The wavelength of neutrino and neutral kaon oscillations

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Abstract

Neutral kaons, and probably also neutrinos, exhibit oscillations between flavor eigenstates, as a result of being produced in a superposition of mass eigenstates. Several recent papers have addressed the question of the energies and momenta of the components of these states, and their effect on the coherence of the states and on the wavelength of the oscillations. We point out that the mass eigenstates need have neither equal momentum nor equal energy, but can nevertheless be coherent, and that a correct treatment of the kinematics recovers the usual result for the wavelength of the flavor oscillations.

1 Introduction

When neutral particles are produced in a flavor eigenstate that is not also a mass eigenstate, the resultant system is, in general, a superposition of mass eigenstates. If so, the system may oscillate between the different flavor eigenstates. This situation has been familiar for many years for the case of neutral kaons; if these are produced in one of the strangeness eigenstates, K^0 ($S = 1$)

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or $\bar{K}^0$ ($S = -1$), the system is a superposition of the mass eigenstates, K_L and K_S, and oscillates between the two strangeness eigenstates. Recently, strong evidence has been found that neutrinos show a similar behaviour [1,2,3,4].

The standard quantum-mechanical treatment of kaon oscillations [5] has been known for many years and results in an expression relating the wavelength of the strangeness oscillations to the mass difference between the mass eigenstates, $\delta m = m_L - m_S$. In the last decade, several papers have appeared which question this treatment, sometimes resulting in a different relation between the wavelength and δm. Srivastava et al. [6] derived a relation that differs by at least a factor of 2 from the standard result. The origin of this factor was studied by Lowe et al. [7] and by Burkhardt et al. [8] who found an error in Refs. [6], and demonstrated that the standard result is recovered when this error is corrected.

Other treatments [9,10,11,12,13,14,15,16] have studied some consequences of differing assumptions about the system and, in particular, the energies and momenta of the mass eigenstates. Since the K_L and K_S states that make up the oscillating $K^0 - \bar{K}^0$ system have different masses, they cannot have *both* the same momentum *and* the same energy. For example, Lipkin and collaborators [9,13] have studied the consequences of assuming either equal momentum or equal energy for the K_L and K_S. In several of the above papers these two kinematic assumptions are examined, and some papers predict a wavelength for the oscillations which differs by a factor of exactly 2 from the standard treatment. A recent paper by Okun et al. [16] gives a concise summary of the situation.

However, we are not free to choose the energy and momentum of the mass eigenstates. Usually, the neutral particles are produced either in a reaction (e.g. $\pi^- p \to \Lambda K^0$) for the case of kaons, or a decay (e.g. $\pi \to \mu \nu_\mu$) for neutrinos. In either case, the mass eigenstates have neither the same energy nor the same momentum for a given center-of-mass energy (mass in the rest frame of the source). The energies and momenta are determined by the kinematics. This was pointed out initially by Boehm and Vogel [17], Goldman [18], Srivastava et al. [6], Dolgov [15] and also by the present authors [7,8], who showed that when this is taken into account correctly, a consistent treatment of the kinematics follows and the standard result for the wavelength of oscillations is recovered.

However, Lipkin et al. [9,13] and Stodolsky [12] claim that the two mass eigenstates must have the same energy. In particular, Lipkin [13] states that mass eigenstates with different energy cannot interfere to produce the oscillations. In sect. 2 we show that this argument is incorrect and that the K_L and K_S states can indeed interfere to give strangeness oscillations. In sect. 3, we show that the mass eigenstates with kinematically correct energies and momenta produce oscillations with the same wavelength as in the standard treatment,

without any additional factors.

2 Coherence of the mass eigenstates

In this section, we examine the coherence of two interfering wave functions. Suppose the wave functions are plane waves, ψ_1 and ψ_2. These might be, for example, two parts of the wave function in a 2-slit optical experiment, or an electron diffraction experiment. If so, they will have the same energy and momentum, but for generality, we keep both energies and momenta distinct for now.

The wave function at the point of interference, (x, t), is

$$\psi = \psi_1 + \psi_2 = \sqrt{\frac{1}{2}} \left[\exp\{i(p_1 x - E_1 t)\} + \exp\{i(p_2 x - E_2 t + \phi)\} \right] \qquad (1)$$

where ϕ is some phase angle introduced by the geometry (for example, the path difference between the two slits to the interference point or some difference induced at the presumed common source of the two components). We assume only that ϕ is fixed and does not vary, e.g., randomly for different times due, for example, to fluctuations in the background medium (e.g., due to index of refraction fluctuations). It is in this sense that we may refer to the two waves as "coherent" [19]. The probability density is

$$\mid \psi \mid^2 = 1 + \cos[(p_1 - p_2)x - (E_1 - E_2)t + \phi] \qquad (2)$$

In general, the second term oscillates in time with a time period characteristic of the energy scale of the system. For the optical case, this is $\sim 10^{-15}$s, and in particle-physics experiments, the characteristic time is much shorter. In either case, this is well below the time resolution of any normal detector, so the second term averages to zero in the measurement, and ψ_1 and ψ_2 are therefore incoherent in the sense of Lipkin et al. [9,13]. There may be other reasons for the cross term in Eq. (2) to vanish (e.g. different spin wave functions or different internal states of particles associated with ψ_1 and ψ_2), but in the absence of any such reason, the rapid time dependence of the energy term is the only reason why ψ_1 and ψ_2 are orthogonal when E_1 and E_2 are different. It is this energy term that seems to be the basis for statement often made (e.g. in [13]) that the kaon mass eigenstates are incoherent, and will not interfere, unless they have the same energy.

However, we must examine Eq. (2) in the particular case where ψ_1 and ψ_2 are K_L and K_S states. Here, $E_1 - E_2$ is of order δm, which is $\sim 3 \times 10^{-6}$eV. The associated time scale is $\sim 2 \times 10^{-10}$s, which is readily measurable. Thus, even in this plane-wave case (that is, in the absence of packeting), the response of a detector to $\mid \psi \mid^2$ at a fixed point x will oscillate measurably in time as given by Eq. (2). Furthermore, in actual experiments, the kaons do not appear in continuous plane waves, but are formed by the scattering in a bounded region of space and time resulting, e.g., from detection of a particle in the incident beam. Since kaon velocities in experiments are usually an appreciable fraction of c, the distance scale is many cm, again readily measurable. Of course, if the measurement averages over time or distance scales large compared with these values, then the cross term vanishes and the states become incoherent.

Thus, although the fact that the kinematics of the kaon (or neutrino) production process preclude equal energy for the two interfering states, the states may nevertheless be coherent, in the sense that we refer to above, and interference may be observable in certain situations.

3   Kinematics

Here, we calculate the kinematics for a specific case. For definiteness, we choose the $K^0 - \bar{K}^0$ example rather than neutrinos because

(i) There are just two states rather than three or more,

(ii) Accurate numerical values are known for the masses and the mass difference,

(iii) For neutrinos, it has been argued that a full treatment requires the inclusion of the detector in the system [13]. However, for kaons, the $\Delta S = \Delta Q$ rule implies that the K^0 and $\bar{K}^0$ components can be identified from the kaon decay, without the need for a specific detector, thus simplifying the problem.

Neutrinos or kaons are produced either by a reaction such as

$$\pi p \to \Lambda K^0$$

or by a decay, for example

$$\pi \to \mu \nu_\mu.$$

For the first of these, the center-of-mass energies and momenta of the mass eigenstates are given by

$$p_i^2 = \frac{(s - m_i^2 - m_\Lambda^2)^2 - 4m_i^2 m_\Lambda^2}{4s}, \quad E_i = \frac{s + m_i^2 - m_\Lambda^2}{2\sqrt{s}} \tag{3}$$

where $i = S$ or L. Similar expressions hold for the neutrinos from pion decay. The quantity $\sqrt{s}$ is the total center-of-mass energy for a reaction or the mass of the decaying particle for a decay. Although there may be a spread in the value of $\sqrt{s}$ for the overall system wavepacket, our analysis proceeds component by component, i.e. at a precise value of $\sqrt{s}$ (within the constraints of the uncertainty principle). Thus $p_S \neq p_L$ and $E_S \neq E_L$. In the following, to make the equations more readable, we ignore CP violation and we omit the widths of the kaon states.

Since the above reaction produces a pure K^0 state, the wave function at the reaction point, where $x = t = 0$, is

$$| K^0 \rangle = \sqrt{\frac{1}{2}} (| K_L \rangle + | K_S \rangle). \tag{4}$$

This state develops in time as

$$\psi(x,t) = \sqrt{\frac{1}{2}} \{ \exp[i(p_L x - E_L t)] \, | \, K_L \rangle + \exp[i(p_S x - E_S t)] \, | \, K_S \rangle \}. \tag{5}$$

Since Eqs. (3) give p_i and E_i in the center-of-mass frame, the x and t in Eq. (5) should also be in this frame. However, Eq. (5) and all following equations involve only invariants, so may be reinterpreted in the lab frame. Note that the two components in Eq. (5) are coherent in the sense that we defined above: no fluctuation of the relative phase occurs at the source. Under the assumption of propagation in a vacuum, there is also no medium to induce phase fluctuations coupled to the medium. At (x,t), the probability amplitude for detecting a K^0 is

$$\langle K^0 \, | \, \psi(x,t) \rangle = \frac{1}{2} \{ \exp[i(p_L x - E_L t)] + \exp[i(p_S x - E_S t)] \} \tag{6}$$

using $\langle K^0 \, | \, K_L \rangle = \langle K^0 \, | \, K_S \rangle = \sqrt{\frac{1}{2}}$. This is just as Eq. (1) above (with $\phi = 0$), giving the probability density as

$$| \langle K^0 \mid \psi(x,t) \rangle \mid^2 = \frac{1}{2} \{ 1 + \cos [(p_L - p_S)x - (E_L - E_S)t] \}. \tag{7}$$

Now the observation time, t, is related to x by the velocity of the wave packet. We must be careful to verify that the wave-packet envelope satisfies the uncertainty principle while still being wide enough to allow Eq. (7); this is never a practical problem for realisable systems. The velocity of the wave packet can be defined, for example, by the average momentum and energy as

$$\beta = \frac{p_L + p_S}{E_L + E_S}, \tag{8}$$

which lies between the velocities p_L/E_L and p_S/E_S. The precise value of β used to define the observation point is not important provided that the point (x,t) is within the wave packet (see [8] for a detailed discussion). It is, however, crucial that the observation is made at a *single* space-time point; no meaning can be attached to the interference of wave functions at different values of x or t (see [8,16]). Thus Eq. (7) becomes

$$| \langle K^0 \mid \psi(x,t) \rangle \mid^2 = \frac{1}{2} \left[1 + \cos \{ (p_L - p_S)x - \frac{(E_L - E_S)}{\beta}x \} \right]. \tag{9}$$

Using

$$\frac{1}{\beta} = \frac{E_L^2 - E_S^2}{(p_L + p_S)(E_L - E_S)} = \frac{p_L - p_S}{E_L - E_S} + \frac{2m\delta m}{(E_L - E_S)(p_L + p_S)}, \tag{10}$$

Eq. (9) becomes

$$| \langle K^0 \mid \psi(x,t) \rangle \mid^2 = \frac{1}{2} \left[1 + \cos \frac{m \, \delta m}{p} x \right] \tag{11}$$

where m and p are the mean neutral kaon mass and momentum. Thus the wave number of strangeness oscillations in space is $k = m \, \delta m/p$. This is often expressed as an oscillation in time, in which case, the angular frequency is $\omega = m \, \delta m/E$. Both of these results are in agreement with the standard results [5], without any assumption about equality of momenta or energies.

4 Discussion

We conclude that a treatment of kaon and neutrino oscillations, with the kinematics treated in full, does indeed give the correct relation between the mass difference and the oscillation wavelength. Most of the recent literature is in agreement with this standard result for the wavelength, though sometimes using incorrect kinematics. If CP violation and the finite kaon lifetimes are incorporated in the above algebra, a more realistic result is obtained, with more cumbersome-looking equations, but the wavelength of the oscillations remains the same. The full equations are given in [7,8].

The algebra presented above treats the wave functions as plane waves. As indicated, a more complete description requires that these waves be used as components of wave packets for each particle. If this is done explicitly, the conclusions are unchanged (see [8] for a detailed discussion).

Lipkin [13] has suggested that the detector should also be included in the wave function of the system, since interaction with it depends on the details of the neutrino or kaon wave function. By choosing the kaon system here, we avoid this problem since the particles resulting from K^0 or $\bar{K}^0$ semileptonic decay identify the strangeness eigenstate, so no kaon detector, as such, is required. However, inclusion of a detector would not change our conclusions; at the zeros of the K^0 oscillation pattern, there are only $\bar{K}^0$ mesons and no K^0 mesons, so no detector could detect a K^0 at such a point.

If the mass eigenstates did in fact have equal momenta or equal energies, then this would imply a failure of 4-momentum conservation in the kaon or neutrino production process. Such a failure would be evident well outside the oscillation region; the K_L, which is the only state left in the asymptotic region, would, in principle, have an energy or momentum inconsistent with 4-momentum conservation.

In a recent preprint by Field [20], the kinematics and other aspects of the neutrino production are treated quite differently, giving a wide range of correction factors to the standard result for the wavelength. However, there is an error in his derivation of the pion decay rates (Eq. (7) of Ref. [20]). When this is corrected, the motivation for his later modifications to the standard treatment, with their bizarre physical consequences, is removed.

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