Girvan-Newman algorithm for community detection

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Main idea:

- Calculate the betweenness of each edge.
- Successively delete the edge(s) with the highest betweenness (and recalculate betweenness).

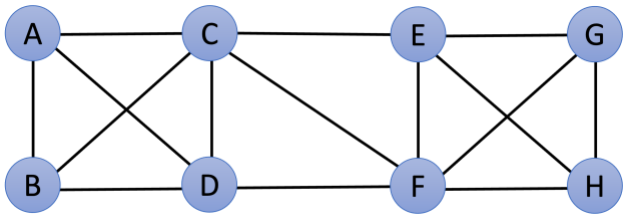
How to calculate the betweenness of an edge:

(Step 1) For <u>each node</u> A do the following:

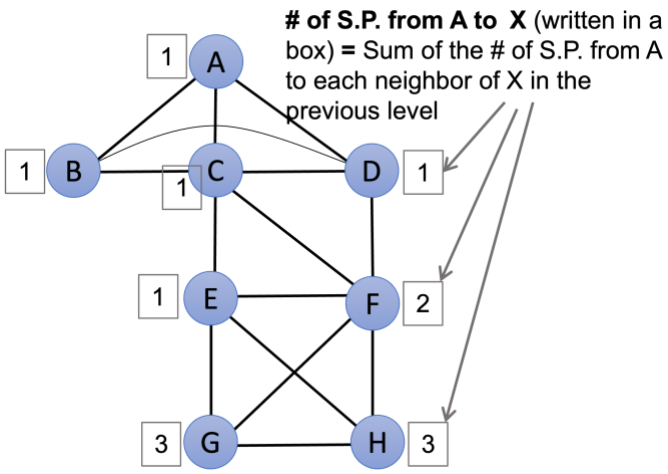
1. Do BFS starting with A.
2. Calculate the # of shortest paths (S.P.) from A to every other node.
   - # of S.P. from A to X = Sum of the # of S.P. from A to each neighbor of X in the previous level of BFS.
3. Calculate the quantity of water flow through each edge.
   - A sends 1 gal. to every other node X. Water from A to X gets evenly split across all S.P. from A to X.

(Step 2) Betweenness of an edge
= sum of all water flow through that edge (i.e., over all the BFS starting with each node).

Input network

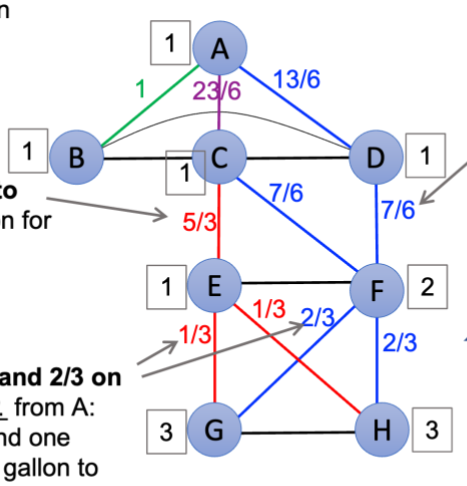


After BFS and #S.P. calculation, starting with A
(Note: showing all edges; it's not a BFS tree!)



# of S.P. from A to X (written in a box) **=** Sum of the # of S.P. from A to each neighbor of X in the previous level

Water flow calculation



Sanity check: Flow on AB + AC + AD = 1 + 23/6 + 13/6 = 7 gal. because there are 7 other nodes.

Why 5/3 gal. from C to E? E demands 1 gallon for itself plus 1/3 + 1/3 to send downstream. 1 + 1/3 + 1/3 = 5/3

Why 1/3 gal. on EG and 2/3 on FG? G has <u>three S.P.</u> from A: two S.P. through F and one through E. A sends 1 gallon to G. It's evenly split across the three S.P. to G.

Why 7/6 gal. from D to F? F demands 1 gallon for itself plus 2/3 + 2/3 to send downstream. This 1 + 2/3 + 2/3 = 7/3 gallons of water is split evenly across the two S.P. from A to F: through CF and DF.

Bottom-up calculation of water flow: <u>Read this figure bottom up.</u> Recall: A sends 1 gallon to every other node. Water flows only on S.P. to a node and splits evenly across all S.P. to it.

Final note: The calculation above is just for node A. We'll need to repeat it for nodes B, C, …, H. Then, for each edge, we'll accumulate the water flow over all these different. The accumulated water flow through an edge gives the betweenness measure for that edge.