A Model of Strategic Behavior in Networks of Influence (Extended Abstract⋆)

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Abstract. We present *influence games*, a class of non-cooperative games that we designed to model behavior resulting from influence among individual entities in large, networked populations such as social networks. While inspired by threshold models in sociology, our model is fundamentally different from models based on contagion or diffusion processes and concentrates on significant strategic aspects of networked populations. In this extended abstract, we focus on the existence and computation of pure strategy Nash equilibria (PSNE) in influence games. We show that influence games are, in fact, equivalent to 2-action polymatrix games modulo the set of pure strategy Nash equilibria (PSNE). One particular application of influence games is finding *the most influential individuals* in a network. We present an approximation algorithm, with provable guarantees, for this problem. We illustrate the overall computational scheme by studying questions such as, who are the most influential senators in the US Congress?

1 Influence Games

The study of influence in networks has so far concentrated mostly on analyzing the diffusion (or "contagion") processes induced by the influences in the network [1, 7, 8]. The notion of "influential nodes" considered in this paper is different, and is aimed at complementing the traditional line of work with a new game-theoretic perspective. Inspired by threshold models in social sciences [2], we define *influence games* as a class of graphical games [6] where each player corresponds to a node in a directed graph encoding"influence factors." The set of *actions* (or *pure strategies*) of each player is $\{1, -1\}$, for "adopt" or "not-adopt" a behavior, respectively. Each player also has an *influence function* f_i mapping each joint-action of the player's parents in the game graph to a real number, and a real-valued *tolerance threshold* b_i. Each player i's *payoff function* is defined such that, given a joint-action $\mathbf{x}_{\mathrm{Pa}(i)}$ of player i's parents $\mathrm{Pa}(i)$ in the graph, player i's best-response is 1 (respectively, -1) if $f_i(\mathbf{x}_{\mathrm{Pa}(i)})$ exceeds (is below) b_i; and *indifferent* if $f_i(\mathbf{x}_{\mathrm{Pa}(i)}) = b_i$. In the special class of *linear influence games*, each f_i is a weighted *sum* of $\mathbf{x}_{\mathrm{Pa}(i)}$. We show that any linear influence game is equivalent to a 2-action polymatrix game [4], modulo the set of PSNE.

2 Formulation of Most Influential Nodes Problem

We define a set S of players in a game as most influential, *with respect to a specified pure strategy Nash equilibrium (PSNE)* $\mathbf{x}^*$, if the players in S to choosing actions according to $\mathbf{x}^*$ enforces all

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others to also choose actions according to $\mathbf{x}^*$. Said differently, the players in S are collectively so influential that they are able to restrict the choice of actions of every other player in a stable solution to a *unique* one. We further extend this definition by allowing for a preference function over all possible sets of the most influential nodes (e.g., a minimum-cardinality set). Departing from the contagion model and rather concentrating on the PSNE of the influence game, we capture significant, basic, and core *strategic* aspects of complex interaction in networks that naturally appear in many real-world problems (e.g., determining the most influential Senators in Congress).

3 Hardness Results

We prove the following (proof omitted in this extended abstract):

1. It is NP-complete to decide the following questions in linear influence games:
   (a) Does there exist a PSNE?
   (b) Given a designated subset of k players, does there exist a PSNE consistent with those k players playing 1?
   (c) Given a number $k \geq 1$, does there exists a PSNE with at least k players playing 1?
2. Given a linear influence game and a number $k \geq 1$, it is co-NP-complete to decide if there exists a set of k players such that there exists a *unique* PSNE with those players playing 1.
3. It is #P-complete to count the number of PSNE, even if the graph of the linear influence game is a star.

4 Algorithmic Results

We study two fundamental algorithmic questions—computing PSNE of influence games and finding the most influential set of nodes. We show how to compute a PSNE of special types of influence games, such as the ones with non-negative influence factors and the ones having tree structures, in polynomial time, if there exists one. We also devise a practically effective heuristic, based on a divide-and-conquer approach with an intelligent backtracking search that incorporates the Nash-Prop algorithm [9], to enumerate all PSNE of influence games. Furthermore, given the set of all PSNE H, we give a $(1 + \log |H|)$-factor approximation algorithm for the most influential nodes selection problem, which is based on a connection we establish to the *minimum hitting set problem* [5].

5 Empirical Results

We illustrate the whole computational scheme empirically, using random influence games and influence games learned from the US Congress voting records using machine learning techniques [3]. For instance, the 101st senate influence game consists of 100 nodes, each representing a Senator, and 936 weighted arcs among these nodes. In these games, each node can play one of the two actions: 1 (yes vote) and -1 (no vote). First, we have applied our heuristic to find the set of all PSNE. We have obtained a total of 143,601 PSNE for the 101st senate game. Note that the number of PSNE in this game is extremely small relative to the maximum possible 2^{100}. Next, we have computed the most influential Senators using the approximation algorithm. We have obtained a solution of size five for the 101st Senate graph, which we have verified to be an optimal solution, indeed. The solution consists of Senator Rockefeller (Democrat, WV), Senator Sarbanes (Democrat, MD), Senator Thurmond (Republican, SC), Senator Symms (Republican, ID), and Senator Dole (Republican, KS).

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