Chapter 2: vectors

[mention typos in ch 3: $\frac{5}{3} \to \frac{4}{3}$ in 3 places on p. 49]

[we've met 2 examples of vectors: $\vec{F}$ and $\vec{v}$]

(Moore C2.1) vector: direction and magnitude

[length prop. to magnitude]

$\vec{u}$

$u = |\vec{u}| = \text{mag}(\vec{u})$

(Moore C2.2) [we've already talked about adding vectors using tail-to-head rule]

change in velocity $= \Delta\vec{V} = \vec{V}_f - \vec{V}_i$

[Q: An object ...]

[to subtract: reverse $\vec{V}_i$ and add]

An object initially has a velocity of 3 m/s due west. After interacting with something else for a while the object ends up with a velocity of 4 m/s due south. What is the direction of the change in velocity?

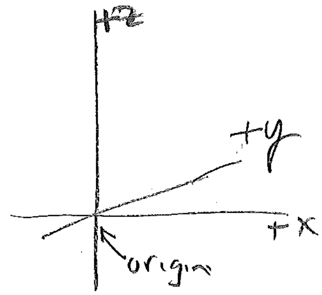
(a) roughly northeast

(b) roughly northwest

(c) roughly southwest

(d) roughly southeast

(move c2-8)



Axes all $\perp$

Right handed coordinate system

[r.h. rule]

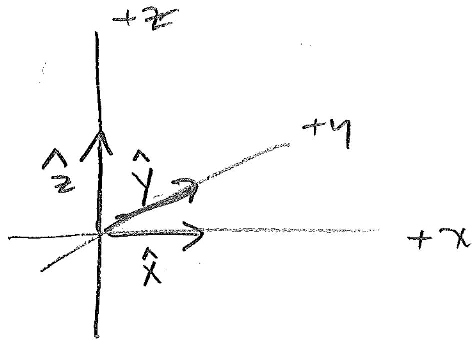
[C2. T3]$_F$

[C2. T5]$_D$

[you should know from reading] Standard orientation: $\begin{cases} +x & \text{East} \\ +y & \text{North} \\ +z & \text{up} \end{cases}$

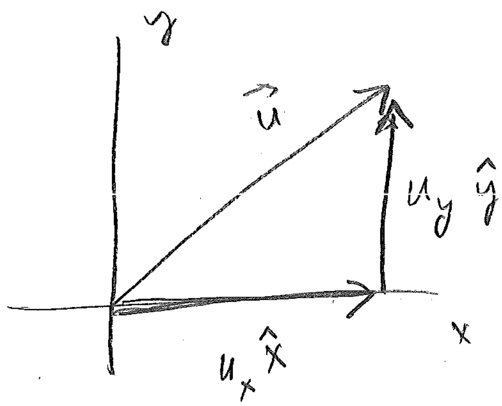
(Moore C2.3) <u>Unit vectors</u> (vectors w/ magnitude = 1)

Denoted by $\hat{u}$ $\qquad |\hat{u}| = 1$



$\hat{x}, \hat{y}, \hat{z}$ point along coordinate axes

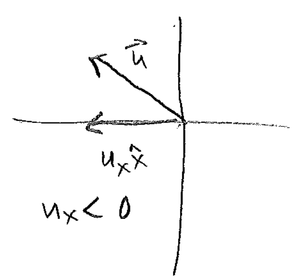
(or $\hat{\imath}, \hat{\jmath}, \hat{k}$)



$$\vec{u} = u_x \hat{x} + u_y \hat{y}$$

u_x, u_y called components of $\hat{u}$

[can be positive or negative]



(Represent vectors as columns or rows of numbers)

$$\hat{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \hat{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Then $\vec{u} = u_x \hat{x} + u_y \hat{y} + u_z \hat{z}$

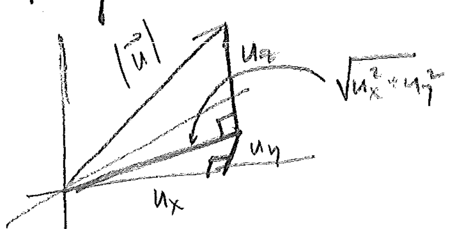
$$= \begin{pmatrix} u_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ u_y \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ u_z \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \text{ or } (u_x, u_y, u_z)$$

Adding vectors using components

$$\vec{u} + \vec{w} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} + \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} = \begin{pmatrix} u_x + w_x \\ u_y + w_y \\ u_z + w_z \end{pmatrix}$$

$$2\vec{u} = 2 \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} 2u_x \\ 2u_y \\ 2u_z \end{pmatrix}$$

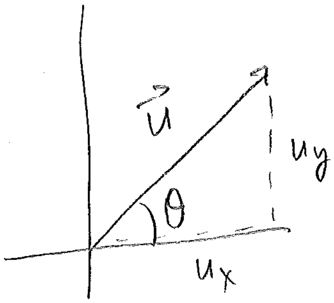
Magnitude using components



$\Rightarrow$

$$|\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

Direction using components

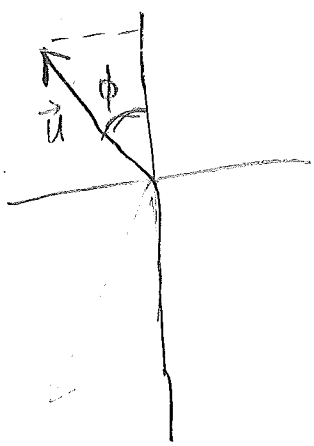


$$\sin\theta = \frac{O}{H} = \frac{u_y}{u}$$

$$\cos\theta = \frac{A}{H} = \frac{u_x}{u}$$

$$\tan\theta = \frac{O}{A} = \frac{u_y}{u_x}$$

specify θ north of east



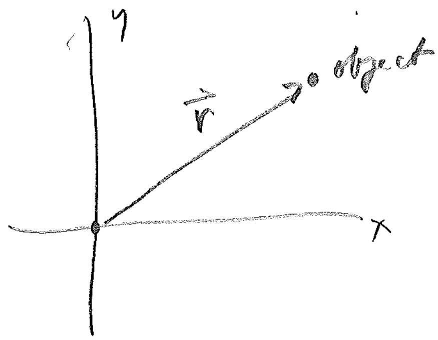
$$u_x = -u\cos\phi$$

$$u_y = +u\sin\phi$$

Not for class!!

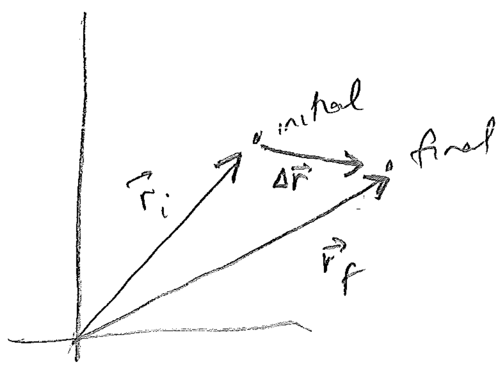
Saddle our horses canter away happily toward the adventure
Some old hippie caught another hippie tripping on acid
Sex on holiday can add delights to our adventures!

Position vector $\vec{r}$ (from origin to object)



$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Displacement $\Delta\vec{r}$ = final position relative to initial position



$\boxed{\text{C2.T6}}$ D

$$\vec{r}_i + \Delta\vec{r} = \vec{r}_f$$

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = \text{change in position}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \begin{pmatrix} x_f - x_i \\ y_f - y_i \\ z_f - z_i \end{pmatrix}$$

$\boxed{\text{Now redo C2.T6}}$

"Small" displacement $d\vec{r}$

[optional past this point]

optional C2-7

[C2.T7]$_F$

[C2.T8]$_T$

Do first

Let $\vec{s} = \vec{u} + \vec{w}$

[C2.T12] then $s = u + w$ (T/F)?

Could also do [C2.T9]

[C2.T10]

[C2.T11]

challenging [C2.T13]

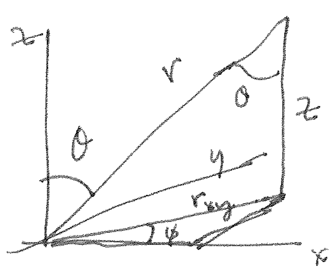
F101

Go over C2 S.5 [Hw for prev class, on which most ppl said $v_f = \sqrt{2}$, not $\frac{1}{\sqrt{2}}$]

Go over C3X.3
C3X.7] ← Lawrence della Bitta had asked abt these

Amy Finkelstein asked how you would specify angles if vector had x, y, + z components

↓ so I explain



$\cos\theta = \frac{z}{r}$

$\tan\phi = \frac{y}{x}$

$r = \sqrt{r_{xy}^2 + z^2}$

$r_{xy} = \sqrt{x^2 + y^2}$

$\Rightarrow r = \sqrt{x^2 + y^2 + z^2}$

C2T.3 The x, y, and z directions of a reference frame point up, west, and north, respectively. Such a coordinate system is right-handed, T or F?

C2T.4 A reference frame drawn on a sheet of paper has its y direction oriented toward the top of the sheet and its x direction toward its right edge. What direction must the z direction point if the frame is right-handed?

A. Outward, perpendicular to the plane of the paper.
B. Inward, perpendicular to the plane of the paper.
C. Diagonally to the lower left, in the plane of the paper.
D. Diagonally to the lower right, in the plane of the paper.
E. Vertically downward in the plane of the paper.
F. Other (specify).

C2T.5 If an object is located 3.0 m north and 1.0 m west of the origin of a frame in standard orientation on the earth's surface, its coordinates in that frame are

A. [3.0 m, 1.0 m, 0 m]
B. [−3.0 m, −1.0 m, 0 m]
C. [1.0 m, 3.0 m, 0 m]
D. [−1.0 m, 3.0 m, 0 m]
E. [−1.0 m, 0 m, −3.0 m]
F. Other (explain)

C2T.6 An object whose initial position coordinates are [1.5 m, 2.0 m, −4.2 m] in a frame in standard orientation is found a short time later at a position whose coordinates are [1.5 m, −3.0 m, −4.2 m]. What is the direction of this object's displacement during this time interval?

A. East.
B. West.
C. North.
D. South.
E. Down.
F. Other (specify).

C2T.7 The components u_x, u_y, and u_z of a certain vector $\vec{u}$ are *all* negative numbers. The vector's magnitude u is then certainly negative as well, T or F?

C2T.8 The *only* way that a vector can have zero magnitude is for *all* its components to be zero, T or F?

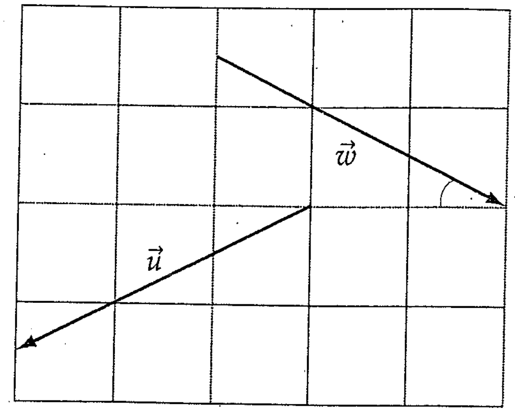


Figure C2.12
Vectors for problems C2T.9 through C2T.11.

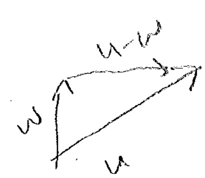
C2T.9 Consider the two vectors shown in figure C2.12. The sum of these vectors points most nearly
A. Up. B. Down. C. Right. D. Left.

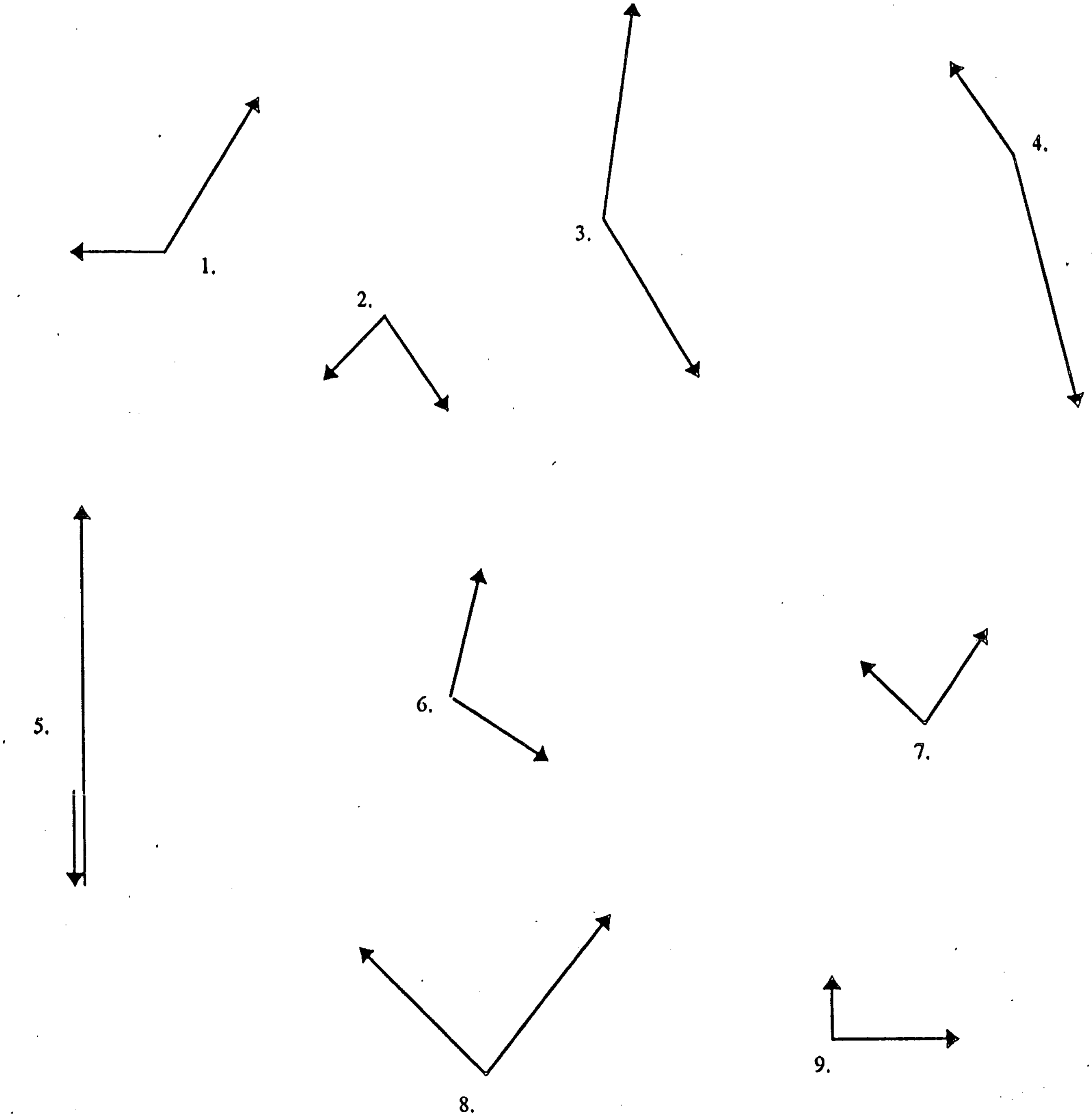
C2T.10 Consider the two vectors shown in figure C2.12. The vector $\vec{w} - \vec{u}$ points most nearly
A. Up. B. Down. C. Right. D. Left.

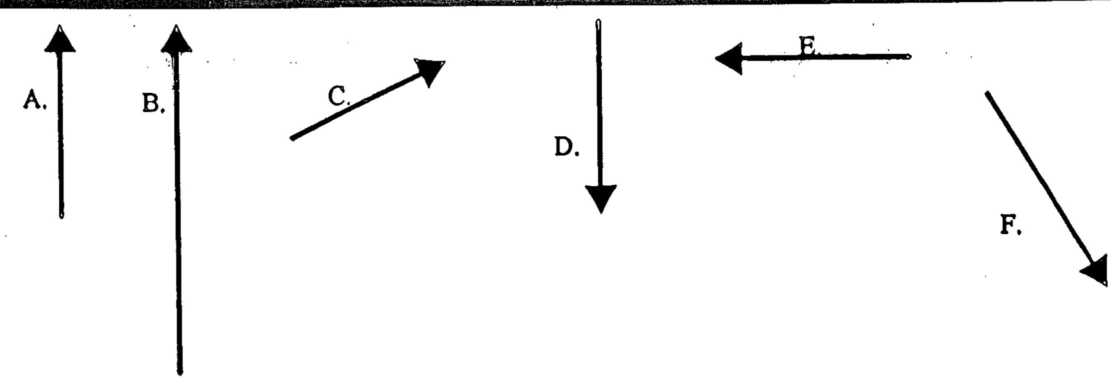
C2T.11 Consider the two vectors shown in figure C2.12. To change $\vec{u}$ into $\vec{w}$, one would have to
A. Multiply by −1.
B. Multiply by 120°.
C. Add the vector $\vec{u} + \vec{w}$.
D. Add the vector $\vec{w} - \vec{u}$.
E. Add the vector $\vec{u} - \vec{w}$.
F. Do none of the above.

C2T.12 For all possible orientations and lengths of $\vec{u}$ and $\vec{w}$, $\text{mag}(\vec{u} + \vec{w}) = \text{mag}(\vec{u}) + \text{mag}(\vec{w})$, true or false?

C2T.13 For all possible orientations and lengths of $\vec{u}$ and $\vec{w}$, $\text{mag}(\vec{u} - \vec{w}) \geq |\text{mag}(\vec{u}) - \text{mag}(\vec{w})|$, true or false?




1.
2.
3.
4.
5.
6.
7.
8.
9.


A.
B.
C.
D.
E.
F.