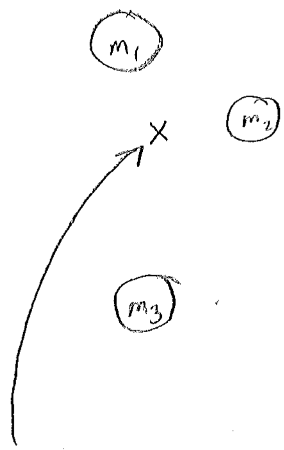
System of objects

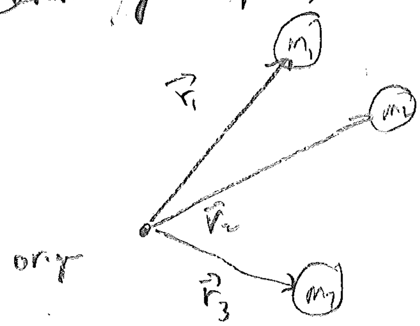


Total mass $M = m_1 + m_2 + m_3 + \dots$

$$= \sum_{j=\text{objects}} m_j$$

[j not i = initial]

Center of mass of system = weighted average of positions. [How define?]



$$\vec{r}_{cm} = \left(\frac{m_1}{M}\right)\vec{r}_1 + \left(\frac{m_2}{M}\right)\vec{r}_2 + \dots$$

$$= \frac{\sum m_j \vec{r}_j}{\sum m_j}$$

[DEMO: earth/moon: where is cm?]

Trick: find cm by balancing it.

Trick find cm of meter stick by moving fingers in

Try putting a weight on one end.

[C4.T3]$_B$

What is velocity of cm?

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{d}{dt}\left(\frac{\sum m_j \vec{r}_j}{M}\right) = \frac{1}{M}\sum m_j \frac{d\vec{r}_j}{dt} = \frac{1}{M}\sum m_j \vec{v}_j = \frac{\sum \vec{p}_j}{M}$$

Total momentum of a system

$$\vec{p}^{\,sys} = \sum \vec{p}_j$$

Therefore

$$\vec{p}^{\,sys} = M\vec{v}_{cm}$$

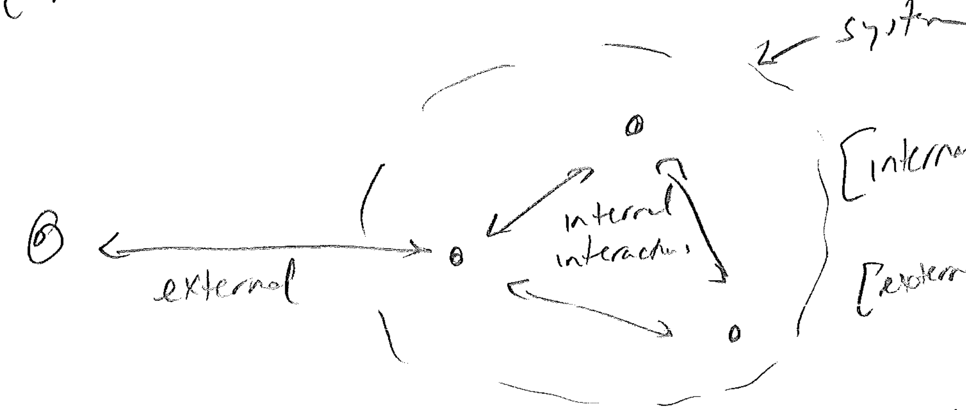
[treat system as if all mass were concentrated at cm & moving w/ $\vec{v}_{cm}$]

Conservation of momentum

The total momentum of an isolated system does not change in time

(because the effects of internal interactions cancel out)

[what do we mean by isolated + internal]



[internal: between objects in the system]

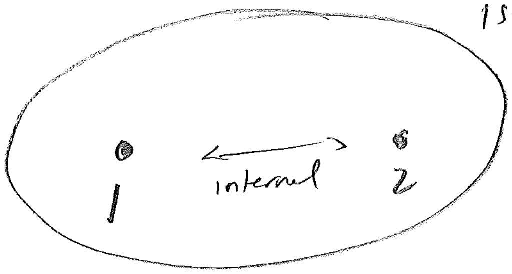
[external: w/ objects outside sys]

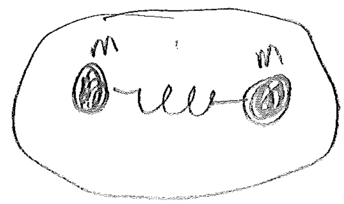
An Isolated system is one w/ no (net) external interactions

[internal interactions xfer momentum around the system, but the total momentum remains unchanged]

[if my wife takes $20 from my wallet, or I take $20 from her purse, household net worth doesn't change]

Isolated system





$$\vec{p}^{\,sys} = \vec{p}_1 + \vec{p}_2$$

Change in momentum of the system

$$d\vec{p}^{\,sys} = d\vec{p}_1 + d\vec{p}_2$$

$$= [d\vec{p}]^{1(2)}_{spr} + \underbrace{[d\vec{p}]^{2(1)}_{spr}}_{-[d\vec{p}]^{1(2)}_{spr}}$$

$$= 0$$

$$\Rightarrow \vec{p}^{\,sys} = \text{const}$$

[Q: I am standing on earth. $\vec{p}^{\,sys}_i = 0$
I jump up $|\vec{p}_I| \neq 0$ but $|\vec{p}_{earth}| = 0$?
Why isn't momentum conserved?] $\vec{p}^{\,sys}_f = 0$
$= \vec{p}_\oplus + \vec{p}_{me}$

$$\vec{p}_\oplus = -\vec{p}_{me}$$

$$M_\oplus \vec{v}_\oplus = -M_{me}\vec{v}_{me}$$

$$\vec{v}_\oplus = -\left(\frac{m_{me}}{m_\oplus}\right)\vec{v}_{me} \ll \vec{v}_{me}$$

Recall $\vec{p}^{\,sys} = M\vec{v}_{cm}$

Since $\vec{p}^{\,sys}$ is conserved, $\vec{v}_{cm}$ for an isolated sys
is constant (Newton's 1st law applied to sys)

[Youtube MIT Push me Pull you: 2 minutes]

If an isolated system is initially at rest ($\vec{v}_{cm}$)
it remains at rest.

ie $\vec{r}_{cm}$ remains fixed in space.

[DEMO: earth/moon: spin it: cm remains fixed]

[discovery of binaries]
↓
Youtube?

Non Isolated System

$$\frac{d\vec{p}^{\,sys}}{dt} = \sum_j \frac{d\vec{p}_j}{dt} = \sum_j \sum_{ext} \left[\frac{d\vec{p}_j}{dt}\right]_{ext}$$ because internal force cancel

$$= \sum_j \sum_{ext} \vec{F}_{j,ext}$$ $\vec{F}_{j,ext}$ = external force on j

change in mom of system is given by

$$= \vec{F}^{\,sys}_{ext} \Rightarrow \vec{F}^{\,sys}_{ext}$$ = vector sum of external force on all objects in the system

$$\vec{F}^{\,sys}_{ext} = \frac{d\vec{p}^{\,sys}}{dt} = \frac{d}{dt}(M\vec{v}_{cm}) = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm}$$

(Newton's 2nd law for system)

The motion of the <u>cm</u> of a non isolated object is determined by the <u>external</u> forces acting on the object

[DEMO: toss ]

[Q4, T5]

Background

Momentum of systems

$$d\vec{p}^{\,sys} = \sum_j d\vec{p}_j$$

$$= \sum_j \sum_A [d\vec{p}_j]_A$$

internal interactions between pairs

$$= \underbrace{\sum_j \sum_{ext} [d\vec{p}_j]_{ext}}_{\vec{F}^{\,sys}_{ext}\,dt} + \underbrace{\sum_j \sum_{k \neq j} [d\vec{p}_{j(k)}]}_{\sum_{j<k} \left(\underbrace{[d\vec{p}_{j(k)}] + [d\vec{p}_{k(j)}]}_{0} \right)}$$

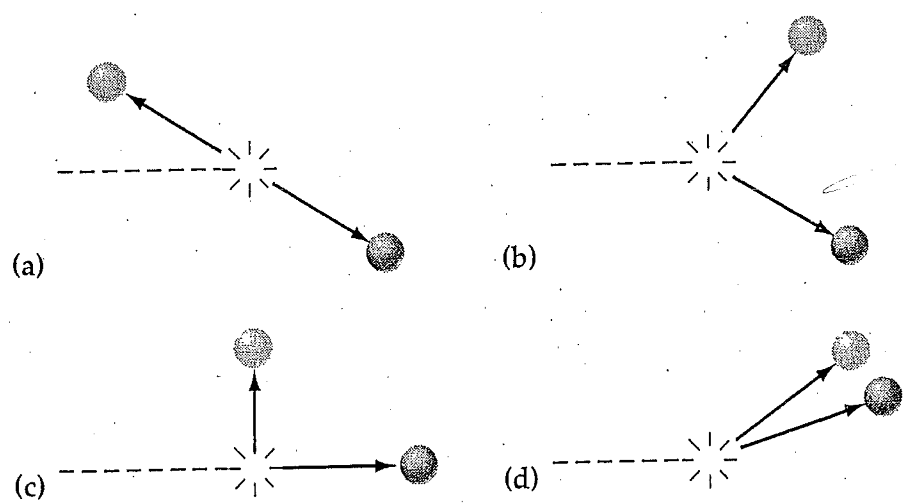
$$\boxed{\frac{d\vec{p}^{\,sys}}{dt} = \vec{F}^{\,sys}_{ext}}$$

Isolated $\Rightarrow$ $\vec{p}^{\,sys} = \text{constant}$

C4T.1 Which of the following things qualifies as being an "extended object"? For each object, answer T if it qualifies, F if it does not, and D if it is debatable.

a. An electron.
b. An atom.
c. A rock.
d. A human being.
e. A swarm of bees.
f. The earth's atmosphere.
g. The water in an ocean current.
h. A cluster of galaxies.

C4T.2 The following diagrams show hypothetical results for collisions between two identical balls floating in space. The white ball was initially moving to the right along the dotted line before it hit the gray ball, which was initially at rest. (The collision is not necessarily head-on.) The arrows depict the balls' final velocities. Which outcomes are physically possible? For each case, answer T if it is, F if not.



C4T.3 Consider a system consisting of two particles, one with 3 times the mass of the other. If the distance between the particles is 1.0 m, the system's center of mass is what distance from the lighter object?

A. $\approx$ 1.0 m
B. 0.75 m
C. 0.66 m
D. 0.33 m
E. 0.25 m
F. Other (specify)

C4T.4 In example C4.1, if we had modeled the atoms as solid spheres instead of point nuclei, we would have gotten the same result for the center of mass, true (T) or false (F)?

C4T.5 In a reference frame fixed on the sun, which do you think follows the straighter path?

A. The earth.
B. The moon.
C. The center of mass of the earth-moon system.

C4T.6 Which of the following would be an appropriate inertial reference frame for analyzing the effects on the earth of the gravitational interaction between the earth and the moon? Answer T if this frame would be okay, F if not.

a. A frame attached to the earth's surface.
b. A nonrotating frame attached to the earth's CM.
c. A nonrotating frame attached to the CM of the earth-moon system.
d. A nonrotating frame attached to the sun's CM.
e. A nonrotating frame attached to the solar system's CM.