Physics 81 Set 3 Solutions

1. Exercise 6 of Appendix 3 (page 225)

   You can assume no albedo. In steady state, the glass comes to a temperature T_g and thus radiates a flux $F_g = \sigma T_g^4$. Similarly, the floor warms to temperature T_f and radiates a flux $F_f = \sigma T_f^4$. Also assume that the incoming solar flux is S. Because 20% of the outgoing IR flux from the floor passes through the glass, we have the following flux balance just outside the glass:

   $$S = 0.2F_f + F_g$$

   Just above the floor, we have the flux balance:

   $$S + F_g = F_f$$

   Note that in both equations, the incoming fluxes are on the left and the outgoing fluxes are on the right.

   We have 2 equations and 2 unknowns, so we can solve for F_f and F_g. If we eliminate F_g from the equations, we get

   $$F_f = \frac{2}{1.2}S = \frac{5}{3}S$$

2. Exercise 1 of Appendix 4 (page 230)

   As the sun beats down on the sand and the ocean it delivers equivalent energy to both materials. Since the specific heat capacity of water is very high, this large energy input only results in a small change in temperature. Since the specific heat capacity of sand is 5 times smaller, the equivalent energy input to the sand warms it 5 times as much (i.e ΔT is 5 times greater, measured in *kelvins*.

   You should also note that because sand has a higher albedo than seawater, the sand doesn't actually absorb as much of the incoming sunlight as the ocean, so the 5-fold warming increase isn't fully realized.

3. Exercise 2 of Appendix 4 (page 230)

   Start with the barometric equation. The form that I gave you in class is:

   $$P = P_0 e^{-\frac{mgz}{kT}}$$

   To solve for z, divide by P_0 and take the natural logarithm (ln) of this expression (since $\ln(e^x) = x$) to get

   $$\ln(\frac{P}{P_0}) = \frac{-mgz}{kT}$$

   Solving for z gives

   $$z = -\frac{kT}{mg}\ln(\frac{P}{P_0})$$

   Now all we have to do is plug in the appropriate values for the constants:

   $$k = 1.38 \times 10^{-23} J/K$$

$$T = 253K$$

$$m = (28.8amu)(1.66 \times 10^{-27}kg/amu) = 4.78 \times 10^{-26}kg$$

$$g = 9.8m/s^2$$

$$P_0 = 100,000Pa$$

Now try out the different values for P. You should get that $P = 100mb$ (i.e. $10,000Pa$) at $17,158m = 17.2km$. Similarly, $P = 10mb$ at $34.3km$ and $P = 1mb$ at $51.5km$.

Note two things:

(a) I have chosen my units very carefully. The Pascal is the appropriate unit for pressure since it is built on the meters-kilogram-seconds system.

(b) Since the fall-off of pressure is exponential, it shouldn't be too much of a surprise that the pressure drops by a factor of 10 at even intervals in altitude.