A Model of Strategic Behavior in Networks of Influence

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We propose *influence games*, a new class of graphical games, as a model of the behavior of large but finite networked populations. Grounded in non-cooperative game theory, we introduce a new approach to the study of influence in networks that captures the strategic aspects of complex interactions in the network. We study computational problems on influence games, including the identification of the most influential nodes. We characterize the computational complexity of various problems in influence games, propose several heuristics for the hard cases, and design approximation algorithms, with provable guarantees, for the influential nodes problem based on a connection we establish to the *minimum hitting set problem* [4].

To date, the study of influence in networks has concentrated mostly on analyzing the diffusion (or "contagion") processes induced by the influences in the network [1, 6, 7]. The notion of "influential nodes" considered in this paper is different, and is aimed at complementing the traditional line of work with a new game-theoretic perspective. Inspired by threshold models in social science [2], we define *influence games* as a class of graphical games [5] where each player corresponds to a node in a directed graph encoding"influence factors." The set of *actions* (or *pure strategies*) of each player is $\{1, -1\}$, for "adopt" or "not-adopt" a behavior, respectively. Each player also has an *influence function* f_i mapping each joint-action of the player's parents in the game graph to a real number, and a real-valued *tolerance threshold* b_i. Each player i's *payoff function* is defined such that, given a joint-action $\mathbf{x}_{\mathrm{Pa}(i)}$ of player i's parents $\mathrm{Pa}(i)$ in the graph, player i's best-response is 1 (respectively, -1) if $f_i(\mathbf{x}_{\mathrm{Pa}(i)})$ exceeds (is below) b_i; and *indifferent* if $f_i(\mathbf{x}_{\mathrm{Pa}(i)}) = b_i$. In the special class of *linear influence games* (LIGs), each f_i is a weighted *sum* of $\mathbf{x}_{\mathrm{Pa}(i)}$.

We define a set S of players in a game as most influential, *with respect to a specified pure strategy Nash equilibrium (PSNE)* $\mathbf{x}^*$, if the players in S to choosing actions according to $\mathbf{x}^*$ enforces all others to also choose actions according to $\mathbf{x}^*$. Said differently, the players in S are collectively so influential that they are able to restrict the choice of actions of every other player in a stable solution to a *unique* one. We further extend this definition by allowing for a preference function over all possible sets of the most influential nodes (e.g., a minimum-cardinality set). Departing from the contagion model and rather concentrating on the PSNE of the influence game, we capture significant, basic, and core *strategic* aspects of complex interaction in networks that naturally appear in many real-world problems (e.g., determining the most influential Senators in Congress).

We study two fundamental algorithmic questions in this setting—computing PSNE of influence games and finding the most influential set of nodes. We show that various versions of these problems (e.g., existence of a PSNE, uniqueness of PSNE, counting the number of PSNE even in star networks, etc.) are intractable, unless P = NP. Nevertheless, on the positive side, we show how to compute a PSNE of special types of influence games, such as the ones with non-negative influence factors and the ones having tree structures, in polynomial time. Furthermore, given the set of all PSNE H, we give a $(1 + \log |H|)$-factor approximation algorithm for the most influential nodes selection problem. We also illustrate the whole computational scheme empirically, using random influence games and influence games learned from the US Congress voting records using machine learning techniques [3].

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