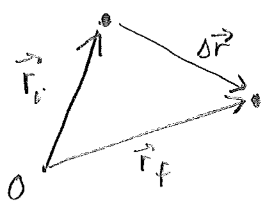
$\boxed{C3}$



Air track

displacement: $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$




<u>Average velocity</u> of a particle between $\vec{r}_i$ and $\vec{r}_f$ is $\dfrac{\text{displacement}}{\text{time interval}}$

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \qquad \Delta t = t_f - t_i$$

$\dfrac{\text{vector}}{\text{scalar}} = \text{vector}$ [graphically, dividing by scalar changes length not direction]


$$\vec{V}_{avg} = \begin{pmatrix} \frac{\Delta x}{\Delta t} \\ \frac{\Delta y}{\Delta t} \\ \frac{\Delta z}{\Delta t} \end{pmatrix} \qquad \begin{aligned} \Delta x &= x_f - x_i \\ \Delta y &= y_f - y_i \\ \Delta z &= z_f - z_i \end{aligned}$$

[Note to self: $|\vec{V}_{avg}| \neq$ avg speed. eg particle in UCM]
↳ problem

$$\Delta\vec{r} = \vec{V}_{avg}\,\Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{V}_{avg}\,\Delta t$$

[While avg velocity between two times might be
e.g. 60 mph, eg driving 150 miles from Brunswick to Boston in 2.5 hrs,
actual velocity will vary (traffic stoppages etc.).
How determine actual velocity?]

## Instantaneous velocity

$\vec{V} = \frac{d\vec{r}}{dt}$ where $dt$ is an interval sufficiently short that velocity doesn't change during it

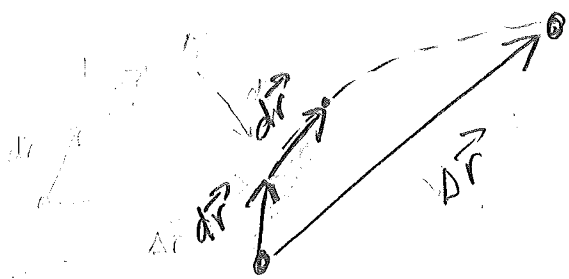
[may seem slightly circular, but keep taking $\Delta t$ smaller until $\vec{v}$ doesn't change (limiting process)]

[Recall $d\vec{r}$ = "small" displacement]

omit → [Speed

$$V = |\vec{v}| = \frac{|d\vec{r}|}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$ ]

[Observe: not only can instantaneous speed vary, so can its direction]



[DEMO: unequal mass carts w/ magnets on air track? Connected w/ rubber band & then burn it w/ a match]

[Recall] net force changes an object's velocity

$$\boxed{2m} \leftarrow \vec{F}_{mag} \qquad \vec{F}_{mag} \rightarrow \boxed{m}$$

[Q: if same forces act on each, why different velocities? A: $m_1 \neq m_2$]

net force is proportional to change in an object's momentum

["quantity of motion" acc. Newton]

momentum $\vec{p} = m\vec{v}$

change in momentum $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

"small" change $d\vec{p}$

## Interactions transfer momentum between pairs of objects

Define $[d\vec{p}]_A$ as "momentum transfer" or "impulse" by an interaction A

$[d\vec{p}]_A = \vec{F}_A \, dt$ = amt of momentum transferred to an object by a force $\vec{F}_A$ acting for a time interval $dt$

$\vec{F}_A = \dfrac{[d\vec{p}]_A}{dt}$ force = rate at which momentum is transferred

[units: $p$ : $\frac{kg\,m}{s}$ , $F$ : $\frac{kg\,m}{s^2} = 1\,N$ ]

$$\overset{1}{\circ} \xleftrightarrow{\text{interaction } A} \overset{2}{\circ}$$

Newton's 3rd law $\quad \vec{F}_A^{\,1(2)} = -\vec{F}_A^{\,2(1)}$

$[d\vec{p}]_A^{1(2)}$ = momentum transferred to 1 from 2 via interaction A

$$= \vec{F}_A^{\,1(2)}\, dt$$

$$= -\vec{F}_A^{\,2(1)}\, dt$$

$$= -[d\vec{p}]_A^{2(1)}$$

[mom xferred from 2 to 1 is equal & opp to mom xferred fr 1 to 2]

[Momentum is like currency. If I give you \$20, I lose \$20.
You are \$20 richer. I am \$20 poorer. :)
Currency doesn't have x & y components though.]

The change in momentum of an object $d\vec{p}$ is vector sum of momentum transfers $[d\vec{p}]_A$ by all interactions A

$$d\vec{p} = \sum_{A=\text{interaction}} [d\vec{p}]_A$$

(Financial: if you all give me \$5, the transfer is an interaction. The change in my net worth is like $d\vec{p}$. If I pay my bill, $[d\vec{p}]_A$ my checking acct ↓ $d\vec{p}$.)

$$= \sum_A \vec{F}_A \, dt$$

$$= \underbrace{\left(\sum_A \vec{F}_A\right)}_{\vec{F}_{net}} dt$$

$$d\vec{p} = \vec{F}_{net} \, dt$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$ : Newton's 2nd law $\left[\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}\right]$

[C3.T2]$_E$

[C3.T3]$_T$ (talk about the other interaction)

[C3.T4]$_A$

[C3.T5]$_T$

be sure to get to this ⟹ [C3.T7]$_T$ about 50-50

[C3.T9]

Tennis ball serve:

$M_{ball} = 57\ g$

$v_f = 100\ mph = 45\ m/s$

$\Delta p = 2.5\ \frac{kg\,m}{s} = [d\vec{p}] = \vec{F}\,\Delta t$

$\Delta t = 5\ ms$

$\Rightarrow \vec{F}_{serve} = 500\,N \qquad \vec{F}_{grav} = 0.5\,N$

C3T.1 "The car rounded the corner at a constant velocity." Would this statement make sense to a physicist?

A. No, the word *velocity* is being used incorrectly.
B. No, a car has to slow down to turn a corner.
C. It could make sense or not, depending on the corner.
D. Yes, this statement is acceptable.

C3T.2 Imagine that a 1.0-kg cart traveling rightward at 1.0 m/s hits a 3.0-kg cart at rest. Afterward, the smaller cart is observed to move leftward with a speed of 0.75 m/s. What impulse did the collision give the smaller cart at the expense of the larger?

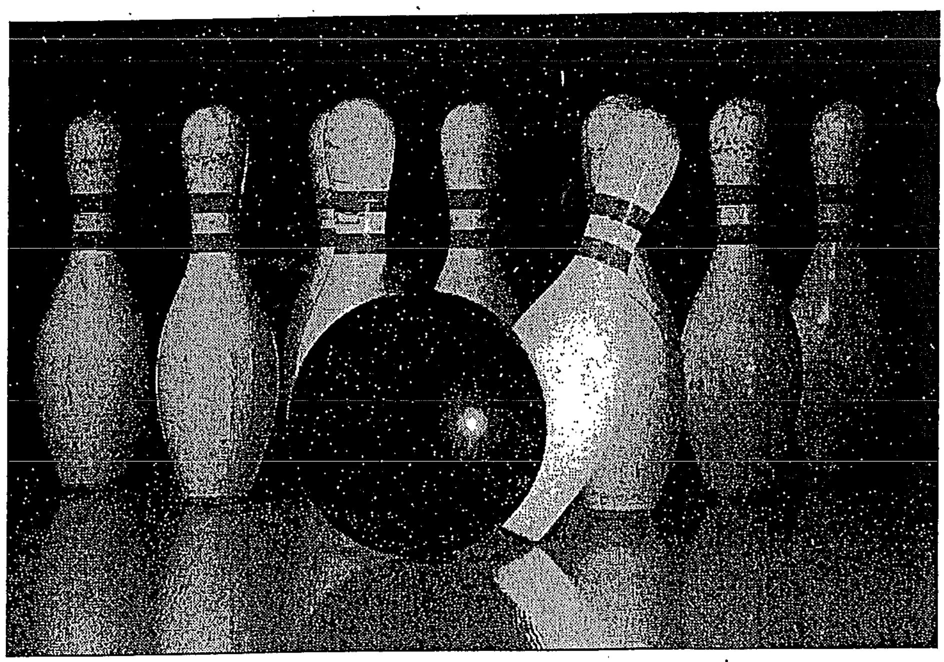
A. None; the larger cart was at rest and so had no momentum to give.
B. None; the lighter cart gave an impulse to the more massive cart, not the other way round.
C. 0.75 kg·m/s leftward.
D. 1.00 kg·m/s leftward.
E. 1.75 kg·m/s leftward.
F. Other (specify).
T. Both A and B are correct.

C3T.3 Imagine that a moving cart (cart *A*) hits an identical cart (cart *B*) at rest. Cart *B* remains at rest, and cart *A* rebounds with a speed equal to its original speed. Cart *B* must have participated in some other interaction during the collision process, true (T) or false (F)?

C3T.4 Imagine that two identical carts traveling toward each other at the same speed collide ~~and come to rest~~. According to the momentum-transfer principle, if one of the carts is observed to be at rest after the collision, the other

A. Must be at rest also.
B. Must rebound backward with its original speed.
C. Must rebound backward with twice its original speed.
D. Must continue forward with twice its original speed.
E. Does none of the above. This process violates the momentum-transfer principle!
F. Other (specify).

C3T.5 An 8.0-kg bowling ball hits a 1.2-kg bowling pin. The force that the contact interaction exerts on the pin has the same magnitude that it exerts on the ball, T or F?



C3T.6 It is possible for a human being to have a weight of 150 kg, T or F?

C3T.7 A cup sitting on a table constantly receives upward momentum from the table, T or F?

C3T.8 A particle is launched horizontally with an initial speed of 5 m/s and subsequently interacts *only* gravitationally with the earth. According to the three-reservoir model, the horizontal component of the particle's velocity after a few seconds has passed is
A. Somewhat greater than 5 m/s.
B. Essentially equal to 5 m/s.
C. Somewhat less than 5 m/s.
D. 0.
E. Other (specify).

C3T.9 Which of the following statements involving vectors are correct? Answer T if it is correct and F if it is not (be prepared to identify the error if you answer F).
a. $\vec{p}_{\text{tot}} = \vec{p}_1 + \vec{p}_2$ implies that $p_{\text{tot}} = p_1 + p_2$.
b. $\vec{p} = m\vec{v}$ implies that $m = \vec{p}/\vec{v}$.
c. If $\vec{v} = [0, -5.0\text{ m/s}, 0]$, then $\vec{v}_y = -5.0\text{ m/s}$.
d. If $\vec{v} = [0, -5.0\text{ m/s}, 0]$, then $v = +5.0\text{ m/s}$.
e. If $\vec{v} = 5.0\text{ m/s}$ and $m = 2.0\text{ kg}$, then $\vec{p} = 10\text{ kg}\cdot\text{m/s}$.

(1-15-2014)

Impulse

tennis serve: Dale says 70 - 140 mph

avg schlep → 70; worldchampion → 140

$$m_{ball} = 57g$$

$$v_f = 100 \text{ mph} = 45 \text{ m/s}$$

$$\Delta p = 2.5 \ \frac{\text{kg m}}{\text{s}}$$

$$\Delta t = 5 \text{ ms empirically}$$

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$$F_{serve} = 500N$$

$$F_{grav} = 0.5N$$

Drop a ball on a racket

HMC osc

$$-kx$$

$$\Delta t = \pi\sqrt{\frac{m}{k}}$$

$$\left(\frac{1}{2}kA^2 = \frac{1}{2}mv^2\right)$$

$$\int F\,dt = \int_{t=0}^{t=\pi/\omega} kx(t)\,dt = \int_0 kA\sin(\omega t)\,dt = \frac{2kA}{\omega} = 2\frac{k}{\omega}\sqrt{\frac{m}{k}}\,v = 2mv = \Delta p$$

← So $\Delta t$ & $\langle F \rangle$ depend on stiffness of string

$$\langle F \rangle = \frac{2mv}{\pi\sqrt{\frac{m}{k}}} = \frac{2v}{\pi}\sqrt{mk}$$

Higher string tension gives more control (less angular error) but also means the ball rather than strings will deform, making the collision less elastic & the ~~p~~ final speed less

# Physics of the tennis racket

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Several parameters concerning the performance of tennis rackets are examined both theoretically and experimentally. Information is obtained about the location of the center of percussion, the time a ball spends in contact with the strings, the period of oscillation of a tennis racket, and the coefficient of restitution of a tennis ball. From these data it may be possible to design a racket with improved playing characteristics.

The physics of the tennis racket is a subject that very few people paid attention to until Head produced his oversized Prince racket. In fact, in reading various tennis magazines, looking at advertisements for tennis rackets, and talking to tennis players, tennis professionals, or sports equipment sales people one gets such conflicting statements that one realizes that no one seems to understand the physics of the tennis racket.

When a tennis racket is examined, one notes that the basic shape and size have not changed in over half a century. It is probable that these parameters were not determined solely by playing characteristics but also by structural considerations imposed by the strength of wood. When metal rackets became popular a few years ago (and composite fiberglass, boron, or carbon filament recently) the manufacturers initially copied the general shape and size of the wooden rackets, since they probably assumed that they were optimum—or that any radical change might not sell. The same might be said for the strings—where gut has successfully withstood the challenge of all the modern synthetic materials for tournament play.

There seems to be no information in the published physics or tennis literature about the optimization of size, shape, weight, etc., of a tennis racket. Since everyone learned to play with essentially the same type of racket, any radical change would feel wrong and require the player to relearn to some degree. Under these conditions the design of rackets might evolve and improve slowly—but there is no way to know if a racket that was radically different might not prove better if a player originally learned with it.

One obvious change was the introduction of the Prince racket with an oversized head. Head, its inventor, was very careful to produce a racket of the same overall length, weight, and balance as a conventional racket so that it would "feel" the same as a normal racket. To prove this, any average player can pick one up and play with it immediately. However to play with it in an optimum manner does take some retraining since it was designed to give its best performance with the ball striking it 5 or 6 cm closer to the handle than on a normal racket. To understand the advantages of the Prince racket (without playing with it) some simple kinematics must be investigated.

Consider a tennis racket of mass $M$ suspended freely in space. If a ball strikes it and imparts a momentum $+\Delta p$ to the racket, the center of mass of the racket will move with a velocity $V = +\Delta p/M$.

If the ball hits the racket at the racket center of mass, the racket will translate but not rotate. If the ball hits the racket at a distance $b$ from the center of mass, the racket will rotate around the center of mass as well as translate. From conservation of angular momentum, the angular velocity of the racket about the c.m. will be $\omega = \Delta pb/I_{\text{c.m.}}$ where $I_{\text{c.m.}}$ is the moment of inertia of the racket about its center of mass. Assume the racket has been struck by a ball, the racket (c.m.) is translating to the right with velocity $V$ and the racket is rotating clockwise about the c.m. with angular velocity $\omega$ [Fig. 1(a)]. There then will be one point in the racket which is instantaneously at rest, if the racket handle is long enough. If this point is a distance a from the center of mass then the condition for that point to have no velocity is $V = \omega\, a$. In other words, the motion due to the rotation exactly cancels the overall translation (of the racket) at that one point. Then

$$a = \frac{V}{\omega} = \frac{\Delta p/M}{\Delta pb/I_{\text{c.m.}}} = \frac{I_{\text{c.m.}}}{b\,M}.$$

The value of a is independent of how hard the ball hits the racket ($\Delta p$) as it should be. Then $ab = I_{\text{c.m.}}/M = k^2$ (radius of gyration squared).

If a ball hits a racket at a distance $b$ from the c.m. (at point $B$) and the racket is held a distance a from the c.m. (at point $A$), no force or impulse from the hand need be imparted to the racket, since, in a frame of reference initially moving with the racket, the point $A$ remains at rest. In the frame of reference with the racket handle being swung with velocity $V'$, the point $A$ continues to move with velocity $V'$ with no external force applied to it.

If the racket is held at point $A$ then point $B$ is called the center of percussion and it is of some interest to determine the distance $a+b$. This distance $a+b$ can be obtained with a simple experiment that uses the racket as a physical pendulum. If the racket is allowed to swing freely about the point $B$, the frequency of the oscillation (for small amplitude) can be calculated and also measured.

In Fig. 1(b), the restoring torque is $-Mg\,b\sin\theta \sim -Mg\,b\,\theta$ (for small $\theta$). Then $\tau = I_B\,(d^2\theta/dt^2)$, where $I_B$ is the moment of inertia about $B$. Using the parallel axis theorem, $I_B = I_{\text{cm}} + Mb^2$ and substituting this into the $\tau = I\alpha$ equation

$$-Mg\,b\,\theta = (I_{\text{cm}} + Mb^2)\,\frac{d^2\theta}{dt^2}.$$

The solution to this equation is a simple harmonic motion with angular frequency $\omega = [Mg\,b/(I_{\text{cm}} + Mb^2)]^{1/2}$ but $I_{\text{c.m.}} = Mk^2$, so $\omega = [gb/(k^2+b^2)]^{1/2}$. However, since $k^2 = ab$, $\omega = [\mathrm{gb}/(\mathrm{ab}+\mathrm{b}^2)]^{1/2} = [\mathrm{g}/(\mathrm{a}+\mathrm{b})]^{1/2}$. Conse-

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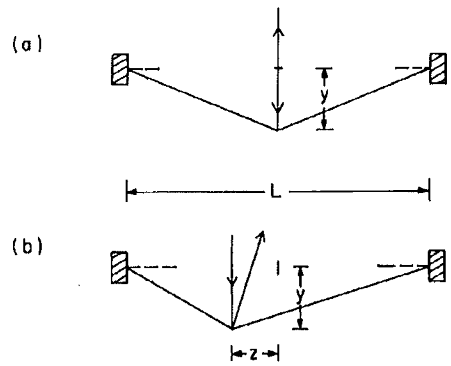



Fig. 2. String deflection when ball hits (a) at center of strings and (b) closer to one side of racket.

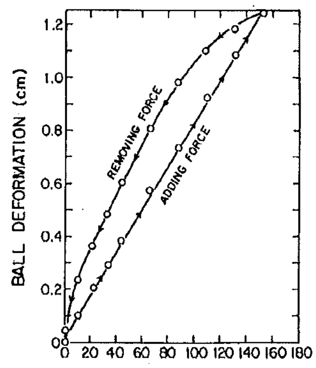



Fig. 4. Deformation of a tennis ball as a function of the applied force. The ball was placed in a rigid hemispherical cup so that only one side deformed.

a doubling of the moment of inertia. It is also possible to increase the diameter of the racket handle so that the torque tending to prevent twisting is increased without increasing the forces applied by hand.

Even if the ball is hit off center and the racket twists a little, there is another effect that will tend to compensate for it. If the ball hits off center, the deflection of the strings is asymmetric. Neglecting spin of the ball, gravity, etc., in the rest frame of the racket, it is expected that relative to the plane of the strings the angle of the ball's rebound will be equal to the angle of its incidence. This will be true only if the strings deflect symmetrically [Fig. 2(a)] which occurs for hits in the center. For off center hits, the asymmetrical string deflection tends to deflect the ball toward the center [Fig. 2(b)]. Head was able to demonstrate this using a high-speed motion picture camera with the racket held in a vise. The data he obtained allowed him to determine the angular error of the rebounding ball as a function of position and map out contours of angular error regions with error less than 10°, between 10° and 20° and greater than 20°.[1]

This angular error depends upon several factors. If a string of length $L$ deflects (perpendicular to the plane of the strings) by a distance $y$ when hit by the ball and the ball misses the center by a distance $z$, the angular error will be proportional to $(z/L)$ $(y/L)$. There is not much a physicist can say about reducing $z$, hence reducing the error, but $y$ and $L$ are subject to analysis. The maximum value of the string deflection can be obtained if the effective spring constant of the strings is known (the slope in Fig. 5), the ball momentum specified and the assumption that the motion is simple harmonic is made. Then $\int F\,dt$ over ½ cycle equals $\Delta mv$. Since $F = -ky$ and $y = A \sin \omega t$, the amplitude $A$ of the deflection can be obtained: $A = (\Delta v/2)(m/k)^{1/2}$, where $m$ is the mass of the ball. (The mass of the strings is neglected.) The value of $k$ is a linear function of $T/L$, the string tension divided by the string length and also depends upon the effective number of strings that deflect. When all of this is put together, the angular error $\Delta\theta = (Cz\Delta v/L)(m/LT)^{1/2}$, where $C$ is a constant.

It is then quite clear that increasing the size of the racket head (increasing $L$) reduces this error, hence increases the effective area of the racket within which this error is tolerable. It is also clear that an increase in string tension will reduce this error somewhat.

Another interesting result is that this error is proportional to $\Delta v$, the change in the ball velocity. Consequently, when the ball is hit hard, unless it is hit dead center on the racket, it may not go exactly where it is aimed. This compounds the difficulty of hard hits, which, due to their kinematics, already have very little margin for error.

The string tension also influences how long a ball spends in contact with the strings (dwell time) and the velocity at which the ball leaves the racket.[2] To optimize these parameters various measurements have to be made, including a determination of some of the properties of tennis balls.

The most surprising thing is that it appears that stringing the racket looser rather than tighter will actually lead to slightly higher ball rebound velocities (more "power"). This is due to the fact that tennis balls have a rather low coefficient of restitution and a dwell time on the strings which is short when compared to half of the natural period of vi-

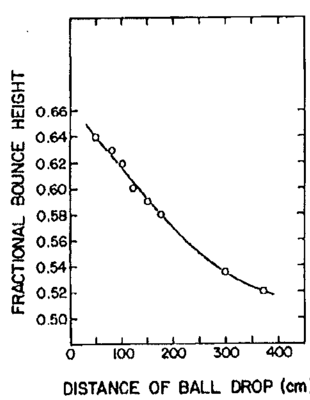



Fig. 3. Ratio of rebound height to drop distance as a function of drop distance. Ball was dropped onto a hard surface.

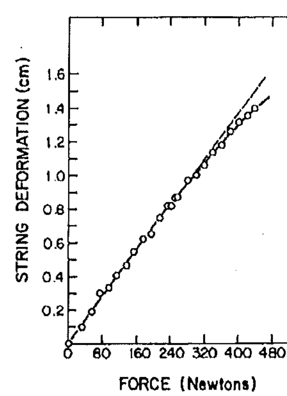



Fig. 5. String deformation as a function of applied force. The racket head was braced and the force was applied over a circular area of 12 cm$^2$.

of a particular point on the racket is a function of its distance from that point. Moving the impact point several inches toward the body should reduce the velocity of the racket at the impact point by less than 10%, so this effect can probably be ignored. (It may be somewhat more significant in the serve when the wrist is snapped—which makes the pivot to impact distance considerably shorter.)

The length of time that a ball spends in contact with the strings of the racket can be determined by direct measurement or it can be approximately calculated from static measurements made on the ball and strings. If the string-ball system is treated as a simple harmonic oscillator, then the dwell time of the ball on the strings will be half the natural oscillation period of the system. The effect "$k$" (spring constant) of the strings is a function of the stringing tension, gauge and type of string, size of the racket head, etc. This k was determined for the strings of several different rackets by measuring the delection of the strings as a function of applied force. Typical measured values ranged from 2 to $3.5 \times 10^4$ N/m (the slope of Fig. 5). A tennis ball has a mass of 0.060 kg, leading to a natural frequency of oscillation of $100h$ [$f = \omega/2\pi = (1/2\pi)(k/M)^{1/2}$] and a dwell time of $4.5 \times 10^{-3}$ sec.

The direct measurement of dwell time can be accomplished in a number of ways, i.e., by using high-speed motion picture photography (greater than 1000 frames/sec), by taking a series of single flash pictures of a repetitive event—each picture at a slightly different electronic delay or by a method using a laser, photodetector, and an oscilloscope that I have developed.

The apparatus for the laser method is shown in Fig. 6. The ball is placed on the racket strings and the laser is adjusted in position so that its beam is parallel to the rebounding surface and is exactly one ball diameter above it (part of the beam is intercepted by the ball at rest). A photodetector (silicon solar cell) is positioned in the beam beyond the ball and the output of the detector fed to an oscilloscope on internal trigger. The ball is then removed.

The ball is then dropped onto or propelled at the rebounding surface. The ball passing through the laser beam blocks the light from the detector and triggers the scope sweep. The light will again hit the detector when the ball has passed completely through the beam. However, due to the position of the laser and rebounding surface, this will only occur while the ball is in contact with the surface (strings). As the ball leaves the surface, it again intercepts the laser beam and cuts off the light to the detector. The resulting scope trace shows a period of time with no light (ball moving toward surface), a period of time with light (ball in contact) and a second period of no light (ball rebounding). This method not only gives the dwell time, but, since the ball diameter is known, it gives the velocity of incidence and velocity after rebounding—hence the coefficient of restitution. If the ball does not cut through the laser beam at a diameter (the center of the ball misses the beam by a distance $\Delta x$) then there will be an error in the timing due to the fact that the beam will be cut by a chord rather than a diameter. The chord, being shorter than the diameter by a distance $4(\Delta x)^2/D$ will cutoff the light for less time and will also increase the apparent dwell time correspondingly. This timing error will be $4(\Delta x)^2/vD$ where $v$ is the velocity of the ball and $D$ its diameter. The fractional error in the time the light is cut off is then $4(\Delta x)^2/D$ and the fractional error in the dwell time ($t$) will be $4(\Delta x)^2/Dvt$.

Table II. Coefficients of restitution.[a]

| Surface | Ratio of rebound height to drop height | COR | COR from velocity measurements |
|---|---|---|---|
| Lead brick | 0.520 | 0.721 | 0.767 |
| Prince racket (70-lb tension) | 0.716 | 0.846 | 0.887 |
| Prince racket (50-lb tension) | 0.730 | 0.854 | 0.901 |
| Spalding Smasher (unknown tension) | 0.726 | 0.852 | 0.895 |

[a]These data were taken with the ball dropped from a height of 3.7 m above the rebounding surface. It is the average of data taken with several types of tennis balls (pressureless Tretorn, Penn ball, and Spalding Australian ball). The COR values obtained by direct velocity measurements (laser method) are consistently higher than the rebound measurements by about 0.05. This is due to the air resistance which reduces the kinetic energy of the ball during both its fall and rise, and therefore give lower values for the COR. For these data the racket head was clamped.

For $\Delta x$ of order 1 cm, $D = 13.2$ cm, $v = 800$ cm/sec and $t = 5 \times 10^{-3}$ sec the time errors are 2% and 7.6%, respectively. To reduce this error, a pair of plane mirrors was set up and the laser beam multiply reflected before hitting the detector. The spacing between adjacent beams was of order 1 cm which leads to a maximum spatial error $\Delta x = 0.5$ cm and a maximum error in dwell time of less than 2%.

A second error in dwell time is present if the laser beam is not exactly one ball diameter above the rebounding surface. If the placement error is $\Delta y$, then the dwell time will be in error by $2\,\Delta y/v$. An error $\Delta y$ of 2 mm then leads to an error in timing of $0.5 \times 10^{-3}$ sec, so an effort was made to have the beam within 1 mm of the ball top. The results of these measurements with several different rebounding surfaces are shown in Table III.

The predicted dwell time of $5 \times 10^{-3}$ sec seems to be confirmed by these measurements. The tennis rackets used were those that were readily available. It is clear that these measurements should be repeated using a number of identical rackets, each strung with different material and a variety of tensions.

The tennis racket frame also has a natural period of vibration which is determined by the mass and mass distribution of the frame, the elasticity or stiffness and the damping of the oscillations. The parameter of interest is

Table III. Typical dwell times (msec).[a]

| Rebounding surface | Penn | Type of ball Spalding Australian | Tretorn (pressureless) |
|---|---|---|---|
| Lead brick | 3.9 | 4.5 | 4.5 |
| Prince (70 lb) | 6.1 | 6.3 | 6.3 |
| Prince (50 lb) | 6.5 | 6.7 | 6.4 |
| Spalding Smasher | 6.4 | 6.8 | 6.6 |

[a]All balls were dropped from a height of 3.7 m. Their measured velocity at impact (8.3 m/sec) was slightly lower than the theoretical value (8.5 m/sec) because of air resistance. One Prince racket was strung with a synthetic at 70-lb tension, the other Price with gut at 50-lb tension and the Spalding Smasher with synthetic at an unknown tension. The racket head was clamped for these data.

Impulse is $m\Delta v$

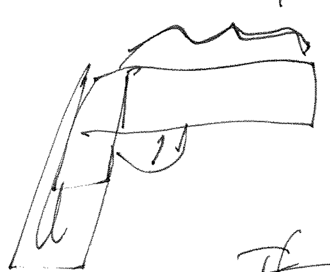
What is force required? $F = m\frac{\Delta v}{\Delta t}$

handgun

What is time?

10 cm

$= 0.1\,m$



~~If bullet leaves at $v_f$~~

If force is const $\left.\begin{array}{l} x_f = \frac{1}{2}at^2 \\ v_f = at \end{array}\right\}$ $t = \frac{2x_f}{v_f}$

$$\therefore\ F = m\frac{v_f}{(2x_f/v_f)} = \frac{mv_f^2}{2x_f}$$

$m = 2g$

$v_f = 200\,m/s$

$$\frac{(0.002\,kg)(200\,m/s)^2}{2(0.1\,m)} =$$

$\Delta p = 0.4\,\frac{kg\,m}{s}$

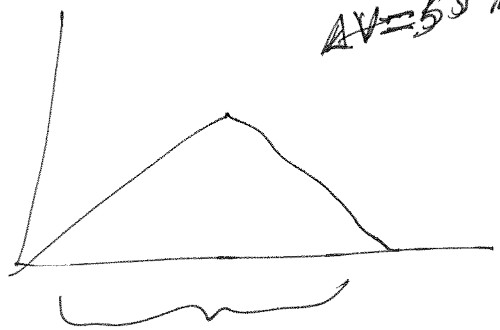
$\Delta t = \frac{2(0.1\,m)}{(200\,m/s)} = \boxed{1\,ms}$

$F = 400\,N$

auto accidents

$F_{compression}$

$\Delta V = 55$ mph



150 ms

[from Dale's technical manual]