motion = kinetic energy K

[C6.S1]

$$K = \frac{1}{2} m v^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$$

scalar quantity

$$K^{sys} = \sum \frac{1}{2} m_j v_j^2$$

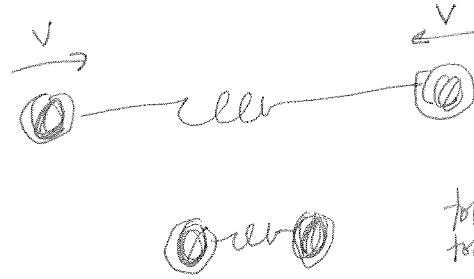
[Q] 50/50

Is K^{sys} of an isolated system conserved, like $\vec{p}^{sys}$?

No!

[DEMO?]

Initial:



final:

total momentum conserved
total kinetic energy is not

[Q: where does energy go?]

C6S.1 A 1000-kg car travels down a road at 25 m/s (55 mi/h). What is its kinetic energy? Now imagine that the car's speed increases to 35 m/s (77 mi/h), which is 40% faster. Is the kinetic energy 40% larger or not? (Note that the severity of a crash is roughly proportional to the kinetic energy that participants bring to it.)

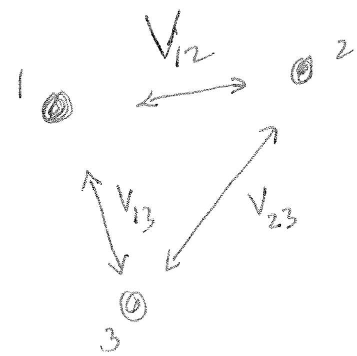
C6S.2 Imagine that if you drop an object from a certain height, its final speed is 20 m/s when it reaches the ground. If you throw the object vertically downward from the same height with an initial speed of 20 m/s, will its final speed be 40 m/s? Carefully explain why or why not.

"Stored energy" = potential energy V

We can associate potential energy with certain interactions, called "conservative" (such as spring, gravity, electrostatic)

$$E^{sys} = K_1 + K_2 + V_{12}^{spr}$$

Since an interaction involves a pair of objects, the potential energy is ~~also~~ associated w/ pairs of objects



Define mechanical energy of a system as sum of kinetic & potential energies

$$E^{sys} = \sum_{\substack{\text{objects} \\ j}} K_j + \sum_{\substack{\text{pairs of} \\ \text{objects } j,k}} V_{jk}$$

The mechanical energy of an isolated system in which only conservative forces act is conserved.

$$E_i^{sys} = E_f^{sys}$$

or $\Delta E^{sys} = 0$

[piece of chalk: is it isolated?
functionally isolated if I am holding it.
Not isolated if I drop it.]

[If gravity is involved, need to include earth in the system]

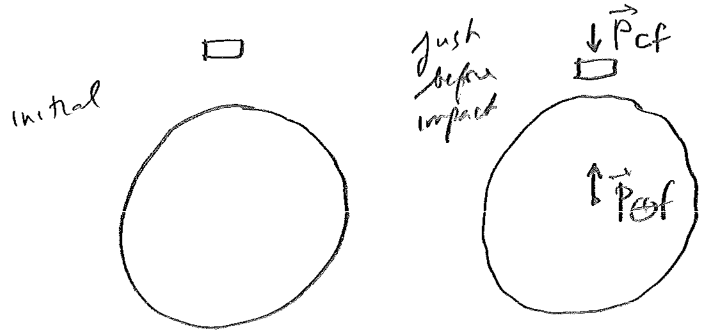
$$E^{sys} = K_c + K_\oplus + V^{grav}_{c\oplus}$$

$$= \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_\oplus v_\oplus^2 + V^{grav}$$

[Q: which is typically larger; K_c or $K_\oplus$
ⓐ K_c
ⓑ $K_\oplus$
ⓒ same] split 3 ways

Drop chalk from rest: $\vec{p}^{\,sys}_c = ?$

[answer = 0]



[Q: which is larger, p_{cf} or $p_{\oplus f}$?
(a) p_{cf}
(b) $p_{\oplus f}$
(c) same]

$$\vec{p}^{\,sys}_f = 0 \quad \Rightarrow \quad \vec{p}_{\oplus f} = -\vec{p}_{cf}$$

$$m_\oplus \vec{v}_{\oplus f} = -m_c \vec{v}_{cf}$$

$$\vec{v}_{\oplus f} = -\left(\frac{m_c}{m_\oplus}\right)\vec{v}_{cf}$$

$m_c = 10\,g$
$m_\oplus = 10^{28}\,g$

$$\frac{m_c}{m_\oplus} = 10^{-27}$$

[Now what about $K_{\oplus}$ and K_c ?]

$$K_{cf} = \frac{p_{cf}^2}{2m_c}, \quad K_{\oplus f} = \frac{p_{\oplus f}^2}{2m_{\oplus}} = \frac{p_{cf}^2}{2m_{\oplus}} \ll K_{cf}$$

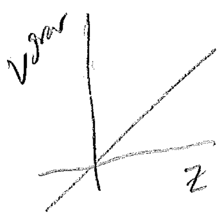
If one object in a system is much more massive than the others, its kinetic energy is very small and can be neglected

[in CM frame!]
non rotating

$$E^{sys} \approx K_c + V^{grav}$$

For objects close to the surface of earth,
gravitational potential energy is approximately

$$V^{grav} = mgz$$

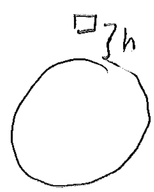


m = mass of object
$g = 9.8\ m/s^2$
z = height of object above earth

[Q: if earth is involved, why doesn't mass of earth affect this?]

[You prob. call this "potential energy of the object" (so do I), but it is energy of interaction bw object and earth]

What is speed of chalk when it hits the ground if dropped from height h?



$$E_f^{sys} = E_i^{sys}$$

$$K_{cf} + V_f^{grav} = K_{ci} + V_i^{grav} \quad \text{(ignore } K_{\oplus}\text{)}$$

$$K_{cf} - K_{ci} = -(V_f^g - V_i^g)$$

$$\Delta K = -\Delta V^g$$

$$\tfrac{1}{2}mv_f^2 - 0 = -mg(z_f - z_i) = -mg(0-h)$$

$$v_f^2 = 2gh$$

$$v_f = \pm\sqrt{2gh}$$

note: choice of origin doesn't affect $z_f - z_i$

(important)

maybe at beginning of next class → [Q: C6.T3] A, C, D

[Q: Two marbles]$_2$.

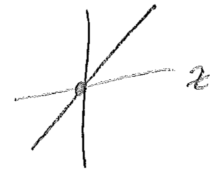
Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

1. as much kinetic energy as the lighter one.
2. twice as much kinetic energy as the lighter one.
3. half as much kinetic energy as the lighter one.
4. four times as much kinetic energy as the lighter one.
5. impossible to determine

[C6.T7]^A

[Since E^{sys} conserved, can choose opportune moment to compute it. Top of trajectory.]

V can be positive or negative



[Can also choose to measure potential energy from bottom of well) but must do so both before or after

[C6.T7 modified]^C

[$E^{sys} > 0$ because $K_i > 0, V_i = 0$ or $K_f = 0, V_f > 0$]

The location that we choose V to be zero (reference pos'n) is arbitrary & does not affect the answer

only the difference of potential energy is physically meaningful

$E_f^{sys} = E_i^{sys} \Rightarrow \Delta E = 0$

$\Delta K = -\Delta V^{grav} = -mg(z_f - z_i)$

[C6.T5]^D

[C6.B4] 6 meters

[Recall: near earth's surface, $V_{grav} = mgz$.]

[C6.T7]^A (results A, B, + some C)

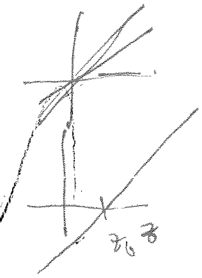
(Since E^{sys} conserved, choose an opportune moment to compute it.
Top of trajectory $\Rightarrow$ $K = 0$, $V = mgz < 0$ $\Rightarrow$ $E^{sys} < 0$)

(NB: while $K \geq 0$, V can be negative)

Reference position = position where V is defined to be zero

eg $V = mgz$ $\Rightarrow$ ref pos is $z = 0$
$V = mg(z - z_0)$ if z_0 = reference position

(Choose ref. position to be bottom of well)



[C6.T7 modified]^C

($E > 0$ because $K_i > 0$, $V_i = 0$ or $K_i = 0$, $V_f > 0$)

[C6.T5]^D

(Either $V = 0$ at $z = -5m$, or $-mgz_0 = 4(10)(5) = 200$)

V can be either mgz or $mgz + C$

$E_i = \frac{1}{2}mv_i^2 + mgz_i + C = E_f = \frac{1}{2}mv_f^2 + mgz_f + C$

(Can shift energy by constant on both sides.)

only changes in energy are physically significant

(if need more [C6.B4] 6 meters)

C6T.1 Which has the larger kinetic energy, a 50-kg person running at a speed of 2 m/s or a 5-g nickel falling at a speed of 200 m/s?

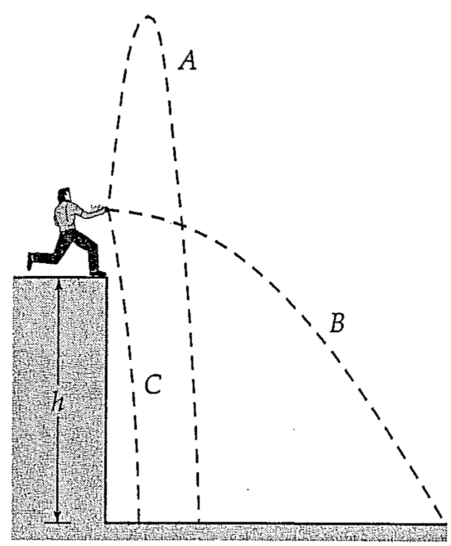
A. The running person.
B. The falling nickel.
C. Both have the same kinetic energy.

C6T.2 How does the kinetic energy K_{fast} of a car traveling at 50 mi/h compare with the kinetic energy K_{slow} of an identical car traveling at 25 mi/h? (The consequences of a collision are in rough proportion to the energy involved!)

A. Both cars are identical, so $K_{fast} = K_{slow}$.
B. $K_{fast} \approx 1.5K_{slow}$.
C. $K_{fast} \approx 2K_{slow}$.
D. $K_{fast} \approx 4K_{slow}$.
E. Other (explain).

do C6S.1 instead

C6T.3 A person throws three identical rocks off a cliff of height h with exactly the same speed v_0 each time (see the drawing). Rock A is thrown almost vertically upward, rock B is thrown horizontally, and rock C is thrown almost vertically downward. Which rock hits the ground with the greatest *speed?* (Ignore air friction.)



A. Rock A.
B. Rock B.
C. Rock C.
D. All rocks hit with the same speed.
E. Other (specify).

C6T.4 Imagine that we know from experiments that when the object moves from point A to point B, the potential energy of its gravitational interaction with the earth *increases* by 24 J, and that when the object moves from point B to point C, the potential energy of its system *decreases* by 18 J. If we define the system's reference separation to be when the object is at point C, what is the value of the system's potential energy when the object is at point A?

A. +6 J
B. −6 J
C. −52 J
D. +52 J
E. 0
F. Other (specify)

C6T.5 In a coordinate system where the z axis is vertical, we choose the gravitational potential energy of a 4-kg rock interacting with the earth to be zero when $z = -5$ m. The formula for the potential energy as a function of z is thus $V(z) = mgz + C$. What is the (approximate) value of C?

A. −50 J
B. +50 J
C. −200 J
D. +200 J
E. 0
F. Other (specify)

C6T.6 There is no way to *experimentally* determine the actual value of a system's total energy, true (T) or false (F)?

C6T.7 Consider a rock interacting gravitationally with the earth. Imagine that we define the interaction's potential energy to be zero if the rock is at ground level. A person standing at the bottom of a well throws the rock vertically upward from 20 m below ground level. The rock makes it all the way up to 1 m below ground level before falling back into the well. The total energy of the rock-earth system is

A. Negative.
B. Zero.
C. Positive (in this particular case).
D. Positive because energy is *always* positive.
E. The answer depends on the rock's mass.
F. The answer depends on the rock's initial speed.
T. The answer depends on something else (specify).