Physics 223

Electric Fields and Circuits

Laboratories

Autumn 2004

Schedule

Group 1: Mondays, 13:00 – 15:55
Group 2: Thursdays, 13:00 – 15:55

Personnel

Name	Task	Office	Phone	Email	Office Hours
M. Battle	Labs	Searles 304	x7129	mbattle	Weds. 13:30 - 15:00, Thurs. 11:30-12:30
D. Syphers	Lectures	Searles 320	x8541	dsyphers	Tues. 9:30-10:30, Weds. 14:30-15:30
Dominica Lord-Wood	Dept Coordinator	Searles 319	x3308	dlord	8:30 – 17:00

Philosophy and objectives

Laboratory work is an integral part of the learning process in the physical sciences. Reading textbooks and doing problem sets are all very well, but there's nothing like hands-on experience to truly understand physics. The laboratory sessions complement your classwork. If you mentally dissociate the two and view the labs as something to be ticked off a list, you are doing yourself a great disservice, missing out on an excellent opportunity to learn more deeply.

In addition to course specific objectives, lab work is meant to develop analytic skills. Various factors may influence the outcome of an experiment, resulting in data differing noticibly from the theoretical predictions. A large part of experimental science is learning to control (when possible) and understand these outside influences. The key to any new advance based on experiment is to be able to draw *meaningful* conclusions from data that do not conform to the idealized predictions.

Requirements

Generalities

Students must complete all laboratory exercises.

Students will work with a lab notebook rather then writing lab reports for each experiment. This is more akin to what is done by a research scientist. In addition to responding to the guiding questions in the lab instructions, students should make whatever notes, observations, and diagrams they think could be useful to them in future labs or if they were asked to reproduce previous results.

When analyzing results, always be as explicit, detailed, and quantitative as possible.

A few specifications

Tables

A table is only useful if it's presented in such a way that the information in it can be used at some later time. This implies that columns (possibly rows also) and the table itself should be clearly labelled. Units must be specified; this can be done either in the column heading or within the table proper, as appropriate.

Graphs

The point of a graph is to visually convey the results of an experiment. If you choose the scale of your graph such that the trend present in the data is not apparent, then your graph is not useful. Axes need not start at zero, choose a scale that allows you to show all your data and for which the trends are easily seen (ask yourself: "What is the smallest range of x and y I could use to show all my data?").

As with tables, a graph isn't very useful if it's not properly labelled. Both axes and the graph itself should be labelled. Always include the units in the axis labels.

References

- *Physics, volume 2* by Halliday, Resnick and Krane
- Electric Fields and Circuits class handouts

Evaluation

Lab books will be collected every week at the end of the lab period. They will be handed back with comments at the beginning of the following lab period. There will not be a grade assigned for each experiment, however, at the midterm, you will be given a grade to give you an idea of where you stand.

The following elements will be considered:

Focus in lab
Problem solving, resourcefulness in lab
Connection to theory
Analysis skills (graphing, uncertainties, etc)
Presentation in lab book

The lab section of this class is worth 20% of your final Phys223 grade.

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List of Figures

1 DC Signals and Meters

1.1 Background

A Direct Current (DC) signal is usually defined as one that does not vary with time. A DC meter will respond to such a signal with a steady reading.

Of course, all signals observed in the laboratory vary to some degree, at least to the extent that they are turned on and off or adjusted during the course of the experiment. If the signal varies very slowly, then the DC meter reading will vary with the signal and will give a true indication of how the signal is changing with time. For instance, in an experiment, the student may vary a voltage and observe how a current, measured with a DC milliammeter, changes with the voltage.

The DC meter is not capable of following rapid variations in a signal. If the signal changes very quickly, then the DC meter will respond only to its average value. Thus if a DC voltmeter is connected across an AC electrical outlet, where the voltage alternates from positive to negative to positive again sixty times per second, then the DC meter reading should be zero.

1.2 Equipment and constraints

In this laboratory exercise, you will be using a pair of DC meters. The DC milliammeter has several ranges for measuring steady currents. The digital multimeter (hereafter DMM) is capable of a variety of electronic measurements. Initially you will be using only its DC voltage ranges. Voltages will be obtained from a DC power supply. They can be varied between zero and some upper limit, $V_{\rm ul}$, approximately 30v for the supplies used this semester. Different models of DC voltage sources are discussed in the next section.

One of the aims of these first exercises is to characterize laboratory voltage sources. To do this, voltage sources, resistors and current meters will be used, and you must understand their limitations. The first part of the laboratory exercise will explore the maximum current and voltages that can be applied to resistors.

In addition to voltage sources, each lab station has a resistor box with a wide variety of resistances. Each resistor obeys Ohm's law, $V = IR$, and has a maximum power rating, $P_{\rm max}$, usually approximately 1 watt.

Pre-lab exercise

For a resistor with power rating $P_{\rm max}$, derive an expression for the maximum voltage $V_{\rm max}$ that can be safely applied to this resistor.

1.3 Laboratory exercises

For resistors with $R > \frac{V_{\rm ul}^2}{P_{\rm max}}$, the maximum voltage allowed to flow through them is greater than the highest voltage that the power supply can provide (i.e. $V_{\rm max} > V_{\rm ul}$). In this case, the resistors are in no danger of being harmed. However, resistors with lesser resistance can be damaged (see the example in the lab). Before using a resistor in a circuit, you should calculate its maximum current and voltage, then make sure that you never exceed these values. *This applies to this laboratory exercise and all subsequent ones (so keep good records!).*

1. Set the DC power supply's voltage and current limit to their greatest possible values (turn knobs fully clockwise).

2. Connect the DMM (on the 200V DC range) across the power supply and measure V_{ul}.

3. For each resistor which could potentially be harmed when using this power supply (i.e. for which $R < \frac{V_{\text{ul}}^2}{P_{\text{max}}}$), tabulate I_{max} (in milliamps) and V_{max} (in volts).

2 DC Voltage Sources

2.1 Background

A device which produces an electrical signal with a non-vanishing DC voltage may be considered a DC voltage source. For an ideal voltage source, the voltage is independent of the current drawn from the device. The characteristic of an ideal voltage source is shown in Figure 1a. Such a source is often termed "well-regulated".

Some voltage sources are well-regulated for small currents, but the voltage drops off if the current drawn from the device becomes too large. A typical curve for such a voltage source is shown in Figure 1b. This type of voltage source is said to be well-regulated up to a current of I_0.

Other voltage sources produce a voltage that drops steadily as more current is drawn from the device. The simplest kind of relationship between voltage and current in such a source is the linear relationship shown in Figure 1c. In this case, the voltage can be expressed as a function of current:

$$V = V_{\rm oc} - R_s I \quad (1)$$

$$= R_s(I_{\rm sc} - I), \quad (2)$$

where $V_{\rm oc}$ is the open circuit voltage, $I_{\rm sc}$ is the short circuit current, and R_s is the internal resistance of the source. Such a voltage source can be represented by an ideal voltage source in series with an internal resistor. For voltage sources of this type the internal series resistance is often called the *output resistance* of the voltage source. A schematic representation of this kind of voltage source is shown in Figure 2.

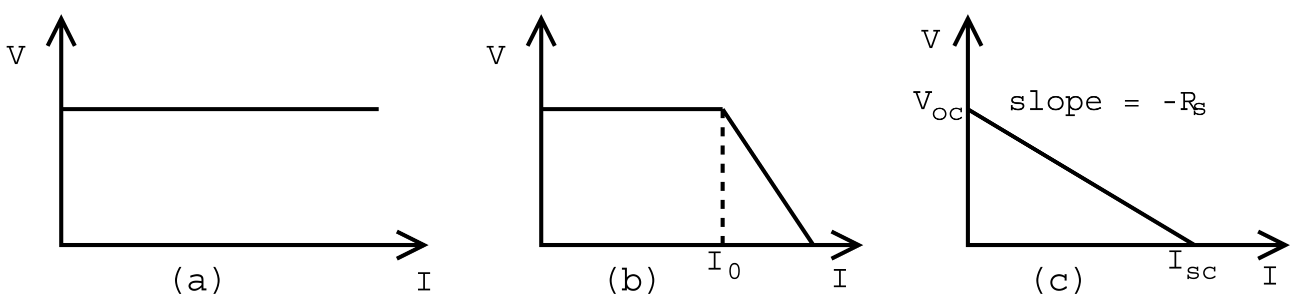

Figure 1: Voltage-current characteristics for different types of voltage sources

2.2 Laboratory exercises

A simple voltage source with which most people are familiar is a battery. This laboratory will characterize both regulated laboratory power supplies and batteries, then compare them.

The characteristic voltage-current relationship of a given DC voltage source can be studied experimentally using the type of circuit shown in Figure 3. As the resistance of the resistor box

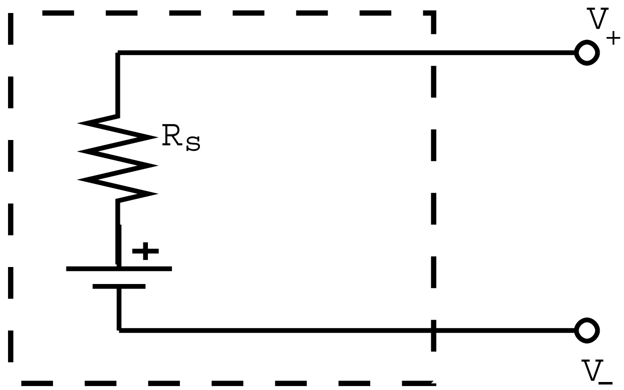

Figure 2: Schematic representation of a voltage source

is changed, the current being drawn from the voltage source will vary. This current is measured by the DC milliammeter at the same time as the voltage output of the voltage source is being measured on the DC voltage ranges of the DMM. A careful plot of these two readings, voltage as a function of current, should produce a curve similar to one of those in Figure 1.

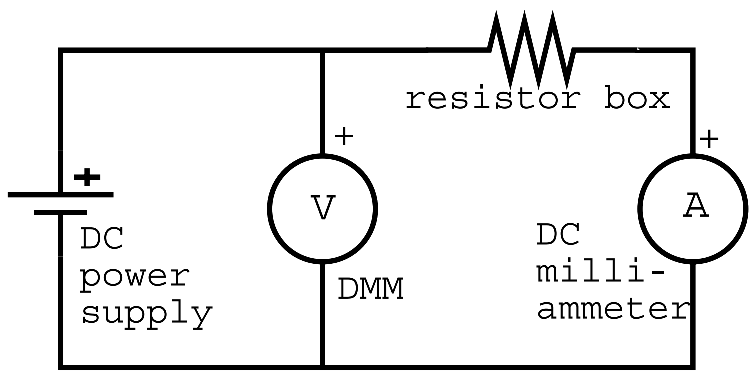

Figure 3: Circuit for evaluation of voltage-current characteristic

Caution! Some care must be taken when performing this type of experiment. The current must not be allowed to exceed the full scale current of the chosen DC milliammeter range. Higher currents could damage the meter. Additionally, the current must not be so large that the power dissipation in the resistor exceeds its maximum rating. In general it is wise to begin with a fairly large value of resistance and to reduce this resistance only when one is quite certain that these limiting currents will not be exceeded.

2.2.1 Regulated power supply

Before starting, it is a good idea to familiarize yourself with the power supply currently being used in this lab: the Instek GPS-3303. The supply should be operated in the INDEPENDENT MODE, where the A supply is independent of the B supply. It does not matter whether you use A or B for the experiment. Above the power switch on these units, you'll find an "output ON/OFF" button. When the output is OFF, the meter shows the *maximum* current and *maximum* voltage settings. When the output is ON, the meters show the *actual* current and voltage the unit is delivering. When the maximum current has not been reached, the unit is in the constant voltage mode and the "c.v." light is green (constant voltage). Despite the name of this mode, when the voltage knob is turned, the output voltage will change, as does the current.

When the maximum current has been reached, the "c.c." red light comes on and the current and voltage are both fixed. This remains true until the current knob is adjusted, regardless of attempts to increase voltage using the voltage knob. This mode, with the "c.c." lamp red, is referred to as "current limited". It is a good idea to set the current limit to a value that will be safely below the power threshold for the resistors in the resistance box.

Time now to begin the experiment:

Adjust the DC power supply's output to the desired values of V_{oc} and I_{sc}. Here's how...

1. Set the voltage of the DC power supply to its maximum value (turn knob fully clockwise), and the current to its lowest possible value (turn knob fully counter-clockwise).

2. Connect the DC milliammeter (on the 100 mA range) directly across the DC power supply and adjust the current limit until the current is about 80 mA.

3. Remove the DC milliammeter and connect the DMM across the DC power supply.

4. Adjust the voltage until it is about 10V.

Evaluate the voltage-current characteristic of the DC power supply, and thereby confirm that the power supply is well regulated. To do this...

5. Add the DC milliammeter and resistor box to obtain the circuit shown in Figure 3.

6. Start with the resistor box at about 10 kΩ and click down a few steps at a time the smallest resistance value in the box, measuring the current and voltage at each step. Present your results in a table.

7. Increase the current limit to about 1.3 Amps and substitute a 20W, 8Ω resistor for the resistance box. Note the current and voltage.

Analyze your results:

- Plot the voltage-current data on a well-labeled graph. Identify the current range over which the DC power supply appears to be well-regulated.

- Is the DC power supply ideal over this range or does it have a small internal resistance? If the latter, estimate the value of this supply resistance.

- How does the DC power supply behave at higher currents, above the well-regulated range?

2.2.2 Battery

Determine the voltage current characteristic of a battery:

1. Substitute the 1.5V battery for the DC power supply in your circuit (Fig. 3).

Q. Why don't you need to avoid any resistance values due to power consideration in this case?

2. Start with the resistor box at about 1 kΩ and click down step by step to the smallest resistance value in the box, measuring voltage and current at each step. Make a table showing your results.

Analyze your results:

- Does your measured voltage equal the current times the resistance? If not, add a column to your table listing the differences between these two parameters. Does the difference vary or is it constant? Explain any observations about differences. Is it due to measurement uncertainty?

- Plot on a well-labeled graph the voltage-current characteristic that you have determined for the battery. Identify the current range over which the battery appears to be well-regulated. Is the battery ideal over this range or does it have a small internal resistance? If the latter, estimate the value of this small internal resistance.

- How does the battery behave at high currents, above the well-regulated range?

- Compare your results with those for the DC power supply.

3 Ammeters

3.1 Ideal ammeters

An ammeter is an instrument for measuring current. It is placed in series with a device for which the current is to be measured in order that all the current flowing through the device of interest should also flow through, and be measured by, the ammeter. An ideal ammeter does not disturb the circuit in which it is placed and therefore must have zero resistance so that there is no voltage across it.

3.2 Real ammeters

Actual ammeters have a small internal resistance, which can be measured using a circuit such as the one shown in Figure 4. Here the DC milliammeter measures the current through itself and the DMM measures the voltage across it. A plot of these two quantities, voltage as a function of current, should yield a straight line through the origin whose slope is the internal resistance of the milliammeter.

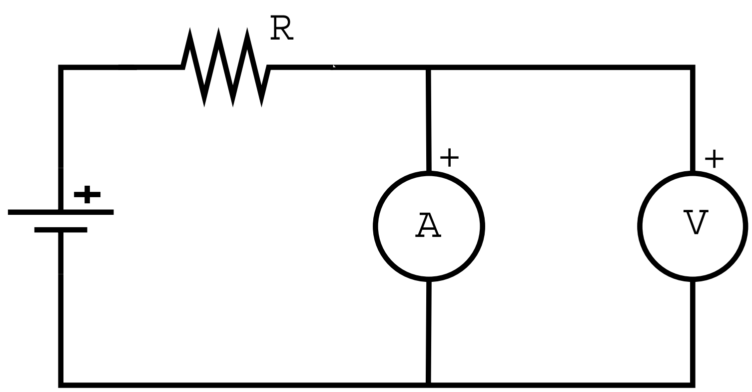

Figure 4: Measuring the internal resistance of an ammeter

The value of the resistor in the circuit should be the largest value you have that is less than both $\frac{V_{\mathrm{ul}}}{I_{\mathrm{fs}}}$ and $\frac{P_{\max}}{I_{\mathrm{fs}}^2}$, where V_{ul} is the upper limit of the DC voltage supply, I_{fs} is the full-scale current of the milliammeter range, and $P_{\max}$ is the maximum power rating of the resistor.

3.3 Laboratory exercises

Determine the internal resistance of the milliammeter for one range:

1. Set up the circuit shown in Figure 4 using the lowest range of the DC milliammeter.

2. Vary the power supply voltage to obtain several different points up to the largest current on the range.

3. Plot voltage versus current.

4. Determine the internal resistance for this range of the DC milliammeter from the slope of the graph.

If you've convinced yourself, based on your graph, that the milliammeter follows Ohm's law, you can determine the internal resistance from a single pair of voltage and current measurements.

5. Determine the internal resistance for the remaining ranges of the DC milliammeter for which you have an appropriate resistor.

Q. How does the internal resistance of the DC milliammeter vary with its range? What does this imply or, in other words, why does the internal resistance vary in this manner?

4 Ohmmeters

4.1 Background

An ohmmeter is an instrument for measuring resistance without having to resort to independant measurements of voltage and current. It is placed directly across the device the resistance of which is to be measured. The ohmmeter's reading will then give you the device's resistance, provided that said device obeys Ohm's law (see Lab#7 for a device that does not obey Ohm's law).

4.2 Equipment and constraints

Make sure your connections are "clean". A poor connection can have a considerable contact resistance in which case the ohmmeter will read this resistance in addition to the desired resistance of the device.

The DMM has several ranges for measuring resistance. For greatest precision, you should generally use the lowest range on which you can make the measurement.

4.3 Laboratory exercises

Familiarize yourself with using the DMM as an ohmmeter by measuring precise values for the resistances in the resistor box:

1. Connect the DMM, used as an ohmmeter, directly across the resistor box.

2. Make a table showing accurate values for every resistor in the box, from its lowest possible resistance to its highest.

Q. Do the values written on the box agree with your measurements? What is the range of percent differences?

Measure the resistance of the milliammeter:

3. Connect the DMM, used as an ohmmeter, directly across the DC milliammeter ('plus' to 'plus', 'minus' to 'minus').

4. On each range, measure the resistance of the milliammeter using the lowest range of the ohmmeter for which the measurement is possible without exceeding the full scale current of the DC milliammeter.

5. When possible, record the current in the milliammeter as well as its resistance.

Q. How do the values you obtained with the ohmmeter for each range of the DC milliammeter compare with the values of resistances which you calculated in Lab#3?

5 Voltmeters

5.1 Background

The experimental determination of the voltage-current characteristic of a power supply using the circuit in Figure 3 can give poor results. The DC milliammeter will measure only the current passing through it, whereas the actual current delivered by the DC power supply is the sum of this current and the current flowing through the voltmeter. Thus the characteristic will be correct only if no current flows through the voltmeter. This requires that it have an infinite resistance. Such a voltmeter is called an ideal voltmeter.

5.2 The internal resistance of the DMM

In previous exercises, the DMM was used to measure the DC voltage because it is very nearly an ideal voltmeter. This fact makes the internal resistance of the DMM somewhat difficult to determine. The DMM cannot be used simultaneously as a voltmeter and as an ohmmeter to measure its own internal resistance. Furthermore, the DC milliammeter is generally not sensitive enough to accurately measure the very small currents that flow through the voltmeter. Therefore a less direct method is needed.

Suppose one connects the DC power supply in series with a resistor of several megaohms and the DMM as shown in Figure 5. The voltmeter reads its voltage, and its current is the same as in the resistor, i.e.

$$I = \frac{V_s - V}{R} \tag{3}$$

where V_s is the source voltage which can be measured by temporarily connecting the DMM directly across the power supply. Given the voltage and the current, one has enough information to determine the internal resistance of the DC voltmeter. This resistance should be very large since, as stated above, the DMM is very nearly an ideal DC voltmeter i.e. it draws very little current.

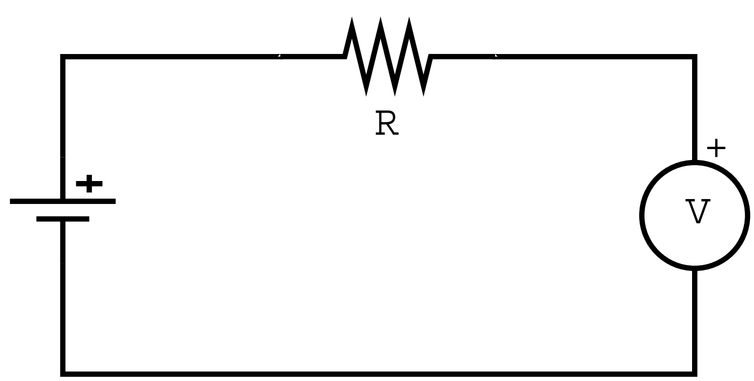

Figure 5: Measuring the internal resistance of a voltmeter

5.3 Laboratory exercises

Determine the internal resistance of the DMM for one of its ranges:

1. Set up the circuit shown in Figure 5 with the DMM on the 20VDC range and R set at about 5 MΩ.

2. Vary the voltage of the power supply and for each setting, measure the supply voltage and the reading of the voltmeter, and calculate the current using Eq.3.

3. Make a graph of voltmeter readings as a function of current. Fit a straight line to your data.

Q. Based on your graph, what is the internal resistance of this range of the DMM?

Calculate the maximum current through the voltmeter:

4. Calculate the current that will flow in the voltmeter when it is reading V_{ul}, the upper limit of the voltage power supply. This is normally the largest current that will flow in the voltmeter.

Q. How does this current compare with the current at the first mark on the most sensitive range of the DC milliammeter? What does this imply?

Determine the internal resistance of the DMM for other ranges:

5. Assuming that Ohm's law holds, calculate the internal resistances for the 200 VDC and 2 VDC ranges of the DMM. Choose for your single measurement the largest source voltage you can produce that is appropriate for the range in question.

Q. Does the internal resistance of the DMM used as a DC voltmeter vary with its range?

6. Make any other observations and draw any other conclusions that you can.

6 Operational amplifiers

6.1 Background

In the introduction to Chapter 9 of *Principles of Electronic Instrumentation (Third edition)*, Diefenderfer & Holton advertise op amps as the device to cure almost all your circuit designing ills:

> As the medicine-show pitchman heralded the magic elixir as the panacea for "whatever ails you," the operational amplifier is prescribed for the novice designer's ills. Just as the elixir wasn't good for every illness, the operational amplifier has its limitations; fortunately, however, these limitations are generally insignificant, particularly when compared with the devices advantages. The operational amplifier offers a direct and immediate entry into design with a high assurance of success.

Operational amplifiers are convenient, functional building blocks for circuit design, requiring knowledge of only a few simple rules to make proper use of them.

A perfect amplifier would have an infinite input impedance such that any signal could be applied, and it would have a null output impedance such that the power output is not limited. As well, in a perfect world, the gain would be infinite. As a result, the characteristics of an ideal op amp are:

1. The current at the inverting ($-$) and non inverting ($+$) inputs are zero.
2. The operational amplifier does whatever is necessary to make the voltages at the two inputs equal to each other.

6.2 Equipment specifications

The pin diagram for the LF 351 N Op Amp is shown in Figure 6. In order to operate, the operational amplifier must be powered; this is the reason for connections 4 and 7. The two balance inputs, 1 and 5, are used with a variable resistor (potentiometer) to help achieve the ideal results as in the "typical connection" diagram in Figure 7.

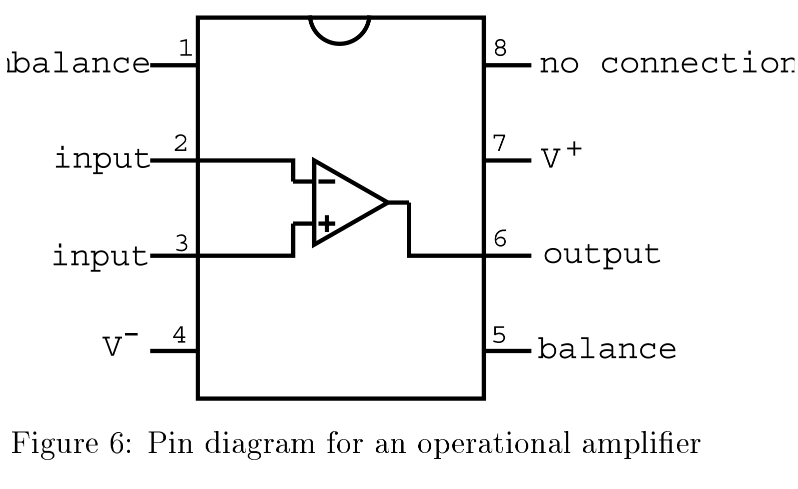

Figure 6: Pin diagram for an operational amplifier

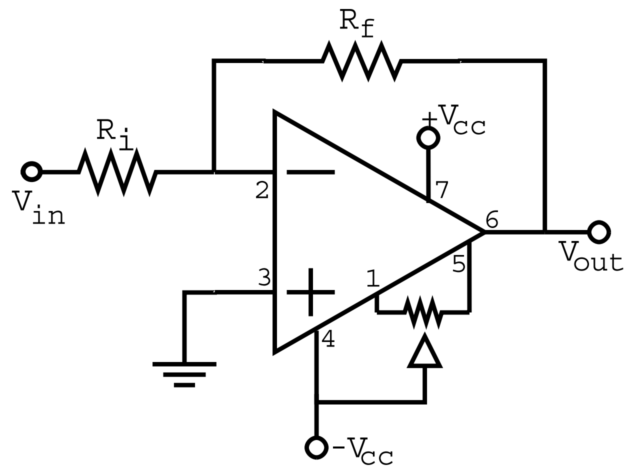

Figure 7: "Typical connection" diagram for an operational amplifier

6.3 Laboratory exercises

In this lab you will explore the use of real operational amplifiers in DC circuits and compare with the ideal case.

6.3.1 Inverting amplifier

Make a unity gain inverting amplifier.

1. Set up the "typical connection" shown in Fig.7 with $R_i = R_f$. Use resistors with a value larger than 10 kΩ for these. Omit the potentiometer in the balance circuit, and leave the balance lines (1 and 5) unconnected.

2. Theoretically solve this circuit to find the expect relationship between V_{in} and V_{out}. Keep the solution general, i.e. leave it in terms of R_i and R_f.

3. Use your power supply to provide a known voltage greater than 1 V at V_{in}. We suggest that you use one voltmeter to measure V_{in} (with greater precision than afforded by the display on the power supply), and a second voltmeter to measure V_{out}. Note the output voltage.

4. Switch the two resistors and note the output voltage again.

Q. How does the average of these two voltages compare to V_{in}? Comment.

5. Check the voltage at the inverting input.

Q. Is it the same as the grounded non-inverting input? Note any differences and comment.

6. Insert your ammeter into the circuit to check the current to the inverting input.

Q. Is it what you expected? Explain your answer.

7. Restore the circuit to its original configuration.

8. Put a 1000 Ω load resistor on the output (between output and ground) and check the output voltage.

Q. Is it unchanged? If it isn't, what could explain the difference?

9. Repeat with a 100 Ω load resistor.

Q. Is the output voltage unchanged? If it isn't, what could explain the difference?

10. Repeat with a high-power 8 Ω resistor as the load resistor.

Q. Is the output voltage unchanged? If it isn't, what could explain the difference?

11. Check the amplification by using $R_i = 100\Omega$ and $R_f = 10\ \mathrm{k}\Omega$ for a range of expected V_{out} from 1V to 20V.

Q. Does the circuit amplify appropriately? Comment.

12. Summarize your results by stating when the op amp acts like an ideal op amp and when it ceases to act like an ideal op amp. What does this tell you about the output impedance of the op amp?

6.3.2 Analog computing

1. Construct the circuit in (Figure 8) and solve theoretically. For R_L, choose a value greater than $1K\Omega$. (Why this lower limit?)

2. Check the output for $R = 10\mathrm{k}\Omega$ (or greater), then again for $R = 1000\Omega$. Use different values of V_1 and V_2, covering a range of expected outputs from $V_{\mathrm{out}} = 3$ V to $V_{\mathrm{out}} = 20$ V (just a few cases). For R_{L}, choose some value greater than 1000Ω (why?).

3. Comment on your observations.

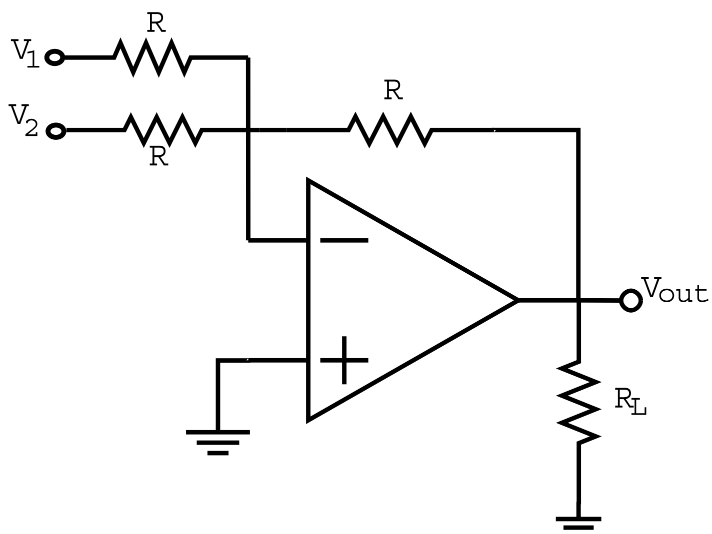

Figure 8: Operational amplifier circuit from lecture

7 Diodes

7.1 Background

Not all two-terminal devices obey Ohm's law. The semiconductor diode is a good example of one that does not. This is a device that allows current to flow only in one direction (called the forward direction). In the opposite direction (called the reverse direction), any currents are usually immeasurably small. Diodes can therefore act like switches, a fact which is important and extremely useful in circuit designs.

An *ideal* diode has zero resistance in the forward direction and therefore there is no voltage drop across it, and it has infinite resistance in the reverse direction and therefore no current flows through it no matter what voltage is applied across it. Of course, a *real* diode has a non-zero resistance in the forward direction. As a result, there is a voltage drop across it when current flows through the diode. This is typically of the order of 0.6 – 0.7 V for Si diodes.

7.2 Measuring forward and reverse characteristics

Circuits for measuring the forward and reverse characteristics of a diode are shown in Figures 9 and 10. Note the different placement of the DC milliammeter in the two circuits. In Figure 9, the milliammeter is actually measuring the sum of the currents in the diode and in the voltmeter. The voltmeter current is usually negligible compared to the large forward current of the diode. However, if the diode is reversed, then the diode current would approach zero, and the small current through the voltmeter would dominate the milliammeter's reading. For this reason, it is preferable to use the circuit shown in Figure 10 for the reverse diode.

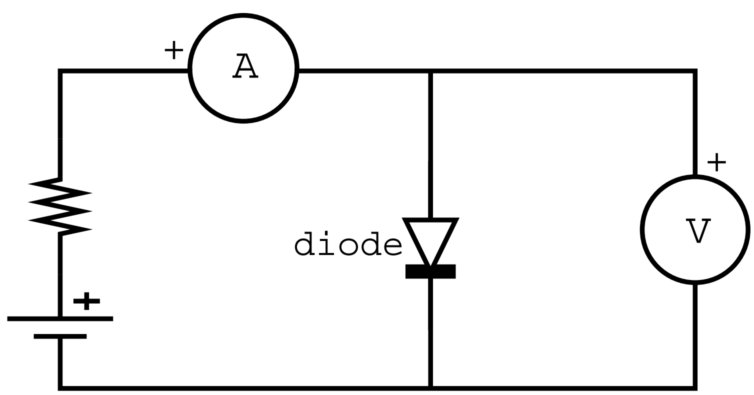

Figure 9: Circuit for measuring the forward characteristic of a diode

The forward characteristic of a semiconductor diode is often essentially exponential. This means that a graph of the logarithm of current as a function of voltage should be a straight line. If your diode is exponential, then the data satisfy

$$I = I_0 \, e^{V/V_0} \tag{4}$$

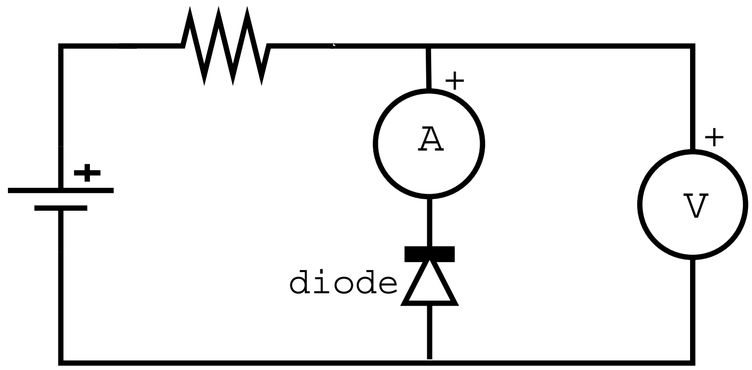

Figure 10: Circuit for measuring the reverse characteristic of a diode

which is equivalent to

$$\ln(I) = \ln(I_0) + \frac{V}{V_0} \qquad (5)$$

where I_0 and V_0 are parametric constants.

7.3 Laboratory exercises

Measure the forward characteristic of the diode:

1. Set up the circuit in Figure 9 to study the forward behavior of a semiconductor diode. Begin with the DC milliammeter on the 10 mA range and the resistor box set to about 1 kΩ.

2. Vary the voltage of the DC power supply to obtain settings for which the current flowing through the diode exceeds 1 mA. For each, measure the voltage across the diode as well as its current.

3. Take a few more readings of current and voltage for each of the lower ranges of the milliammeter.

∗ To enable you to better control the current, you may want to increase the resistance as you decrease the range. (Do you understand why this helps?)

Q. What is the lowest voltage for which you can get a measurable forward current in the diode (a current at the first mark of the most sensitive range of the DC milliammeter)?

4. Make a graph of current as a function of voltage for the forward direction of the diode.

5. Graph $\ln I$ as a function of V, where I is the forward current through the diode in *microamperes* (in which case values of $\ln I$ will be positive), and V is the voltage across it.

Q. Is your diode exponential?

6. Plot a best fit straight line through your data points.
7. From your graph, determine the parameters I_0 and V_0 for your semiconductor diode.

Measure the reverse characteristic of the diode:

8. Set up the circuit shown in Fig. 10 and observe the inverse behaviour of the diode.

Q. What current flows in the reverse direction of the semiconductor diode?

8 Metal Oxide Semiconductor Field-Effect Transistors

8.1 Background

There are two basic types of Field-Effect Transistors (FETs): the junction FET (JFET) and the Metal Oxide Semiconductor FET (MOSFET). The operating principles are similar but the MOSFET is simpler and more common these days.

The MOSFET is a sandwich composed of a semiconductor substrate (such as Si), an insulating spacer (such as SiO_2, a thin layer of glass grown on a silicate substrate), and a metal layer completing the sandwich as shown in Figure 11. The metal layer on top is referred to as the gate because the voltage on this layer determines whether the transistor switch is open or closed. The contact regions complete the device, they are referred to as the source and the drain. Current can flow through a small channel at the semiconductor-insulator interface when the appropriate charge accumulates at this interface. In the absence of an appropriate charge at the interface no current will flow.

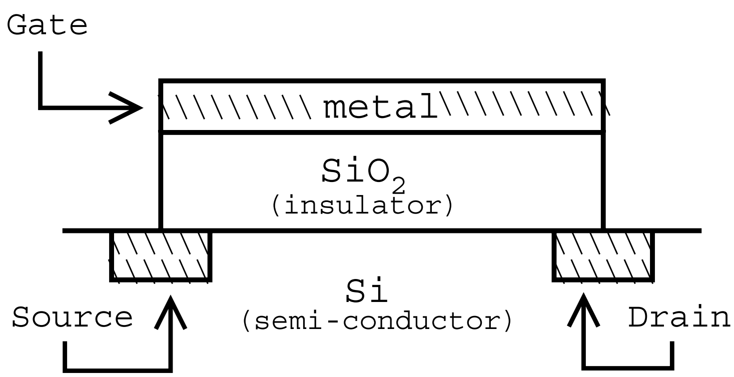

Figure 11: Schematic diagram of a MOSFET

The substrate can be doped with impurities so that it is either p-type, with holes the dominant carrier, or n-type, with electrons the dominant carrier. This is accomplished by doping the material with either acceptor (for p-type) or donor (for n-type) impurities. The dominant carrier in the contact regions will be the dominant carrier in the conducting channel when the device is on. The two different types of FETs, n-channel and p-channel are shown in Figure 12. The voltage on the gate (G) controls the state of the conducting channel, one end of which is the Source (S) and the other, the drain (D). The n-channel device is turned on when a positive voltage is applied to the gate and electrons in the conduction bands are attracted toward the n-channel interface between Si and SiO_2. The p-channel device is normally on with no voltage applied to the gate, due to the surplus of holes in the valence band as a result of doping. When positive voltage is applied to the gate, electrons fill these valence band vacancies, turning the p-channel device off.

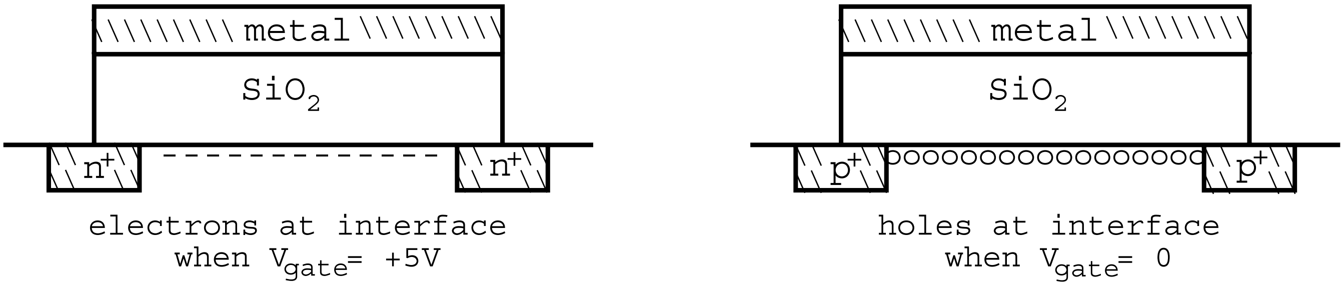

Figure 12: n-channel and p-channel MOSFETs

8.2 Equipment specifications

In this lab, you will be using one of two n-channel MOSFETs, either the MPF960 or the VN2222LL. The schematics for these devices, along with pin diagrams, are shown in Figure 13. As you can see in the diagram, the devices also have an internal diode (zener diode) to protect the MOSFET device (zener diodes will conduct current in either direction if the voltage across it is high enough, but the current will be nearly zero for small voltages — less than 0.5V in the forward direction and less than 10V in the reverse direction). This diode is not part of the MOSFET itself, it is constructed nearby on the silicon chip to prevent damage to the MOSFET.

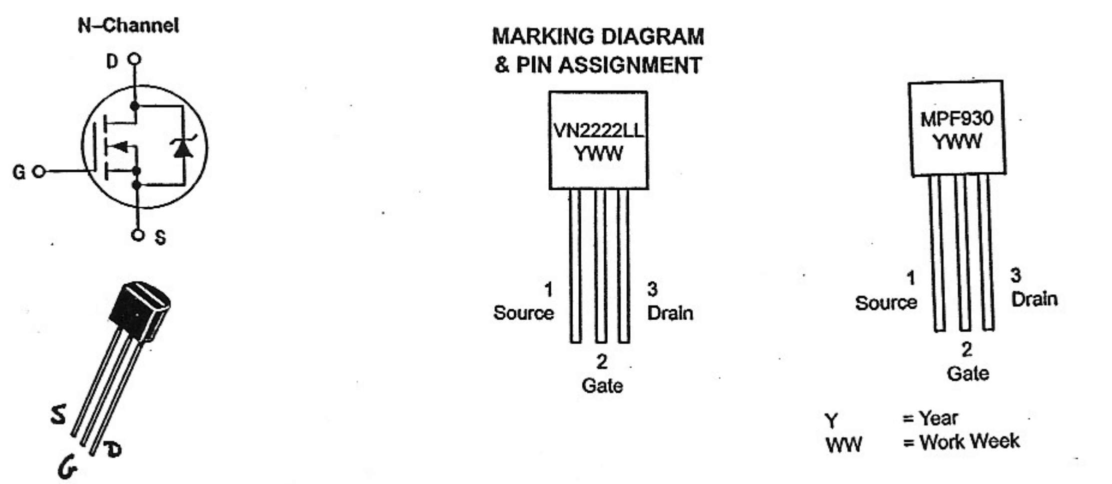

Figure 13: The n-channel MOSFETs MPF960 and VN2222LL

The MOSFET is typically connected with the Source grounded and the Drain connected to the part of the circuit whose current you wish to control by turning the MOSFET on or off. The beautiful thing about the MOSFET is that the control voltage is totally isolated from the circuit you are controlling, by at least 10^{10} ohms (usually by about 10^{14} ohms) since SiO_2 is a wonderful insulator.

8.3 Laboratory exercises

1. Connect the circuit shown in Figure 14.

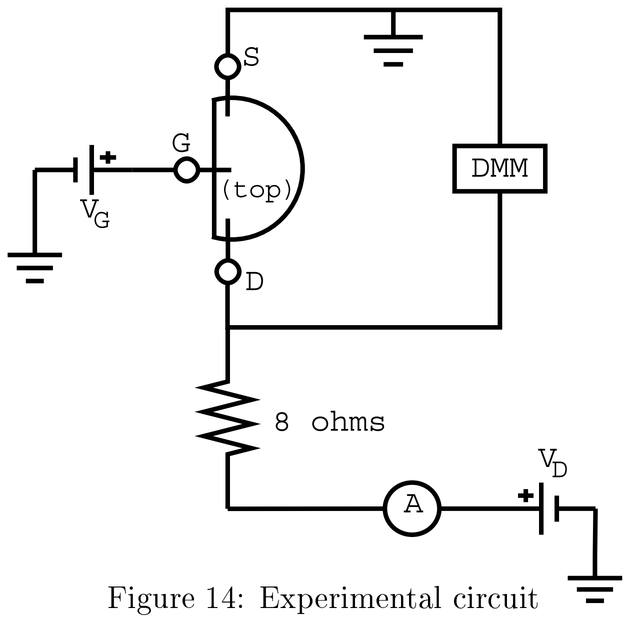

Figure 14: Experimental circuit

The digital multimeter measures the voltage difference between the source and the drain, V_{SD}, and the ammeter measures the current that flows from source to drain, I_{SD}.

Collect data to look at the relationship between the current through the source-drain channel and the voltage across the channel:

2. Set $V_G = 3.0$ V.

3. Vary V_D to get currents between 1 mA and the maximum current the device will allow (but *NEVER* exceed 100 mA for more than a brief period of time). Start by taking data at $I_{SD} = 1$ mA, then at successive factors of 2 higher in current (1,2,4,8,...). Record your data in a table (V_{SD}, I_{SD}, $R_{SD} = \frac{V_{SD}}{I_{SD}}$).

4. Plot R_{SD} versus V_{SD} and determine if you need any more data to be able to draw a smooth curve. Add new data to the table and plot if need be.

Q. What is the maximum current that the device will switch at this gate voltage?

5. Repeat the above steps for $V_G = 5$ V.

Q. Was there a maximum current below 100 mA this time?

6. Briefly switch the ammeter to the 1 A current scale and vary V_D to determine if there is a maximum current for this configuration. Record the maximum I_{SD} and the associated V_{SD}, then promptly reduce the current to below 100 mA (*NEVER* exceed a current of 1 A).

7. Describe the behaviour of the transistor as a function of V_{SD}. Consider and state possible physical explanations for these observations.

Examine the role of the gate voltage, V_G.

8. Set V_D at 0.5 V.

9. Vary the gate voltage from 0 V to 8 V, tabulating data for every 0.5 V increment (V_{SD}, I_{SD}, $R_{SD} = \frac{V_{SD}}{I_{SD}}$).

10. Plot R_{SD} versus V_G.

11. Based on this graph and on your other two curves, describe the behaviour of the device as the gate voltage is increased. Provide a physical interpretation.

9 Transistors

9.1 Background

The bipolar transistor is a semiconductor device which is akin to the FET in many ways, but differs in other important aspects. Current is carried simultaneously by both holes and electrons (hence "bipolar"), rather than only one of the two as was the case for the FET (which is unipolar).

The bipolar transistor has three terminals: the emitter, the base, and the collector. An NPN transistor (Figure 15) has a sandwich-like construction, wherein two n-type semiconductor regions (the collector and the emitter), lie on either side of a p-type semiconductor (the base). The electrons from V_E are injected into the emitter and drift unhindered through the base-emitter junction. A small amount of current is lost in this region due to electron-hole recombination, resulting in the weak current I_B. However, as the base in quite thin, most of the electrons reach the base-collector junction. Because the collector is held positve relative to the base by V_C, the electrons cross the junction. In essence, the base-to-emitter and the base-to-collector act like diodes; this can be verified by testing them as two-terminal devices.

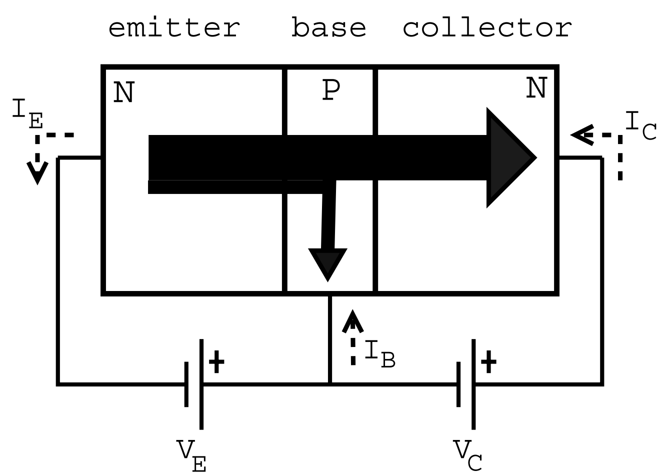

Figure 15: Schematic representation of an npn transistor

The usefulness of a transistor lies in the fact that it acts much like a valve. When current flows through the base-to-emitter part of the transistor, which is acting like a diode, the p-region has its bands bent such that electrons accumulate and conduct current when properly biased. This small base-to-emitter current, causing electrons to accumulate in the p-region, now allows current to flow between the collector and the emitter. Without the base-to-emitter current, the p-region effectively blocks any conduction between the collector and the emitter. With a small base-to-emitter current, the p-region is biased to allow electron flow and now allows a much larger current to be switched on from the collector to the emitter.

9.2 Laboratory exercises

In this laboratory, you will set up a circuit like that shown in Figure 16, which shows the common-emitter configuration, to examine some of the common emitter characteristics of an NPN transistor. The configuration derives its name from the fact that the emitter is tied to the common, or "ground", in the circuit.

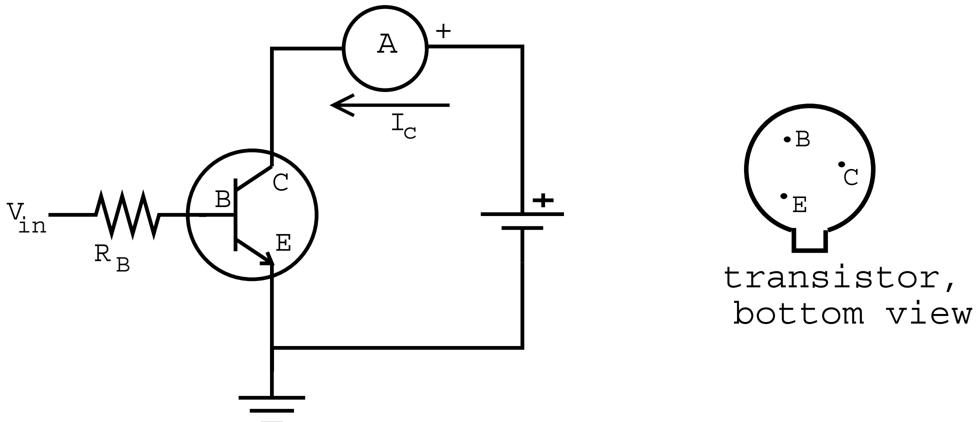

Figure 16: Circuit for the study of an NPN transistor and a bottom view of a transistor

Using a 2N2222 NPN transistor, you will explore how the transistor switches currents. The extensive data characteristics of this device can be looked up in the Semiconductor Data Library on page 2-252, available on the shelves in the Edwin Hall Laboratory.

1. Set up the circuit shown in Fig.16. Choose a power supply voltage of about 5 V and a base resistance, R_B, of roughly 3 kΩ.

2. Vary the input voltage ($V_{\rm in}$) in steps of about 0.1 V, measuring $V_{\rm in}$, the base voltage ($V_{\rm BE}$), and the collector current (I_C) for each setting until the input voltage reaches 2.0 V or the collector current reaches 10 mA.

3. Calculate the base current in each case:

$$I_B = \frac{V_{\rm in} - V_{\rm BE}}{R_B} \quad (6)$$

4. Make a graph of the collector current, I_C, as a function of base current, I_B. These two currents should be roughly proportional and the constant of proportionality is the current gain β. From your graph, determine the current gain of your transistor.

5. Make a graph of the base current (I_B) as a function of base voltage ($V_{\rm BE}$).

6. Repeat the above measurements with power supply voltages of $\sim$3V and $\sim$7V.

7. Add the curves for your new data to your two graphs.

* The set of curves showing collector current versus base current for different power supply voltages applied to the collector is called the forward characteristic of the transistor. The set of curves showing the base voltage versus the base current is called the input characteristic.

Q. What do you conclude about the dependence of the transistor characteristics on collector voltage? How is the transistor responding to changes in the collector voltage? What is it controlling?

10 Oscilloscope

10.1 Background

10.1.1 Periodic voltages

A periodic voltage is one that varies such that

$$V(t + nT) = V(t) \tag{7}$$

where t is time, T is a constant, and n is an integer. The smallest value of T for which equation 7 holds is called the period of the voltage. The reciprocal of the period is the frequency, ν, measured in cycles per second or hertz (Hz). The periodic voltage continually repeats itself, changing through a complete cycle in one period. Most of the voltages that one observes in the laboratory are essentially periodic.

Some examples of periodic voltages are shown in Figure 17. Figure 17a shows a sinusoidal voltage described mathematically by

$$V(t) = \frac{V_{pp}}{2} \sin\left(\frac{2\pi t}{T}\right) \tag{8}$$

where V_{pp} is the peak-to-peak amplitude of the signal. Figure 17b shows a square wave and Figure 17c shows a triangular wave. Each of these periodic voltages can be produced in the laboratory and observed on the oscilloscope.

10.1.2 Oscilloscopes

The cathode ray oscilloscope is an instrument in which an electron beam is allowed to hit a fluorescent screen. The position at which the beam strikes the screen can be moved horizontally and vertically by voltages applied to deflection plates. In each case, the deflection is proportional to the applied voltage. The persistence of the fluorescent screen is such that a figure is traced on the screen as the position at which the beam strikes the screen is moved, in response to voltages at the deflection plates. Liquid crystal display (LCD) oscilloscopes, like the modern Tektronix TDS 210, perform an analogous function electronically with signal processing chips, and display the waveform on the LCD screen.

Built into the oscilloscope are circuits which generate a sawtooth wave, like that shown in Fig.17d. This sawtooth voltage drives the horizontal deflection plates. The linearly increasing voltage sweeps the electron beam across the screen, from left to right, at a constant rate. The beam is returned to the left side by the rapidly descending part of the sawtooth. For the LCD oscilloscope, the sawtooth voltage provides the position for the leading edge of the displayed waveform, and the horizontal position of the leading point is proportional to time. The period of the sawtooth wave is adjustable, and the time at which the sawtooth wave starts the electron beam across the screen can be set by means of trigger controls to a selected voltage value. By synchronizing the start of the sawtooth wave with the vertical input voltage, a stationary display of the input waveform can be made to appear on the screen.

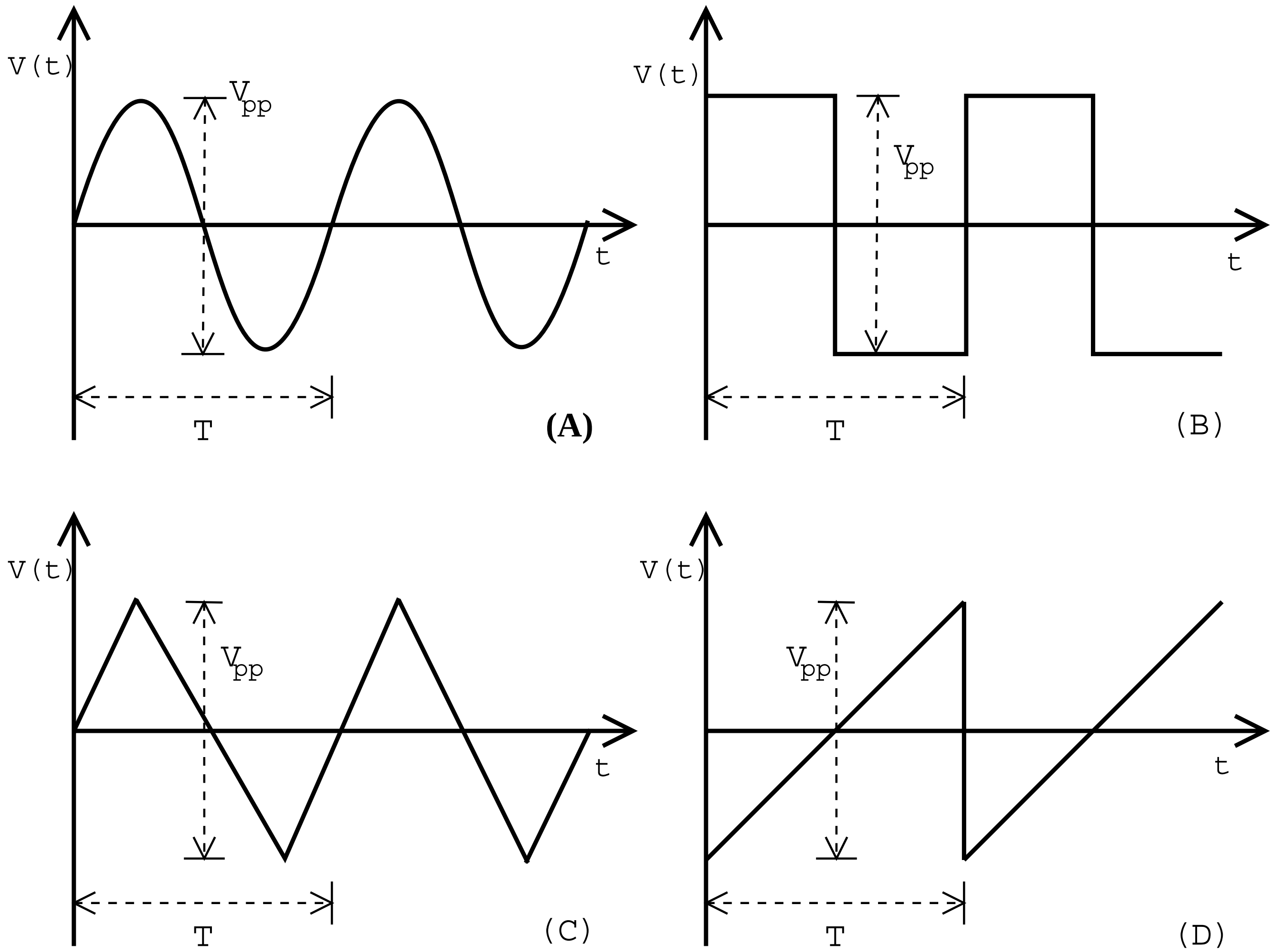

Figure 17: Periodic voltages

10.2 Laboratory exercises

Investigate the workings of the TDS-210 oscilloscope using the bench AC power supply. The power company supplies a voltage source that varies sinusoidally with a frequency of 60 Hz. One terminal of a standard electrical outlet is at earth ground potential. The other alternates between plus and minus 160 V. This voltage is too large for most modern applications in electronics and so the voltage is "stepped down" using a transformer. In the laboratory, the transformer output is located on the lower right of the built-in AC power supply. The transformer has a center tap with an electric potential midway between the potentials of the two output terminals. Any of these three terminals (but only one at a time) may be connected to earth ground, which is also provided as a reference terminal. A diagram of the bench AC power supply and the arrangement of its four terminals is shown in Figure 18.

The objective is to familiarize yourself with the oscilloscope, an instrument which you will be using in all the other lab exercises this term. The steps below are designed to expose you to some of its basic attributes and quirks, but you should explore beyond this and make any notes that seem appropriate to you. You will need to refer to these notes repeatedly during the rest of term.

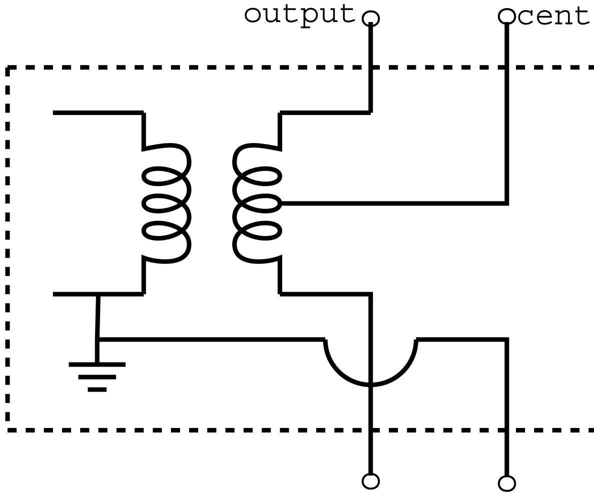

Figure 18: The bench AC power supply

1. Connect the bench AC power supply to the channel-1 input of the oscilloscope.
2. Push the AUTOSET button so that the waveform can be observed.
3. Note the vertical reference position, the horizontal trigger position, the trigger level, the main time base setting, and the vertical scale for channel-1.
4. Observe the actions of the vertical and horizontal position knobs, the VOLTS/DIV and the SEC/DIV knobs, and of the trigger level knob on the waveform displayed. Describe all these effects in your lab notebook, making sketches where appropriate.
5. Adjust the SEC/DIV knob until one or two periods of the waveform are displayed.
6. Use the trigger menu to observe the effect of changing the trigger slope from rising to falling.
7. Change the volts/div setting from coarse to fine and note how this changes the action of the VOLTS/DIV knob and the vertical scale factor.
8. Press the MEASURE button and investigate the options. Set the oscilloscope to measure the frequency, period, and peak-to-peak voltage for channel 1.
9. Vary the horizontal scale such that the display shows from roughly 5 cycles down to roughly one quarter cycle. Note how the readings (frequency, period, peak-to-peak voltage) change.
10. Set the horizontal scale such that a couple of cycles are shown on the screen. Vary the vertical scale such that the display shows the entire voltage range of the waveform, down to roughly one quarter of the voltage range. Note how the readings (frequency, period, peak-to-peak voltage) change.

Q. What important conclusion can you draw from the two previous steps concerning the values displayed by the MEASURE function?

11 AC voltmeters

11.1 Background

The oscilloscope allows one to see a periodic voltage as a function of time. Since the oscilloscope is calibrated in volts/division, its display is a direct measurement of voltage. Other voltmeters simply provide a single number that characterizes the voltage.

11.1.1 Characterizing a periodic voltage

One important number characterizing a periodic voltage is its average value or arithmetic mean, defined by

$$V_{\text{DC}} = \frac{1}{T} \oint V(t)\, dt \tag{9}$$

where the symbol $\oint$ indicates integration over one complete cycle of the function. If the frequency is sufficiently high that the voltmeter cannot respond to the rapid variations of the voltage, then the meter will read only this average value. Such a meter is called a DC voltmeter and the reading of such a meter is called the DC value of the voltage.

The alternating or AC portion of a periodic voltage is the voltage less its DC value:

$$V_{\text{AC}} = V(t) - V_{\text{DC}}. \tag{10}$$

This implies that the AC voltage is periodic with an average value of zero. The periodic voltages shown in Fig.17 were all AC voltages. As suggested by the figure, one parameter for characterizing the magnitude of an AC voltage is its peak-to-peak amplitude. Another useful measure of an AC voltage is its "root-mean-square" or RMS value, defined by

$$V_{\text{rms}}^2 = \frac{1}{T} \oint V_{\text{AC}}^2(t)\, dt. \tag{11}$$

The relationship between the RMS value and the peak-to-peak value depends on the shape of the waveform. For a sinusoidal wave

$$V_{\text{rms}} = \frac{V_{pp}}{2\sqrt{2}} \tag{12}$$

$$\simeq 0.35\ V_{pp}. \tag{13}$$

For a square wave

$$V_{\text{rms}} = \frac{V_{pp}}{2} \tag{14}$$

$$= 0.5\ V_{pp}. \tag{15}$$

And for a triangular or sawtooth wave

$$V_{\text{rms}} = \frac{V_{pp}}{2\sqrt{3}} \tag{16}$$

$$\simeq 0.29\ V_{pp}. \tag{17}$$

Although some AC voltmeters read the true rms value of an AC voltage, many do not. A typical AC voltmeter will read the RMS value of the AC voltage only if that voltage is sinusoidal. For instance, the AC voltage ranges of the DMM can be used to measure the RMS value of a sinusoidal voltage, but other voltages must be observed on the oscilloscope.

One final parameter that is sometimes used to characterise a periodic voltage is the ripple factor. This, in a way, quantifies the variation of the signal relative to its mean value:

$$\text{ripple factor} = \frac{V_{\rm rms}}{V_{\rm DC}}. \tag{18}$$

11.1.2 Periodic signal with DC offset

One means of obtaining a periodic signal with a DC voltage is to include a diode as shown in Figure 19. When the input sinusoid is negative, the diode allows no current to flow so that the output voltage is as shown in Figure 20. The peak-to-peak amplitude is easily measured with the oscilloscope and the DC voltage can be measured either with the DMM or the oscilloscope. The RMS value is dependent on the shape of the waveform which in turn depends on the voltage across the diode when current is flowing. A useful formula is

$$V_{\rm rms}^2 = V_{\rm DC}(0.8V_{pp} - V_{\rm DC}) \tag{19}$$

which assumes that the waveform is parabolic. Note that this equation is only true for a $\frac{1}{2}$-rectified sine wave.

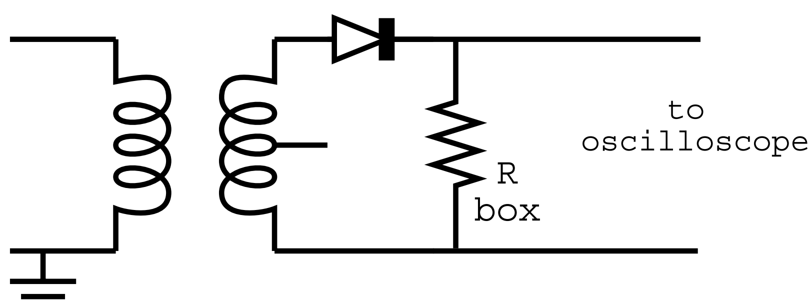

Figure 19: Circuit for periodic signal with DC offset

11.2 Laboratory exercises

1. Connect the transformer output of the bench AC power supply across the channel-1 input of the oscilloscope.

2. Measure the peak-to-peak amplitude based on the height of the signal on the screen and the vertical scale factor.

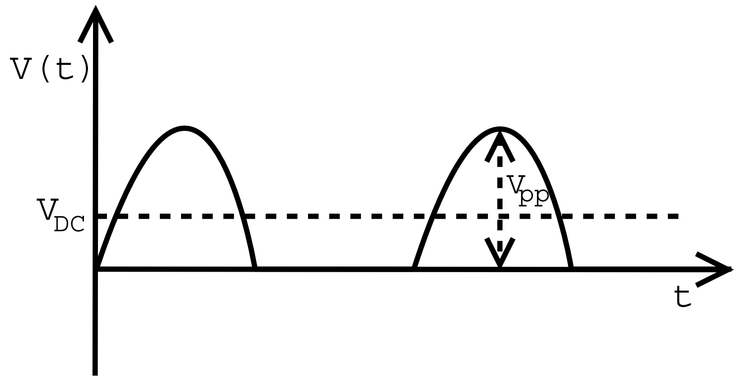

Figure 20: Periodic signal with DC offset

3. Calculate the RMS value of the sinusoidal voltage.

4. Connect the DMM across the same voltage source and measure the RMS voltage on the appropriate AC range of the DMM.

5. Compare these two values ($V_{\rm rms}$ based on the oscilloscope measurement of V_{pp} and $V_{\rm rms}$ measured with the DMM).

6. Use the oscilloscope's MEASURE function to determine attributes of the signal. Set the oscilloscpe to measure Pk-Pk and Cyc RMS for channel 1. The first value is the the peak-to-peak amplitude. The second value is the effective value of the entire voltage defined by

$$V_{\rm eff}^2 = \frac{1}{T}\oint V^2(t)\ dt \tag{20}$$

$$= V_{\rm DC}^2 + V_{\rm rms}^2. \tag{21}$$

   In this case, as $V_{\rm DC} = 0$, it is also the RMS value.

Q. Do these values of V_{pp} and $V_{\rm rms}$ agree with your previous measurements of these quantities, i.e. do they agree with the results from Steps 2, 3 and 4?

7. Set up the circuit shown in Fig.19 with a resistance of about 10 kΩ.

8. Use the oscilloscope's MEASURE function to measure Pk-Pk, Mean and Cyc RMS. The first two values should be the peak-to-peak amplitude and the DC value, as shown in Fig.20. The third value is the effective voltage, which is no longer equal to the RMS value because the DC voltage is non-zero. Make sure that your oscilloscope is set to DC coupling.

9. Check the DC value using the appropriate DC range of the DMM. Compare and comment.

10. Estimate the RMS voltage based on your measurement of $V_{\rm eff}$ and also on equation 19. Do so for both of your $V_{\rm DC}$ values if they are significantly different. (Suggestion: probably best to tabulate these results)

Q. How well do the (2 or 4) values agree?

Q. What is the ripple factor for this signal?

11. Add a 20 μF capacitor in parallel with the resistor to convert the circuit to a half wave rectifier with load (Figure 21). *Be sure the plus side of the capacitor is connected to the high side of the output.*

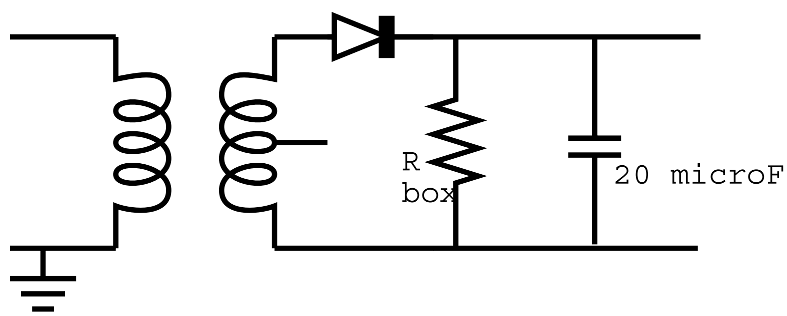

Figure 21: Half wave rectifier with load

12. Again measure the peak-to-peak voltage and the DC voltage.

13. Estimate the RMS voltage using equation 16 (your AC signal should be reasonably triangular).

14. Change the coupling to AC and use the MEASURE function to determine the peak-to-peak and RMS values of this AC waveform.

Q. How do these results compare to the values obtained for the previous signals?

Q. What is the ripple factor?

15. Vary the resistance in steps down to about 1.5 kΩ. For each value of resistance, use the DC Coupling to measure the DC voltage and the AC Coupling to measure the RMS voltage.

Q. What is the ripple factor in each case?

16. Make a graph of ripple factor as a function of the reciprocal of resistance.

Q. How does the ripple factor depend on the resistance? (be quantitative)

12 Half wave rectifier

12.1 Background

A basic use for the semiconductor diode is in a rectifier circuit for converting an alternating voltage to a steady one. The simplest rectifier circuit is the half-wave rectifier shown in Figure 22. When the transformer output voltage is positive, the diode conducts thereby charging the capacitor. The capacitor cannot discharge because the diode does not conduct in the reverse direction. Thus the capacitor charges up to the peak voltage at the transformer output then maintains a steady value.

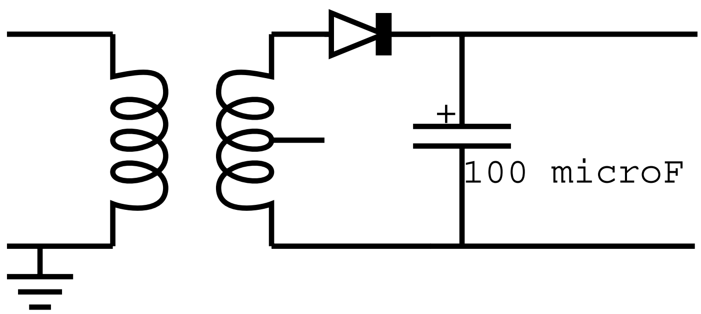

Figure 22: Half wave rectifier circuit

Because its output is constant, a rectifier can be used as a DC voltage source. When current is drawn from the rectifier, however, the capacitor loses charge and the voltage decreases. Thus the rectifier is not a well regulated source. Its voltage-current characteristic can be determined using a circuit like that of Figure 23, which is similar to what was used in the voltage sources lab.

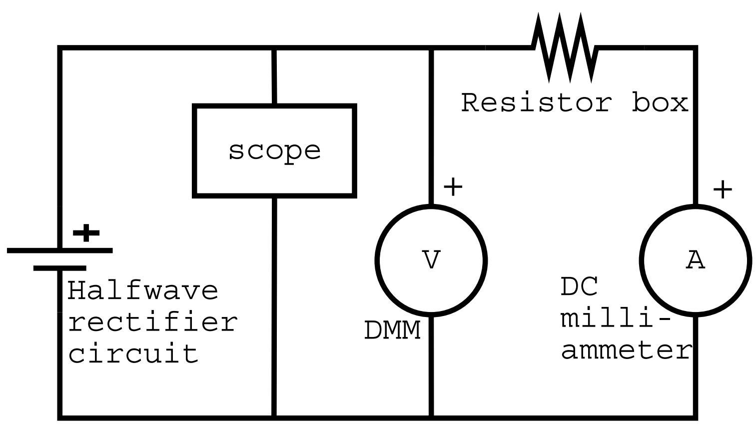

Figure 23: Circuit to determine voltage-current characteristic

The greater the capacitance used in the rectifier circuit, the smaller the voltage drop when a given current is drawn. Therefore, one way to improve the regulation of a rectifier is to increase the capacitance used. The largest values of capacitance require electrolytic capacitors. These capacitors function properly only if one terminal is always maintained at a positive voltage with respect to the other, therefore *rectifier circuits must be wired such that this is the case.*

12.2 Laboratory exercises

Q. Why will the voltage drop less for a given resistance if a greater capacitance is used in the rectifier circuit in Fig.22?

1. Set up the circuit shown in Fig.22 except use a 10 kΩ resistor instead of the capacitor. Connect the oscilloscope to the output and draw the resulting voltage function in your notebook.

2. Replace the resistor with a 100 μF capacitor and measure the open circuit voltage of the rectifier.

3. Add the resistor box (set to 10 $k\Omega$) and the DC milliammeter to your rectifier to obtain the circuit shown in Fig.23. The DC output of your rectifier now serves as a DC power supply.

4. Monitor the DC output with both the DC voltmeter and the oscilloscope (making sure to use DC coupling on your oscilloscope. Note the difference between AC and DC couplings. What is the oscilloscope doing?).

5. Starting with R = 10 kΩ then gradually reducing the resistance, at every second setting measure the output voltage and output current of your rectifier. To protect the diode, do not exceed ~50 mA of current. Sketch the traces seen on the oscilloscope for R = 10 kΩ and for the lowest lowest resistance used.

6. Plot the V-I characteristic of your half wave rectifier and determine its internal source resistance.

7. Compare the oscilloscope trace at 10 kΩ with that from the lowest resistance.

Q. Is the behavior what you expected? Explain your expectations and any differences between them and the results.

Q. Is the voltage decay linear during the part of the cycle that is not charging the capacitor? Should it be?

13 Full wave rectifier

13.1 Background

In the half wave rectifier the capacitor is charged only on the positive half of each cycle. In a full wave rectifier, the capacitor is also charged on the negative half of the cycle. Since the capacitor is charged twice in each cycle, there is a shorter time between chargings and hence a smaller voltage drop when the current is drawn from the rectifier. Thus its internal resistance is decreased.

A simple full wave rectifier uses two diodes and the centre tap of the transformer. This circuit is shown in Figure 24. When the transformer output is positive, one of the diodes conducts, charging the capacitor. When the transformer output is negative, the other diode conducts also charging the capacitor.

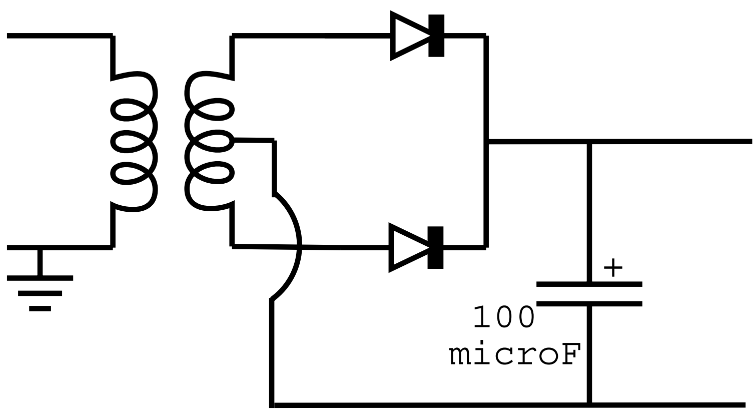

Figure 24: Simple full wave rectifier circuit

One drawback of the simple full wave rectifier is that the capacitor is charged to only one half of the peak voltage of the transformer output. The bridge full wave rectifier obtains the full peak value of the output by eliminating the centre tap and utilizing four diodes. The bridge rectifier circuit is shown in Figure 25. When the transformer output is positive, two of the diodes conduct, charging the capacitor. When the output goes negative, the other two diodes conduct, also charging the capacitor.

13.2 Laboratory exercises

1. Set up the bridge full wave rectifier circuit of Fig.25 except use a 10 kΩ resistor instead of the capacitor. Note that this figure is shown below (not above). Connect the oscilloscope to the output and draw the resulting voltage function in your lab book.

2. Replace the resistor with a 100 μF capacitor and measure the open circuit voltage. Monitor the voltage with the DC voltmeter and the oscilloscope, as done in the half wave rectifier lab.

3. Add the resistor box ($R = 10k\Omega$) and milliammeter to the circuit, as in the half wave rectifier lab. Measure and plot the voltage-current characteristic (for currents up to about 50 mA)

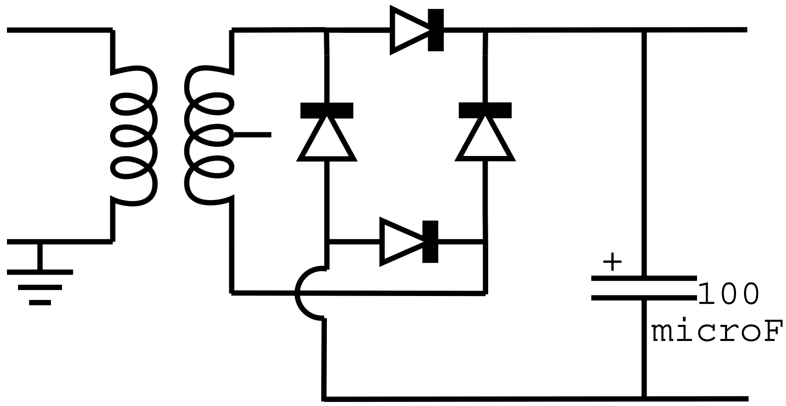

Figure 25: Bridge full wave rectifier circuit

and determine the internal source resistance.

4. Compare the open circuit voltage and source resistance with those of the half wave rectifier.

5. Compare the same two oscilloscope traces for the full wave rectifier with those of the half wave rectifier.

6. Repeat the measurements for the simple full wave rectifier of Fig.24.

7. Compare its open circuit voltage and source resistance with those of the half wave rectifier and those of the bridge full wave rectifier. Also compare the two oscilloscope traces.

14 RC circuits

14.1 Background

In the half-wave rectifier lab you created a DC source by charging a capacitor during one half of the AC cycle and using the diode to block the other half of the cycle, during which the capacitor otherwise would have discharged. As you decreased the DC current drawn from the capacitor by decreasing the value of the load resistor, you noted that the current drained the charge away from the capacitor during the half-cycle that the capacitor was not being charged. The more charge you drained away by drawing current, the lower the voltage became during that half-cycle. However, you noticed that there was not a linear relationship between this voltage decrease and the current drawn, as summarized in the plot of source resistance for this voltage source, which is characteristic of RC circuits.

14.1.1 Response to a voltage step

One possible configuration of a basic RC circuit is shown in Figure 26.

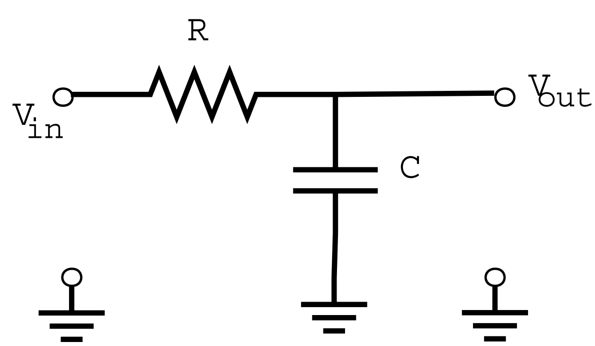

Figure 26: Basic RC circuit

If a DC voltage, V_{in}, is applied to the input at a time t=0, we can apply Kirchoff's laws to obtain an equation governing the voltage-current relationships in the circuit:

$$V_{\text{in}} - IR = \frac{Q}{C} = V_{\text{out}}. \tag{22}$$

where Q is the charge on the capacitor.

Remembering that $I = \frac{dQ}{dt}$ we see that this is actually a differential equation governing the charge across the capacitor. Equation 22 can be rewritten as

$$V_{\text{in}} = R\,\frac{dQ}{dt} + \frac{Q}{C}. \tag{23}$$

Or, rearranging further,

$$\frac{R\,dQ}{Q - V_{\text{in}}C} = -\frac{dt}{C}. \tag{24}$$

All the quantities on the left-hand side of Eq.24 are independent of time, and all the quantities on the right-hand side are independent of charge. Keeping in mind the condition that there be no charge at time zero (i.e. Q(t=0) = 0), we can integrate to find a solution to the above differential equation:

$$Q(t) = CV_{\text{in}}\left(1 - e^{-t/RC}\right) \tag{25}$$

From this equation for charge as a function of time we can obtain current as a function of time:

$$I(t) = \frac{dQ}{dt} = \left(\frac{V_{\text{in}}}{R}\right) e^{-t/RC}. \tag{26}$$

This gives us an exponentially decreasing voltage drop across the resistor and an output voltage which asymptotically approaches V_{in} (from Q/C).

You can see from the form of the exponential that the quantity RC has the units of time and is the natural time scale, or time constant, for the exponential. When $t = RC$, the exponential has reduced to $\frac{1}{e}$ of its initial value, about 37%.

14.1.2 Response to a sinusoidal voltage

If the input voltage is not constant but rather sinusoidal, we have the same differential equation to solve (Eq.23), however the solution takes a different form since the input voltage is now time-dependent ($V_{\text{in}} = V_0 \sin(\omega t)$). We expect a sinusoidal response and try a solution of the form

$$Q = Q_0 \sin(\omega t + \phi). \tag{27}$$

Substituting into Eq.23,

$$V_0 \sin(\omega t) = R\omega Q_0 \cos(\omega t + \phi) + \frac{Q_0}{C}\sin(\omega t + \phi). \tag{28}$$

Using the trigonometric identities

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \tag{29}$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \tag{30}$$

we can rewrite the previous equation as

$$V_0 \sin(\omega t) = \left(\frac{Q_0}{C}\cos\phi - R\omega Q_0 \sin\phi\right)\sin(\omega t) + \left(R\omega Q_0 \cos\phi + \frac{Q_0}{C}\sin\phi\right)\cos(\omega t). \tag{31}$$

Since it is not possible to have the $\cos(\omega t)$ term equal to the $\sin(\omega t)$ terms for all times, the $\cos(\omega t)$ must be 0. Therefore,

$$\tan\phi = -\omega RC. \tag{32}$$

All that is left is

$$V_0 \sin(\omega t) = \left(\frac{Q_0}{C}\cos\phi - R\omega Q_0 \sin\phi\right)\sin(\omega t). \tag{33}$$

Therefore,

$$V_0 = \frac{Q_0}{C}\left(1 + R^2C^2\omega^2\right)\cos\phi. \tag{34}$$

Since V_0 is something we specify, equation 34 is really giving us the charge on the capacitor (Q_0) as a function of V_0, ω, R and C. In order to predict the output voltage (that is, the quantity you observe on your scope), you will need to manipulate equation 34 a little.

Note also that equation 32 specifies the phase shift, ϕ, of the output signal. If ω happens to equal $\frac{1}{RC}$, then $\phi = -\frac{\pi}{4}$. At this particular frequency value, the output voltage amplitude is $\frac{1}{\sqrt{2}}$ times input voltage amplitude (V_0). This frequency is referred to as the 3 dB falloff point since $20\log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -3$ dB.

We can see from the form of $\tan\phi = -\omega RC$, which must be a unitless number, that the quantity ωC has units of impedance $^{-1}$. The capacitor in this circuit can be represented as if it had a complex impedance (real and imaginary parts) that is a function of frequency. The need for a complex impedance can be seen by noting that the phase shift between the output and input could never be accomplished with a purely resistive impedance. The magnitude of this complex impedance is $Z = \frac{1}{\omega C}$.

14.2 Laboratory exercises

In this lab you will explore the quantitative response of RC circuits, first under discrete changes in voltage, then with sinusoidal voltages.

14.2.1 Square wave response

1. Connect the circuit of Figure 26, using a function generator for the input voltage, a 10 $k\Omega$ resistor, and a 100 nF capacitor.

2. Adjust the function generator such that the experimental situation is the same as the one for which the equations were solved above. Set the function generator to a square wave output with a frequency of 30 Hz. Adjust the DC offset of the function generator until the lowest voltage of the square wave is at zero volts with respect to the ground.

3. Monitor the input on Ch 1 of the oscilloscope, making sure to use the DC coupling for all measurements.

4. Adjust the oscilloscope trigger and display until you get a voltage step displayed. Note the value of the step-voltage change.

5. Monitor the output taken across the capacitor on Ch 2 of the oscilloscope. Sketch the output in your lab notebook.

6. Using the display, determine the actual RC time constant.

Q. Do the actual and expected RC time constants agree?

7. Modify the frequency of the signal generator to look at the response of the circuit for 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, and 30 kHz. Describe what you see and explain the response of the output signal, noting any behavior for which you do not have an explanation.

8. Remove the DC offset of the function generator and repeat a few of the measurements.

Q. Why doesn't the DC offset affect the character of the response?

14.2.2 Sinusoidal response

When a sinusoidal voltage is applied to the input, we get a sinusoidal voltage across the capacitor in Fig. 26, but this voltage is out of phase with the input voltage.

1. Use the same set up as before except with a sinusoidal wave. Adjust the DC offset until the input and output voltages have a mean values of zero (i.e. the input wave must not have a DC component, only AC).

2. Monitor the response of the circuit for frequencies of 30 Hz, 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, and 30 kHz. For each, determine and tabulate
   (a) the peak-to-peak output voltage,
   (b) the ratio of the peak-to-peak output voltage and the peak-to-peak input voltage,
   (c) the phase difference between input and output voltages.

3. Compare to the expected values.

4. Graph b) and c).

5. Find the –3dB point experimentally by varying the frequency. Compare this to the expected value.

Q. Does this circuit pass high frequencies or low frequencies to the output?

6. Calculate the magnitude of the complex impedance, Z, for the frequencies used to make your table. Comment on your findings.

Q. Clearly this circuit can't be viewed as a simple voltage divider where the capacitor is replaced by a frequency-dependent resistor of impedance Z, because the phase would have to be the same. Can Z for this inductor be thought of as a "phase-delayed" voltage divider? Use the data from your table to support your answer.

7. Summarize how the capacitor works in this circuit using standard English prose.

15 RL circuits

15.1 Background

In this lab, we will explore an RL circuit, which has the same setup as the RC circuit but with an inductor replacing the capacitor (see Figure 27).

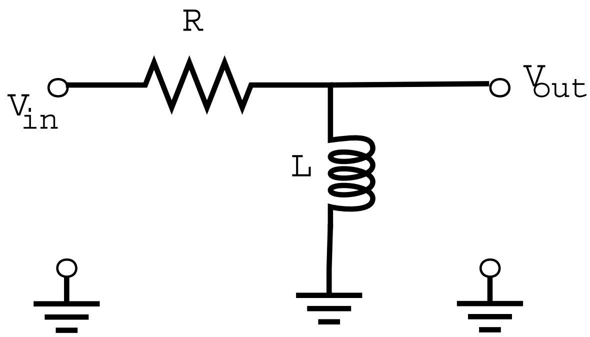

Figure 27: Basic RL circuit

15.1.1 Response to a voltage step

If a DC voltage, V_{in}, is applied to the input at a time $t = 0$, we can apply Kirchoff's laws to obtain an equation governing the voltage-current relationships in the RL circuit:

$$V_{\mathrm{in}} - IR = L\,\frac{dI}{dt} = V_{\mathrm{out}}. \tag{35}$$

This is in fact a differential equation governing the current through the inductor. Rewriting to make this more obvious:

$$V_{\mathrm{in}} = IR + L\,\frac{dI}{dt}. \tag{36}$$

Following a procedure similar to that which we used for the capacitor, we can find a solution for the current, $I(t)$, and for the output voltage, V_{out}:

$$I(t) = \frac{V_{\mathrm{in}}}{R}\left(1 - e^{-tR/L}\right) \tag{37}$$

$$V_{\mathrm{out}} = V_{\mathrm{in}}\, e^{-tR/L}. \tag{38}$$

This gives us an exponentially decreasing voltage drop across the inductor, decaying from the value of the input voltage step.

You can see from the form of the exponential that the quantity $\frac{L}{R}$ has units of time, and is the natural time scale, or time constant, for the exponential. When $t = \frac{L}{R}$, the exponential has been reduced to $\frac{1}{e}$ of its initial value, about 37%.

15.1.2 Response to a sinusoidal voltage

If the input voltage is not constant but rather is sinusoidal, we have the same differential equation to solve, namely Eq.36, however, since the input voltage is now time-dependent ($V_{\text{in}} = V_{\text{out}} \sin(\omega t)$), the solution takes a different form. We expect a sinusoidal response, e.g.

$$I = I_0 \sin(\omega t + \phi). \tag{39}$$

Substituting Equation 39 and the explicit expression for V_{in} into Eq.36:

$$V_0 \sin(\omega t) = RI_0 \sin(\omega t + \phi) + L\omega I_0 \cos(\omega t + \phi). \tag{40}$$

Using the trigonometric identities written down in Eq.29 and Eq.30, the above equation can be rewritten as

$$V_0 \sin(\omega t) = [(RI_0)\cos\phi - (L\omega I_0)\sin\phi]\sin(\omega t) + [(L\omega I_0)\cos\phi + (RI_0)\sin\phi]\cos(\omega t). \tag{41}$$

As was the case for the RC equation, it is not possible to have the $\cos(\omega t)$ term equal to the $\sin(\omega t)$ terms for all times and therefore the $\cos(\omega t)$ term must be zero. As a result,

$$\tan\phi = -\frac{\omega L}{R}. \tag{42}$$

Now all that is left is

$$V_0 \sin(\omega t) = [(RI_0)\cos\phi - (L\omega I_0)\sin\phi]\sin(\omega t). \tag{43}$$

Therefore,

$$V_0 = -(I_0 \omega L)\left[1 + \frac{R^2}{L^2\omega^2}\right]\sin\phi. \tag{44}$$

Since V_0 is something we specify, equation 44 is really giving us the current in the circuit (I_0) as a function of V_0, ω, R and L. In order to predict the output voltage (that is, the quantity you observe on your scope), you will need to manipulate equation 44 a little.

Note also that equation 42 specifies the phase shift, ϕ, of the current, relative to the input voltage. If ω happens to equal $\frac{R}{L}$, then $\phi = -\frac{\pi}{4}$. At this particular frequency value, the output voltage amplitude is $\frac{1}{\sqrt{2}}$ times input voltage amplitude (V_0). This frequency is referred to as the 3 dB falloff point since $20\log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) = -3$ dB.

There is a pleasing symmetry of this result and the corresponding calculation for capacitors. As ω goes to ∞, the $\sin\phi$ phase factor goes to -1, and we get the result expected from applying a time varying voltage to an inductor alone (i.e. the impedance of the inductor is much greater than the impedance of the resistor as $\omega \to \infty$). Whereas with a capacitor, the $\cos\phi$ phase factor goes to $+1$ at $\omega = 0$, and we get the result expected from applying a time varying voltage to a capacitor alone (i.e. the impedance of the capacitor is much larger than the impedance of the resistor as $\omega \to 0$).

We can see from the form of $\tan\phi = -\frac{\omega L}{R}$, which must be a unitless number, that the quantity ωL has units of impedance. The inductor in this circuit can be represented as if it had a complex impedance (real and imaginary parts) and was a function of frequency. The need for a complex impedance can be seen by noting that the phase shift between the output and input could never be accomplished with a purely resistive impedance. The magnitude of this complex impedance is $Z = \omega L$.

15.2 Laboratory exercises

15.2.1 Square wave response

1. Connect the circuit shown in Figure 27 using a 1 kΩ resistor and an 18-gauge air core inductor with a nominal inductance of 5.5 mH (±2%). Use a function generator for the input voltage, setting it to a square wave output with a frequency of 1 kHz.
2. Monitor the input on Ch 1 of the oscilloscope, making sure to use the DC coupling for all measurements. Adjust the oscilloscope trigger and display until you get a voltage step displayed. Note the value of the step-voltage change.
3. Monitor the output taken across the inductor on Ch 2 of the oscilloscope. Sketch the output in your lab notebook.
4. Using the display, determine the actual $\frac{L}{R}$ time constant.

Q. Do the expected and actual $\frac{L}{R}$ time constants agree?

5. Modify the frequency of the signal generator to look at the response of the circuit for 1 kHz, 3 kHz, 10 kHz, 30 kHz, and 100 kHz.
6. Describe what you see and explain the response of the output signal, noting any behavior for which you do not have an explanation. You are encouraged to discuss this with your colleagues.

15.2.2 Sinusoidal response

When a sinusoidal voltage is applied to the input, we get a sinusoidal voltage across the inductor in Fig. 27, but this voltage is out of phase with the input voltage.

1. Use the same set up as before except with a sinusoidal wave. Adjust the DC offset until the input and output voltages have a mean value of zero.
2. Monitor the response of the circuit for frequencies of 30 Hz, 100 Hz, 300 Hz, 1 kHz, 3 kHz, 10 kHz, and 30 kHz. For each, determine and tabulate
   (a) the peak-to-peak output voltage,
   (b) the ratio of the peak-to-peak output voltage and the peak-to-peak input voltage,

(c) the phase difference between input and output voltages.

3. Compare to the expected values. Note that Eq.42 specifies the phase shift for the current, whereas you have measured the phase shift for the voltage which, for an inductor, is proportional to the derivative of the current.

4. Graph b) and c).

5. Find the –3dB point experimentally by varying the frequency, and compare this to the expected value.

Q. Does this circuit pass high frequencies or low frequencies to the output?

6. Calculate the magnitude of the complex impedance, Z, for the frequencies used to make your table. Comment on your findings.

Q. Clearly this circuit can't be viewed as a simple voltage divider where the inductor is replaced by a frequency-dependent resistor of impedance Z, because the phase would have to be the same. Can Z for this inductor be thought of as a "phase-delayed" voltage divider? Use the data from your table to support your answer.

7. Summarize how the inductor works in this circuit using standard English prose.

16 Filters

16.1 Background

For the RC and RL circuits, it was apparent that the output voltage varied considerably as the frequency was changed. Any circuit that allows some frequencies through while greatly attenuating signals at other frequencies is called a filter. There are three basic types of filters: low pass filters (left side of Figure 28), high pass filters (right side of Figure 28), and bandpass filters.

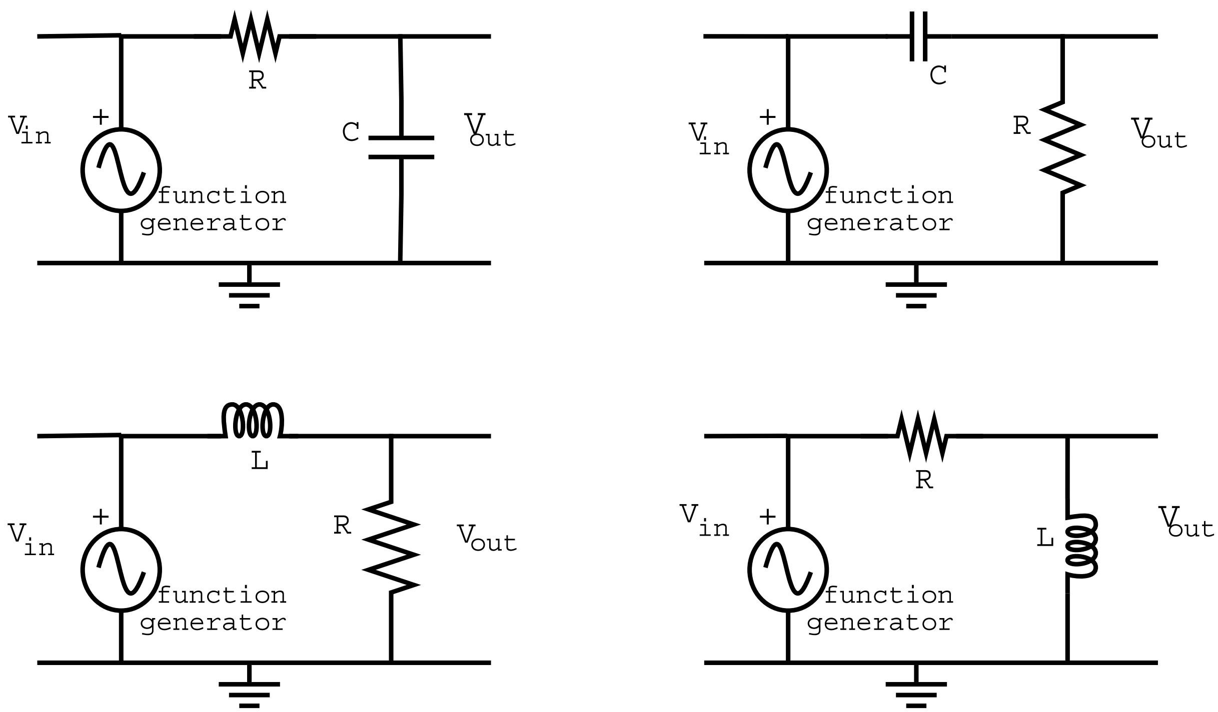

Figure 28: Low-pass and high-pass filters

The circuits on the left side are low-pass filters and the circuits on the right are high-pass filters.

16.1.1 High-pass and low-pass filters

The RC circuit that you considered in Lab #14, where you measured V_{out} across the capacitor, acted as a low pass filter. If you had been measuring V_{out} across the resistor instead, you would have found that it acted as a high-pass filter.

If the capacitor in the circuit is replaced by an inductor, the inverse is found. The circuit that you considered in Lab #15, where you measured V_{out} across the inductor, was a high pass filter. If the resistor and inductor are interchanged, the circuit becomes a low pass filter.

The low-pass filter has a phase shift that is always negative. The phase shift is nearly zero at low frequencies and approaches $-\frac{\pi}{2}$ radians as the frequency becomes large.

16.1.2 Resonant filters

If we now use an inductor and a capacitor in series, as shown in the circuit of Figure 29, then the result is a resonant filter. The capacitor blocks the low frequencies and the inductor blocks the high ones, allowing only a band of intermediate frequencies to pass through.

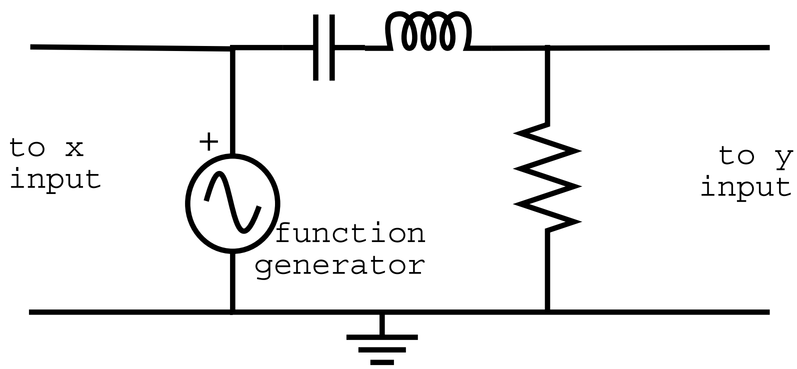

Figure 29: Resonant filter circuit

If the resistance in the circuit is sufficiently small, then the circuit is a narrow resonance (with a high Q value) and only a narrow band of frequencies will pass through with minimal attenuation. The phase shift varies from $+\frac{\pi}{2}$ at very low frequencies to $-\frac{\pi}{2}$ at very high frequencies, passing through zero at the resonant frequency.

Two parameters are used to characterize a resonant filter: its resonant frequency and its bandwidth. The resonant frequency is the frequency for which $\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}$ is greatest (nominally 1), for which the phase shift between input and output signals is zero, and where the impedance of the capacitor and the impedance of the inductor are equal. In other words, at this frequency

$$\frac{1}{\omega C} = \omega L, \tag{45}$$

which leads to

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}. \tag{46}$$

The bandwidth is defined as the difference between the half power points. In addition to being the frequencies at which the power in the circuit has dropped to half of its peak value, at these points, Im(Z) = Re(Z) = R. Furthermore, at these two frequencies (on either side of the resonant frequency) the phase shift is $\pm\frac{\pi}{4}$ radians. The bandwidth can be written as

$$\nu_{\mathrm{bw}} = \frac{R + R_{\mathrm{coil}}}{2\pi L} \tag{47}$$

where R_{coil} is the inherent resistance of the inductor. Note that the resonant frequency is determine by the inductance and the capacitance whereas the bandwidth depends on the inductance and the resistance.

16.1.3 Relationship between phase shift and amplitude

From the above discussion we see that there must be a relationship between the phase shift and the amplitude of the output voltage for a filter circuit. Define the amplitude ratio, A, as the amplitude of the output signal divided by that of the input. For the resonant filter, the angular frequency, the phase shift and the amplitude ratio satisfy the relation

$$\frac{R\omega}{A}\sin\phi = \frac{1}{C} - L\omega^2. \tag{48}$$

A graph of $\frac{R\omega}{A}\sin\phi$ as a function of ω^2 should yield a straight line with a y-intercept of $\frac{1}{C}$ and a slope of $-L$.

A second relation relates the amplitude ratio and the cosine of the phase shift:

$$\cos\phi = \left(1 + \frac{R_{\text{coil}}}{R}\right) A. \tag{49}$$

This relation allows one to determine R_{coil} at frequencies near the resonant frequency given measurements of R and A. Note the interesting fact that if the resistance of the coil is negligible, then the cosine of the phase is equal to the ratio of the amplitudes.

Combining the previous two equations, the relationship between the phase and the impedances is found

$$\tan\phi = \frac{Z_C - Z_L}{R + R_{\text{coil}}}. \tag{50}$$

16.2 Laboratory exercises

1. Measure the DC resistance of your coil using an ohmmeter.
2. Measure the values of L (18-gauge air-core inductor with nominal inductance of 5.5 mH), C (nominal capacitance of 0.1 μF), and your 470 Ω (nominal) resistor using the LCR meter.
3. Calculate the resonant frequency and bandwidth using equations 46 and 47.
4. Set up the circuit of Fig.29 using a resistor of about 470 Ω.
5. Find the frequency at which the phase shift is zero. This is the resonant frequency.
6. Measure this frequency and the amplitude ratio.
7. Vary the frequency in such a way as to have 5-6 data points both below and above the resonant frequency until you reach a frequency where the phase shift is greater than $+\frac{\pi}{3}$ radians at the low end and less than $-\frac{\pi}{3}$ radians at the high end. The point is to have a roughly symmetrical distribution of data points on plots where the x-axis is $\ln(\nu)$. Carefully measure the frequency, the amplitude ratio and the phase shift.

8. Plot two graphs: one showing amplitude ratio, and the other showing the phase shift as functions of logarithm of the frequency.

9. Determine the bandwidth from your phase shift graph.

10. Determine the bandwidth from your $V_{\text{out}}/V_{\text{in}}$ graph. Remember that the definition of bandwidth refers to the *half power*, not the half voltage point, so you first have to remember how power and voltage are related.

11. Compare with your experimentally determined values of resonant frequency and bandwidth.

12. Calculate the expected phase shift for the frequencies you considered. Add these values to your graph for ease of comparison.

13. Determine resistance of the coil based on your measurements of $V_{\text{out}}/V_{\text{in}}$ at resonance and of R. (note that the resistance of the coil is so small compared to the resistance of the resistor that you might not have the precision to determine this. If this is the case, simply make note of it in your lab book).

14. Describe how this circuit works in English prose.

17 Wave shaping

17.1 Background

When a sinusoidal signal is used to drive a circuit containing capacitors and/or inductors, the output signal is also a sinusoid, although usually with a different phase angle. However, signals other than sinusoids are generally distorted in circuits of this type. Sometimes this distortion can be useful, allowing one to change the shape of the input signal into a more desirable waveform at the output.

Figure 30 shows a simple wave shaping circuit with a square wave input. The device in the rectangle in this figure may be either a capacitor or an inductor or both in series. Depending on the frequency of the input, the output may be a slightly distorted square wave or a completely different waveform.

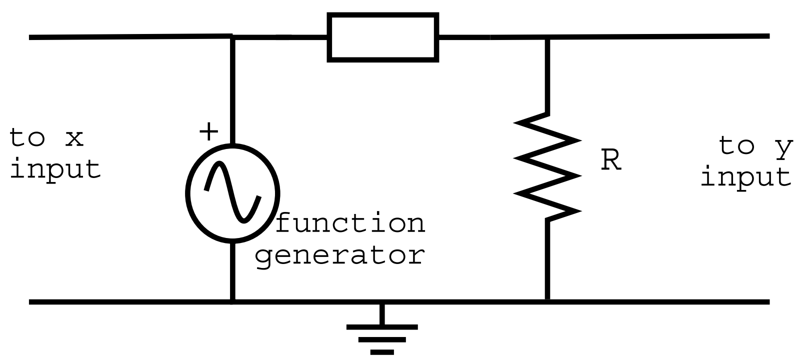

Figure 30: Wave shaping circuit

17.1.1 Using a capacitor

If a capacitor is in the rectangle the circuit is a high-pass filter. When the input jumps discontinuously from its negative to its positive value, the output makes a similar jump. The output does not then remain constant however, but rather falls exponentially toward zero. The fall time is defined as the time for the output to drop from 90% of its maximum value to 10%. If the decay is exponential, the fall time is ln(9) times the time constant, RC:

$$t_{\text{fall}} = \tau \ln(9) = RC \ln(9) \tag{51}$$

$$\simeq 2.20\,\tau = 2.20RC. \tag{52}$$

When the frequency is high, so that $T \ll \tau$, then the output is nearly square. For low frequencies, where $T \gg \tau$, sinusoids are shifted in phase by $\frac{\pi}{2}$ radians which is the same as taking a derivative of the sinusoidal function. Thus one might expect that for a low frequency square wave, the output would be proportional to its derivative. Of course, one cannot take the derivative of a discontinuous function like a square wave, but if the input were a low frequency triangular wave then the output would be nearly square.

17.1.2 Using an inductor

With an inductor in the rectangle, the circuit is a low-pass filter. When the input drops discontinuously from its positive to its negative value, the output takes a similar drop but exponentially rather than discontinuously. Again the fall time is the time for the signal to fall from 90% of its ultimate drop to 10%, and this time is ln(9) times the time constant, $\frac{L}{R}$:

$$t_{\text{fall}} = \tau \ln(9) = \frac{L}{R} \ln(9) \tag{53}$$

$$\simeq 2.20\ \tau = 2.20 \frac{L}{R}. \tag{54}$$

When the frequency is low, the output is essentially a square wave, and when the frequency is high the output is the integral of a square wave, i.e. a triangular wave.

17.1.3 Using a capacitor and an inductor

Adding a capacitor in series with the inductor produces a bandpass filter and if the resistance is sufficiently small the circuit is resonant. In this case, the output is generally a sinusoid at essentially the resonant frequency, but with an amplitude that decays exponentially toward zero. Thus there are two times that characterize the output: the period of the sinusoidal oscillation and the time constant of the exponential amplitude decay. When the frequency is very low the output approaches that of a high pass filter, and the output at very high frequencies approaches that of a low pass filter.

17.2 Laboratory exercises

1. Refresh your memory concerning the behaviour of RC circuits as high pass filters. Review your notes from that lab and/or construct an RC circuit to examine the response to a square wave at different frequencies.

2. If you have not already done so, construct a high-pass filter as in Fig.30, using a capacitor as the un-named device.

3. Use a a triangular wave and vary the frequency in steps of ten from about 2.0 MHz down to 20 Hz, observing and sketching one or two cycles of the output waveform at each frequency.

4. Replace the capacitor with an inductor to obtain a low pass filter and repeat the steps described above.

5. Compare and contrast the resulting outputs in these two different filters.

6. Add a capacitor in series with the inductor to obtain a resonant band pass filter.

7. With a square wave input at about 20 Hz, measure the period of the sinusoidal oscillation and the exponential amplitude decay.

8. Increase the frequency by factors of ten until you reach about 2.0 MHz, observing one or two cycles of the output at each frequency and making an appropriate sketch in your lab book.

9. Replace the square wave with a triangular wave and vary the frequency in steps of ten from about 2.0 MHz down to 20 Hz, observing and sketching one or two cycles of the output waveform at each frequency.

10. Observe the behavior of the resonant band pass filter for input square waves whose frequencies are near the resonant frequency or one of its odd submultiples (1/3, 1/5, etc). Describe what you observe.