Approximate path planning

Computational Geometry
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Outline

Path planning

- Combinatorial
- Approximate

Combinatorial path planning

- **Idea: Compute free C-space combinatorially (= exact)**
- **Approach**
  - (robot, obstacles) => (point robot, C-obstacles)
  - Compute roadmap of free C-space
    - any path: trapezoidal decomposition or triangulation
    - shortest path: visibility graph

- **Comments**
  - Complete
  - Works beautifully in 2D and for some cases in 3D
    - Worst-case bound for combinatorial complexity of C-objects in 3D is high
  - Unfeasible/intractable for high #DOF
    - A complete planner in 3D runs in $O(2^{n\wedge\#DOF})$

Approximate path planning

- **Idea: Since you can't compute C-free, approximate it**
- **Approaches**
    - Graph search strategies
        - A*, weighted A*, D*, …
    - Sampling-based + roadmaps
        - probabilistic roadmaps, rrt, …
    - Potential field
    - Hybrid

- **Comments**
    - local minima, performance guarantees, completeness? optimality?

Approximate path planning

The concept of completeness is relaxed

- A planner is **resolution complete**:
  - finds a solution, if one exists, with probability —> 1 as the resolution of the sampling increases

- A planner is **probabilistically complete**:
  - finds a solution, if one exists, with probability —> 1 as computation time increases

Graph-search strategies

- Sample C-space with uniform grid/lattice
    - refined: quadtree/octree
    - This essentially “pixelizes" the space (pixels/voxels in C-free)
- Graph is implicit
    - given by lattice topology: move +/-1 in each direction, possibly diagonals as well
- Search the graph for a path from start to end
    - use heuristics to guide the search towards the goal
- Graph can be pre-computed (occupancy grid), or computed incrementally
    - one-time path planning vs many times
    - static vs dynamic environment

Graph-search strategies

- Dijkstra's algorithm
  - computes SSSP(vertex s)
  - priority-first search
    - d[v] = cost of getting from s to v
  - initialize
    - d[v] = inf for all v, d[s] = 0
  - greedily select the vertex with smallest priority, and relax its edges
    - use a priority queue to find smallest priority

Graph-search strategies

Dijkstra(vertex s)

- initialize
    - d[v] = infinity for all v, d[s] = 0
- for all v: PQ.insert(<v, d[v]>)
- while PQ not empty
    - u = PQ.deleteMin()
    - //claim: d[u] is the SP(s,u)
    - for each edge (u,v):
        - if v not done, and if d[v] > d[u] + edge(u,v):
            - d[v] = d[u] + edge(u,v)
            - PQ.decreasePriority(v, d[v])

no need to check if v is done,
because once v is done,
no subsequent relaxation can improve its d[]

usually not implemented

Graph-search strategies

Dijkstra(vertex s)

- initialize
    - d[v] = infinity for all v, d[s] = 0
- PQ.insert(<s, d[s]>)
- while PQ not empty
    - u = PQ.deleteMin()
    - for each edge (u,v):
        - if isFree(v) and d[v] > d[u] + edge(u,v):
            - d[v] = d[u] + edge(u,v)
            - PQ.insert(<v, d[v]>)

insert only the start

insert it
(even if it's already there)

isFree(v): is v in C-free

Graph-search strategies

- Dijkstra's algorithm
  - if only a path to a single vertex is required, a heuristic can be used to guide the search towards the goal

- A*
  - best-first search
  - **priority f(v) = g(v) + h(v)**
    - g(v): cost of getting from start to v
    - h(v): estimate of the cost from v to goal
  - Theorem: If h(v) is "admissible" (h(v) < trueCost(v—>goal)) then A* will return an optimal solution.
  - Dijkstra is (A* with h(v) = 0)
  - In general it may be hard to estimate h(v)
    - path planning: h(v) = EuclidianDistance(v, goal)

Graph-search strategies

- A* explores fewer vertices to get to the goal, compared to Dijkstra
  - The closer h(v) is to the trueCost(v), the more efficient
- Example
  - https://www.youtube.com/watch?v=DINCL5cd_w0

- Many A* variants
  - weighted A*
    - c x h() ==> solution is no worse than (1+c) x optimal
  - real-time replanning
    - if the underlying graph changes, it usually affects a small part of the graph ==> don't run search from scratch
    - D*: efficiently recompute SP every time the underlying graph changes
  - anytime A*
    - use weighted A* to find a first solution ; then use A* with first solution as upper bound to prune the search

Graph-search strategies

- Comments
  - Not complete
  - The paths may be longer than true shortest path in C-space
  - Resolution of lattice may not be sufficient to find a solution

Sampling

- When dimension of C-space is high => hard to construct C-obstacles exactly

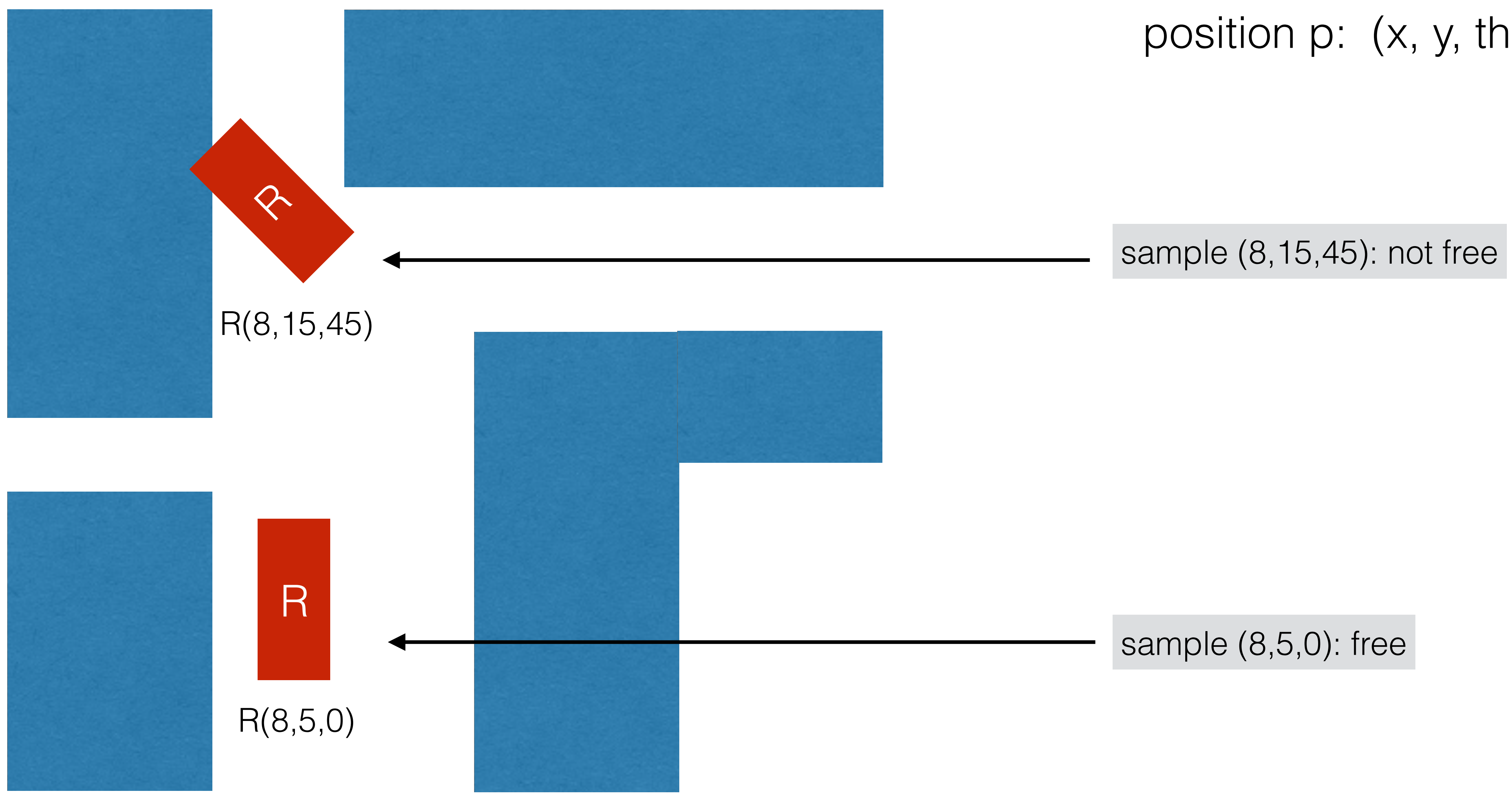
- Much easier to “sample”

    - sample(p)= isFree(p): would my robot , if placed in this configuration, intersect any obstacle?

robot can translate and rotate in 2D

C-space: 3D

position p: (x, y, theta)



How would you write: isFree((x,y,theta)) ?

Sampling

- You are not given the representation of C-free: Imagine being blindfolded in a maze
- Sampling: you walk around hitting your head on the walls
- Left long enough, after hitting many walls, you have a pretty good representation of the maze
- However the space is huge
    - e.g. DOF= 6: 1000 x 1000 x 1000 x 360 x 360 x 360
- So you need to be smart about how you chose the points to sample

Sampling-based planning

- Roadmap
  - Instead of computing C-free explicitly, sample it and compute a roadmap that captures its connectivity to the best of our (limited) knowledge
- Roadmap construction phase
  - Start with a sampling of points in C-free and try to connect them
  - Two points are connected by an edge if a simple quick planner can find a path between them
  - This will create a set of connected components
- Roadmap query phase
  - Use roadmap to find path between any two points

Sampling-based roadmap construction

```
• Generic-Sampling-based-roadmap:
  • V = p_start + sample_points(C, n); E = { }
  • for each point x in V:
      • for each neighbor y in neighbors(x, V):
            //try to connect x and y
            • if collisionFree(segment xy): E = E + xy
  • return (V, E)
```

- Algorithms differ in
  - sample_points(C, n) : how they select the initial random samples from C
    - return a set of n points arranged in a regular grid in C
    - return random n points
  - neighbors(x, V) : how they select the neighbors
    - return the k nearest neighbors of x in V
    - return the set of points lying in a ball centered at x of radius r
  - Often used: samples arranged in a 2-dimensional grid, with nearest 4 neighbors (d, 2^d)

Probabilistic Roadmaps (Kavraki, Svetska, Latombe, Overmars et al , 1996)

- Start with a *random* sampling of points in C-free
- Roadmap stored as set of *trees* for space efficiency
  - trees encode connectivity, cycles don’t change it. Additional edges are useful for shortest paths, but not for completeness
- Augment roadmap by selecting additional sample points in areas that are estimated to be “difficult”

```
(1)   N ← ∅
(2)   E ← ∅
(3)   loop
(4)      c ← a randomly chosen free
          configuration
(5)      N_c ← a set of candidate neighbors
           of c chosen from N
(6)      N ← N ∪ {c}
(7)      for all n ∈ N_c, in order of
          increasing D(c,n) do
(8)        if ¬same_connected_component(c,n)
            ∧Δ(c,n) then
(9)              E ← E ∪ {(c,n)}
(10)             update R's connected
                  components
```

- Components
  - sampling C-free: random sampling
  - selecting the neighbors: within a ball of radius r
  - the local planner delta(c,n): is segment cn collision free?
  - the heuristical measure of difficulty of a node

Probabilistic Roadmaps (Kavraki, Svetska, Latombe, Overmars et al , 1996)

- Comments
  - Roadmap adjusts to the density of free space and is more connected around the obstacles
  - Size of roadmap can be adjusted as needed
  - More time spent in the “learning” phase ==> better roadmap
  - Shown to be probabilistically complete
    - probability that the graph contains a valid solution —> 1 as number of samples increases

```
N ← ∅
E ← ∅
loop
    c ← a randomly chosen free
     configuration
    N_c ← a set of candidate neighbors
      of c chosen from N
    N ← N ∪ {c}
    for all n ∈ N_c, in order of
     increasing D(c,n) do
      if ¬same_connected_component(c,n)
       ∧Δ(c,n) then
            E ← E ∪ {(c,n)}
            update R's connected
             components
```

Probabilistic Roadmaps

- One of the leading motion planning technique
- Efficient, easy to implement, applicable to many types of scenes
- Embraced by many groups, many variants of PRM's, used in many type of scenes.
  - PRM*
  - FMT* (fast marching tree)
  - …
- Not completely clear which technique better in which circumstances

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Incremental search planners

- Graph search planners over a fixed lattice:
    - may fail to find a path or find one that's too long
- PRM:
    - suitable for multiple-query planners
- Incremental search planners:
    - designed for single-query path planning
    - incrementally build increasingly finer discretization of the configuration space, while trying to determine if a path exists at each step
    - probabilistic complete, but time may be unbounded

Incremental search planners

- Idea: Incrementally grow a tree rooted at “start” outwards to explore reachable configuration space
- RRT (LaValle, 1998)
- https://personalrobotics.ri.cmu.edu/files/courses/papers/Kuffner00-rrtconnect.pdf

```
BUILD_RRT(q_init)
  T.init(q_init);
  for k = 1 to K do
      q_rand ← RANDOM_CONFIG();
      EXTEND(T, q_rand);
  Return T
```

```
EXTEND(T, q)
  q_near ← NEAREST_NEIGHBOR(q, T);
  if NEW_CONFIG(q, q_near, q_new) then
      T.add_vertex(q_new);
      T.add_edge(q_near, q_new);
      if q_new = q then
          Return Reached;
      else
          Return Advanced;
  Return Trapped;
```

Figure 2: The basic RRT construction algorithm.

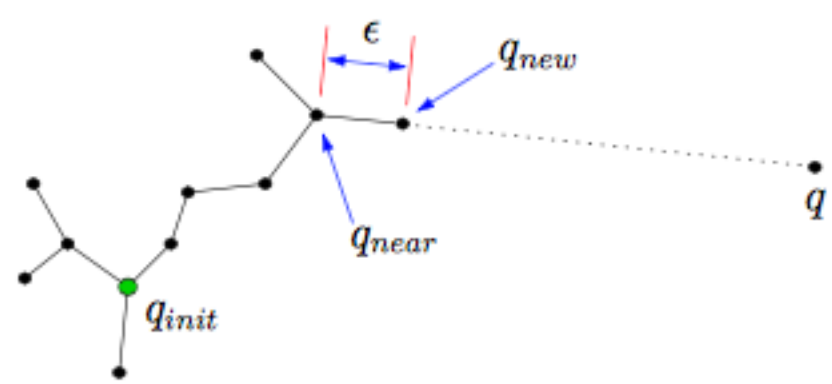

Figure 3: The EXTEND operation.

http://kevinkdo.com/rrt_demo.html

https://www.youtube.com/watch?v=MT6FyoHefgY

https://www.youtube.com/watch?v=E-IUAL-D9SY

https://www.youtube.com/watch?v=mP4ljdTsvxI

Potential field methods

- Idea [Latombe et al, 1992]
  - Define a potential field
  - Robot moves in the direction of steepest descent on potential function
- Ideally potential function has global minimum at the goal, has no local minima, and is very large around obstacles
- Algorithm outline:
  - place a regular grid over C-space
  - search over the grid with potential function as heuristic

https://www.youtube.com/watch?v=r9FD7P76zJs

Potential field methods

- Pro:
    - Framework can be adapted to any specific scene
- Con:
    - can get stuck in local minima
    - Potential functions that are minima-free are known, but expensive to compute

- Example: RPP (Randomized path planner) is based on potential functions
    - Escapes local minima by executing random walks
    - Succesfully used to
        - performs riveting ops on plane fuselages
        - plan disassembly operations for maintenance of aircraft engines

Self-driving cars

- Both graph search and incremental tree-based
- DARPA urban challenge:
  - CMU:
    - lattice graph in 4D (x,y, orientation, velocity); graph search with D*
  - Stanford: incremental sparse tree of possible maneuvers, hybrid A*
  - Virginia Tech: graph discretization of possible maneuvers, search it with A*
  - MIT: variant of RRT with biased sampling

A Survey of Motion Planning and Control Techniques for Self-driving Urban Vehicles, by Brian Paden,

Michal Cˇáp, Sze Zheng Yong, Dmitry Yershov, and Emilio Frazzoli

https://arxiv.org/pdf/1604.07446.pdf