

# **Algorithms for GIS:**

Computing visibility on terrains

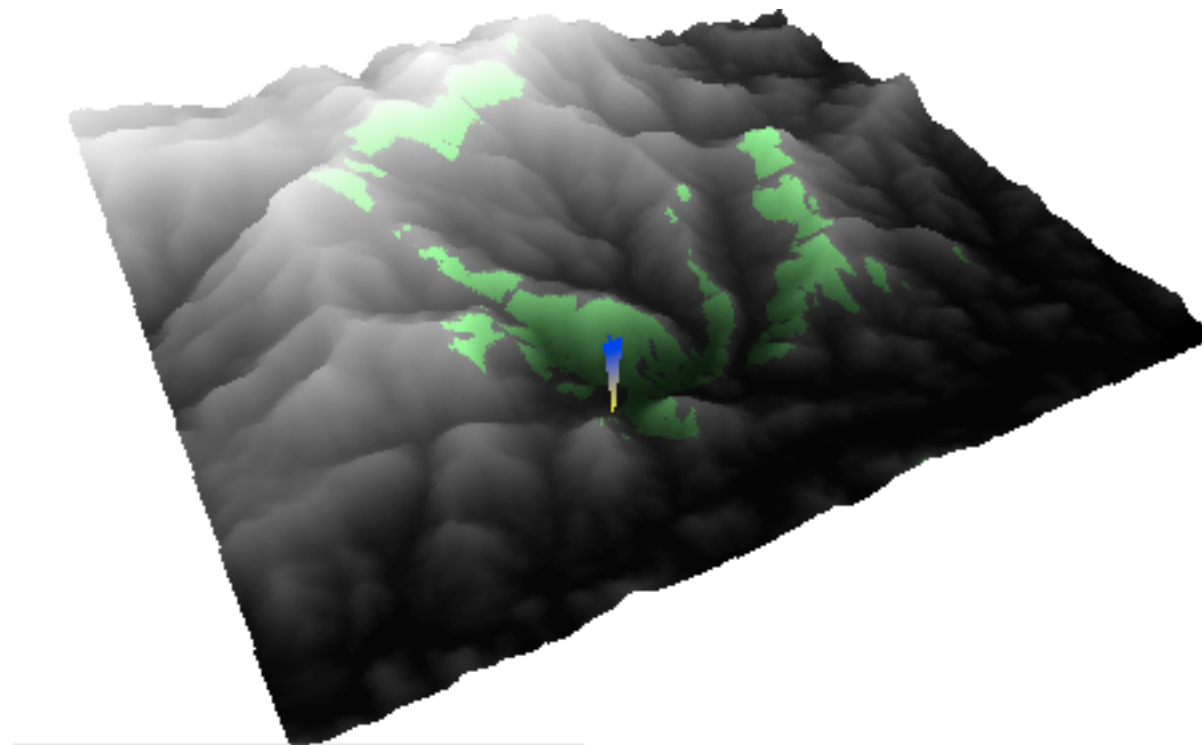
# Visibility on terrains

- Are two points (on a terrain) visible to each other?
- What can one see from a given point (on a terrain)?
- How much does the visible area increase if we stand on a 10ft ladder?
- What is the point with largest visibility?
- What is the point with lowest visibility?
- How to place an ugly pipe in a scenic area?
- How to place a scenic highway?
- What is the cumulative visible area from these set of cell towers?
- Find a set of tower locations to cover the terrain
- ...

# Visibility on terrains

## Problem:

- Terrain model  $T$  + viewpoint  $v$
- Compute the **viewshed** of  $v$ : the set of points in  $T$  visible from  $v$



Sierra Nevada, 30m resolution

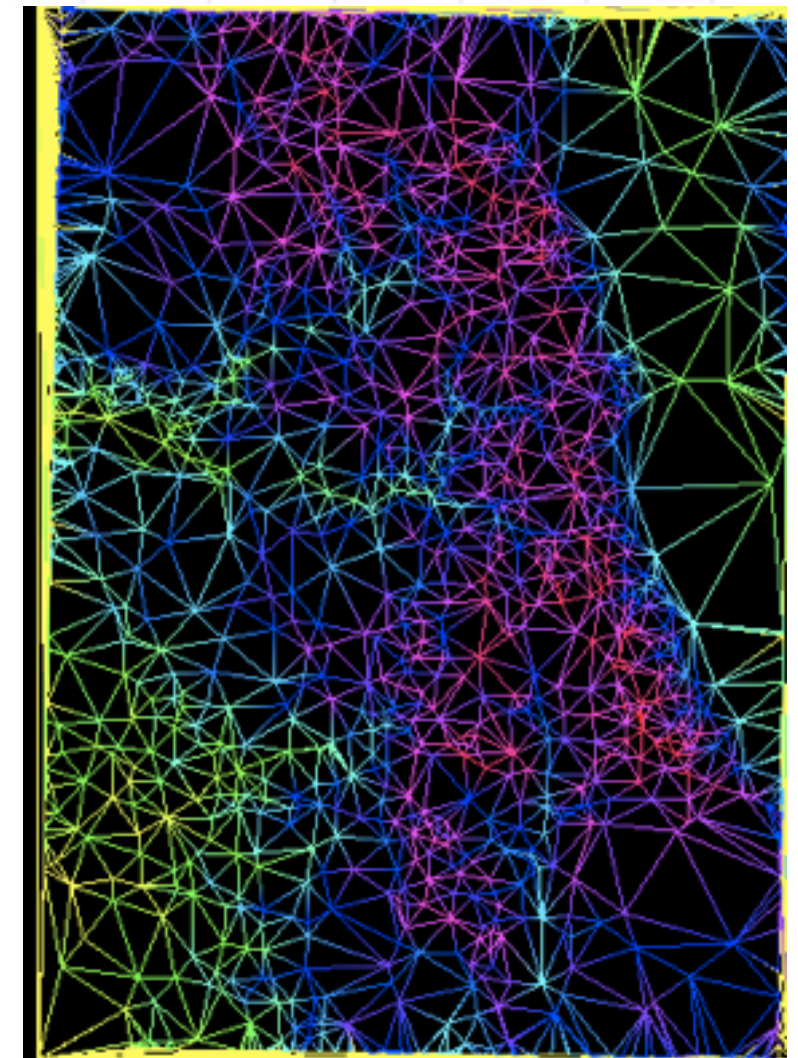
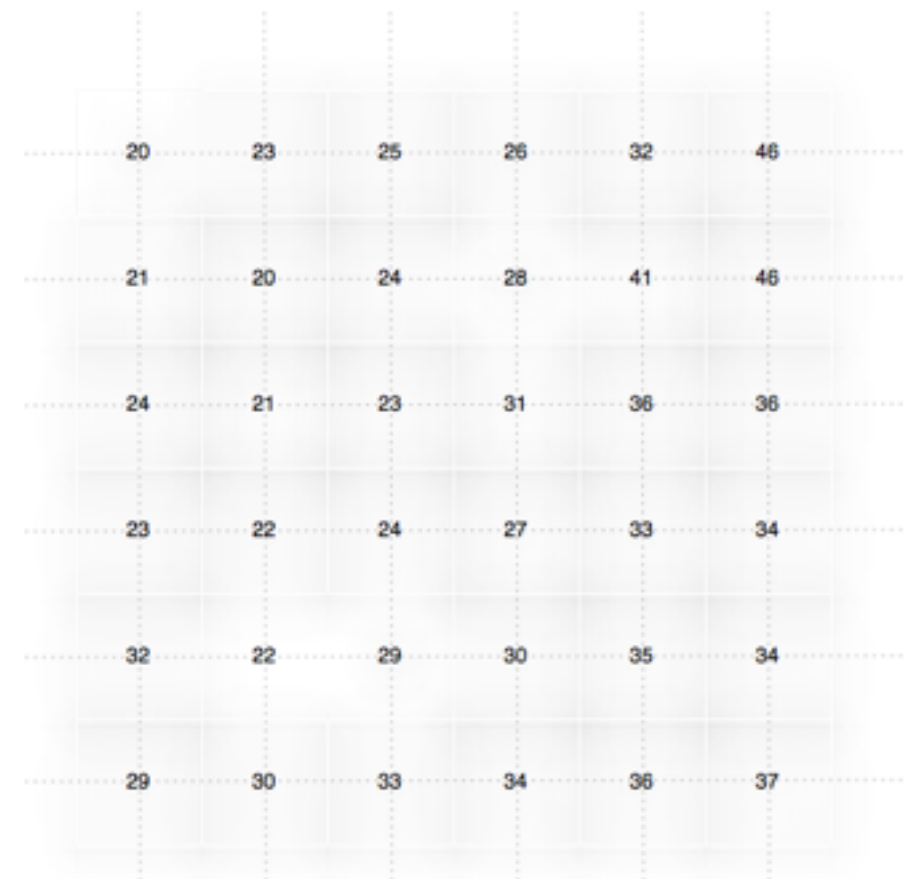
# Visibility on terrains

Input: terrain model

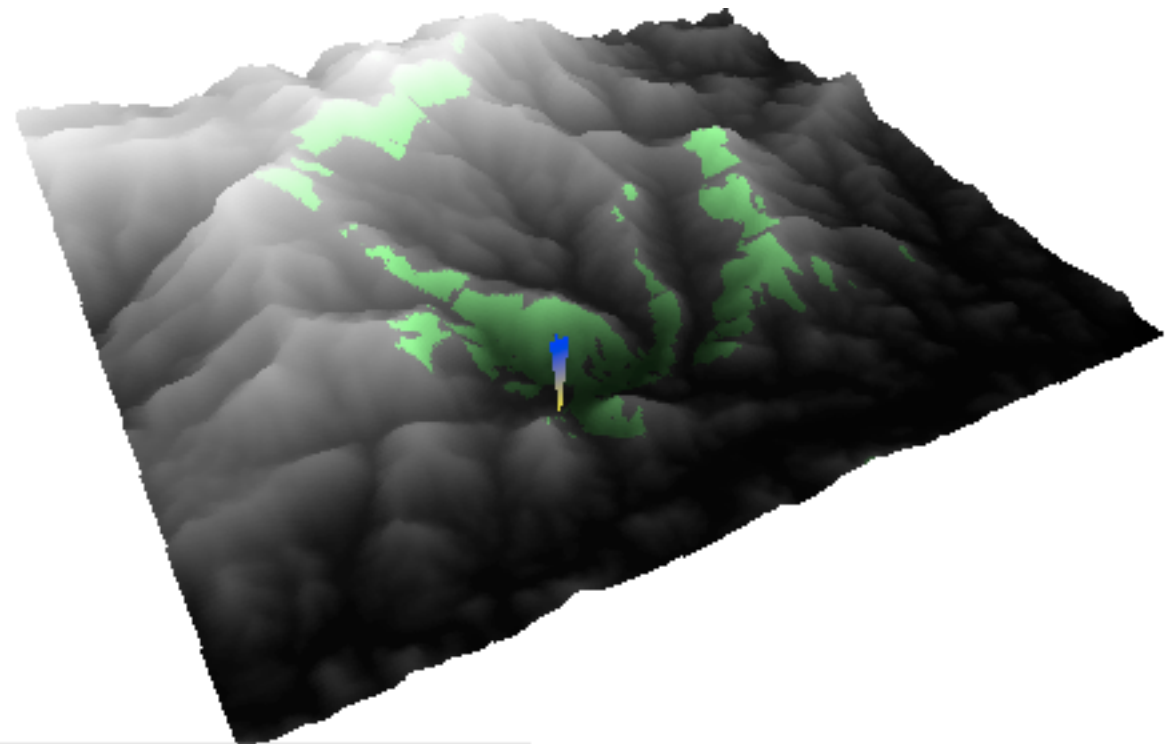
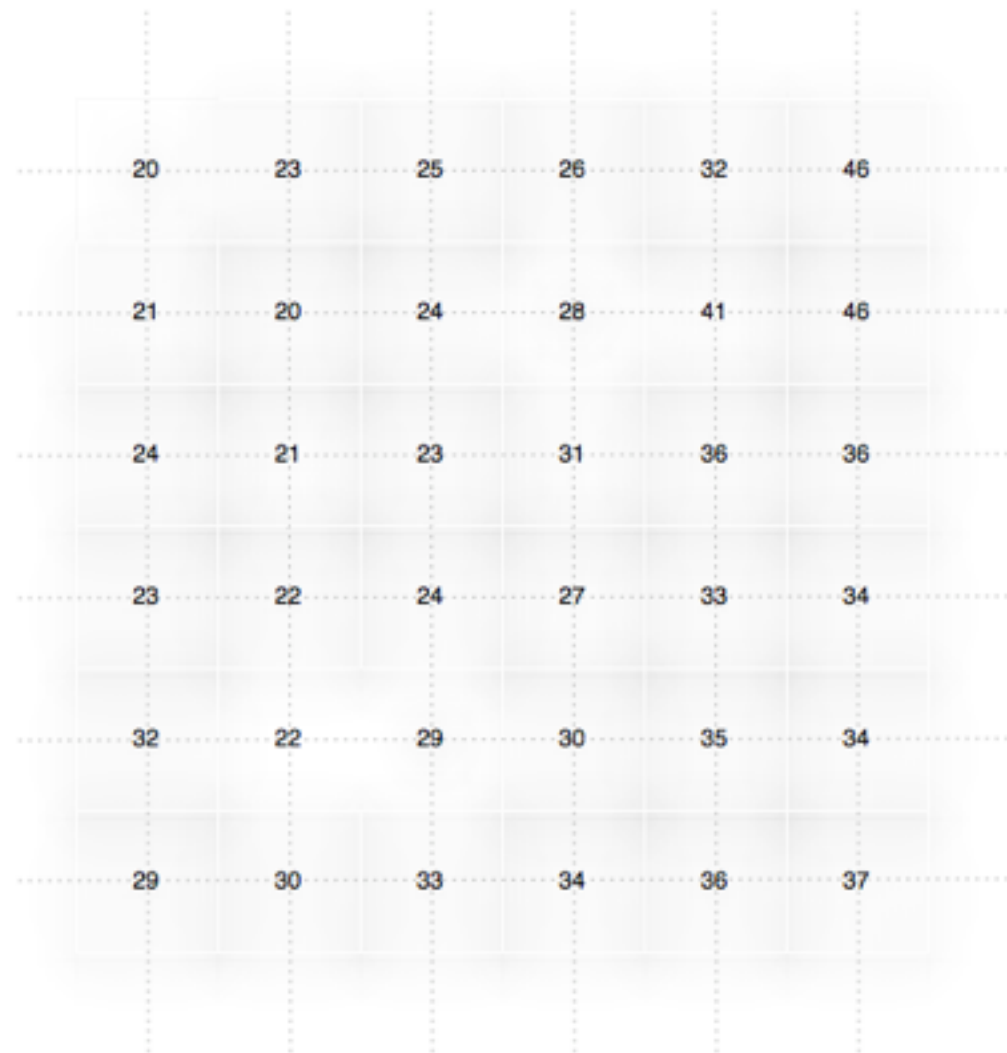
- grid or TIN (triangulation)

Output: viewshed model

- grid elevation model ==> grid viewshed
- TIN elevation model ==> TIN viewshed (...more later...)

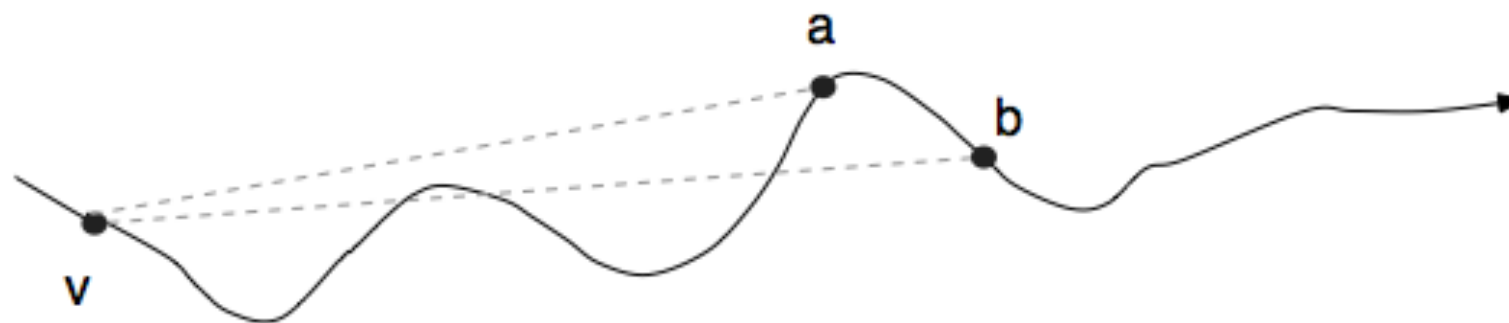
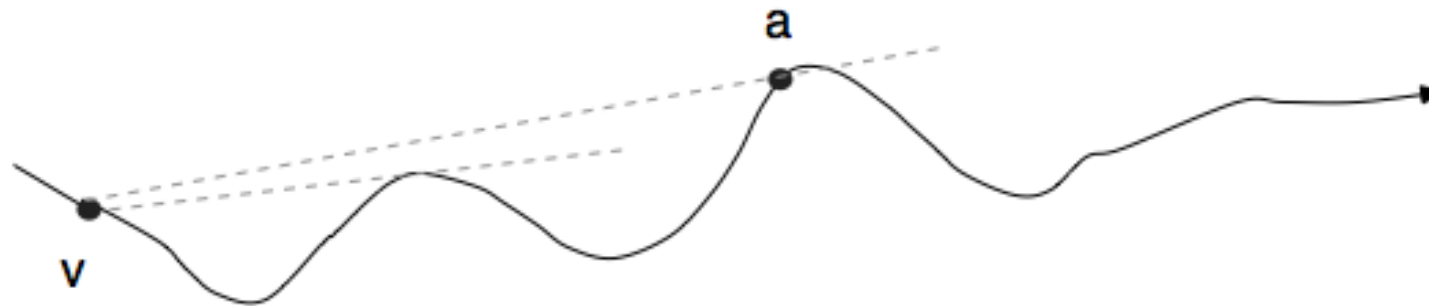


# Visibility on grid terrains



Sierra Nevada, 30m resolution

# Visibility



$(u,v)$  visible iff segment  $uv$  does not intersect  $T$

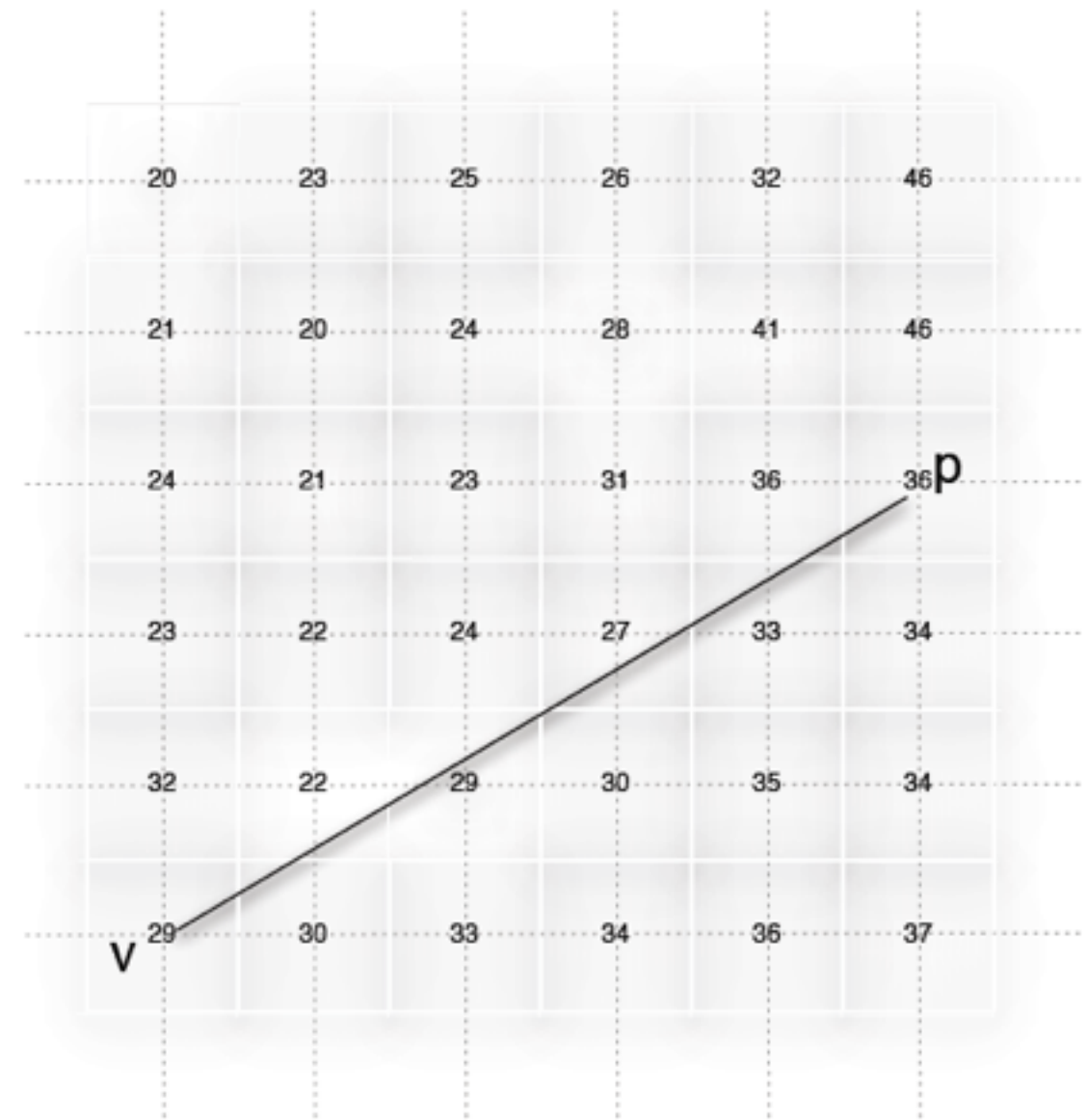
$uv$  is called line-of-sight (LOS)

# Basic viewshed algorithm

Input: elevation grid

Output: visibility grid, each point marked visible/invisible

- For each  $p$  in grid
  - compute intersections between  $vp$  and grid lines
  - if all these points are below  $vp$  then  $p$  is visible



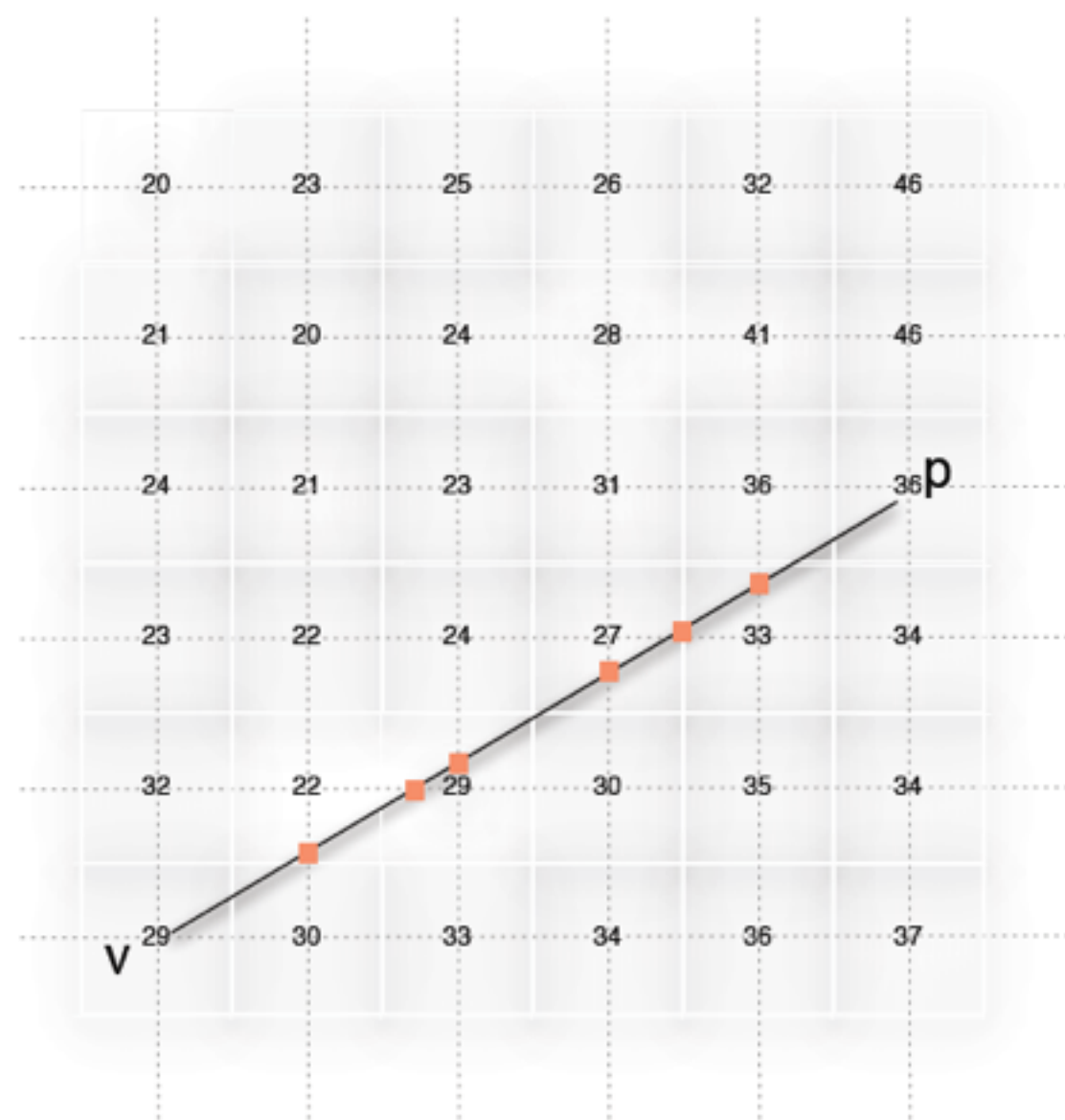


# Basic viewshed algorithm

Input: elevation grid

Output: visibility grid, each point marked visible/invisible

- For each  $p$  in grid
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# Basic viewshed algorithm

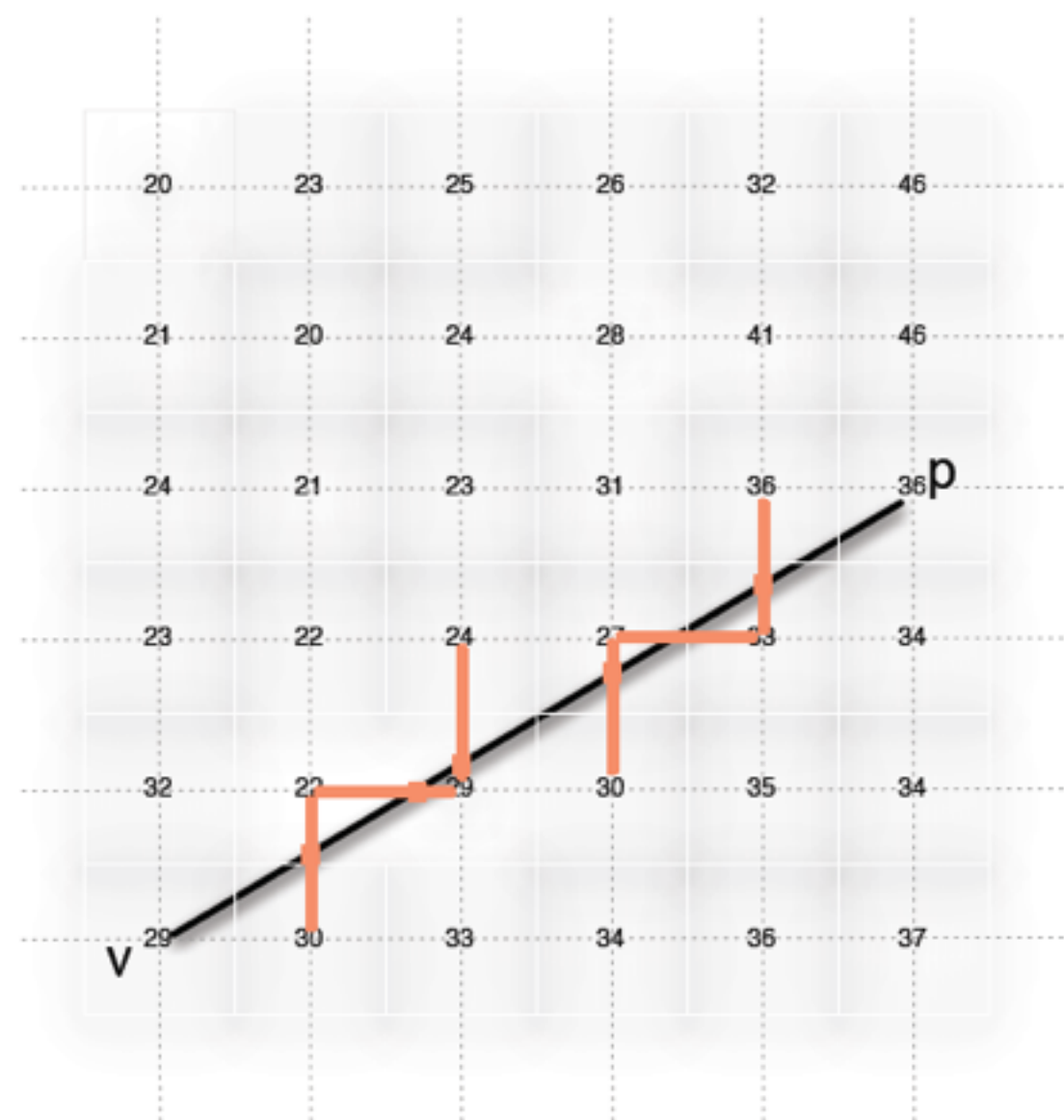
Input: elevation grid

Output: visibility grid, each point marked visible/invisible

- For each  $p$  in grid
  - compute intersections between  $vp$  and grid lines
  - if all these points are below  $vp$  then  $p$  is visible

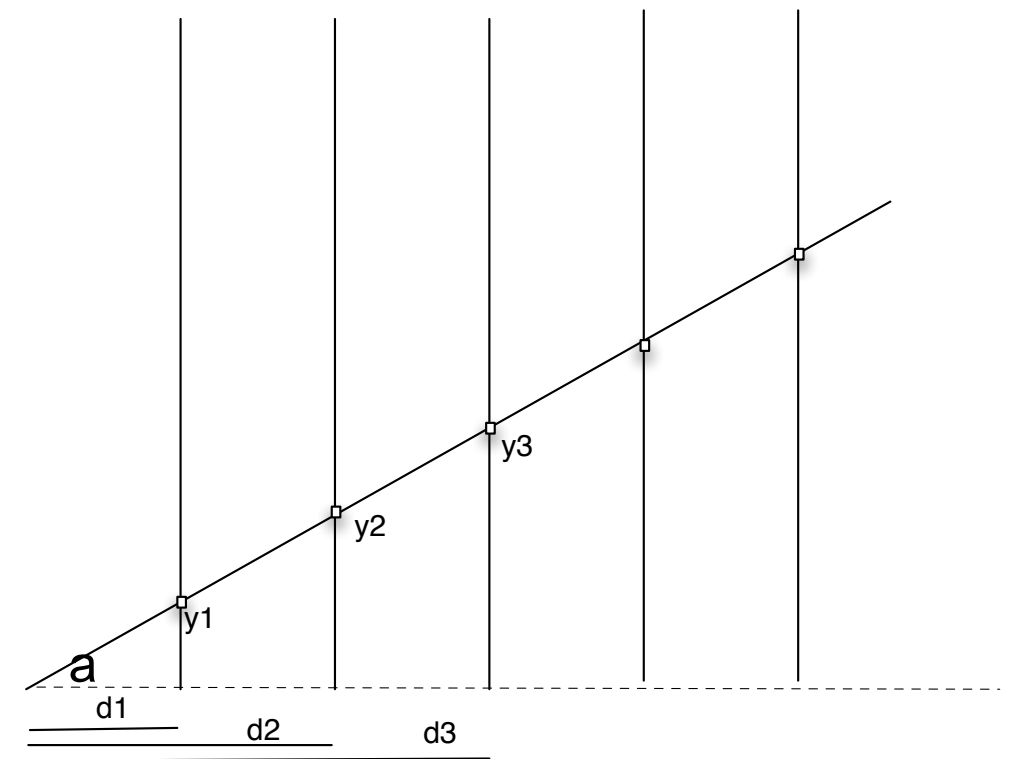
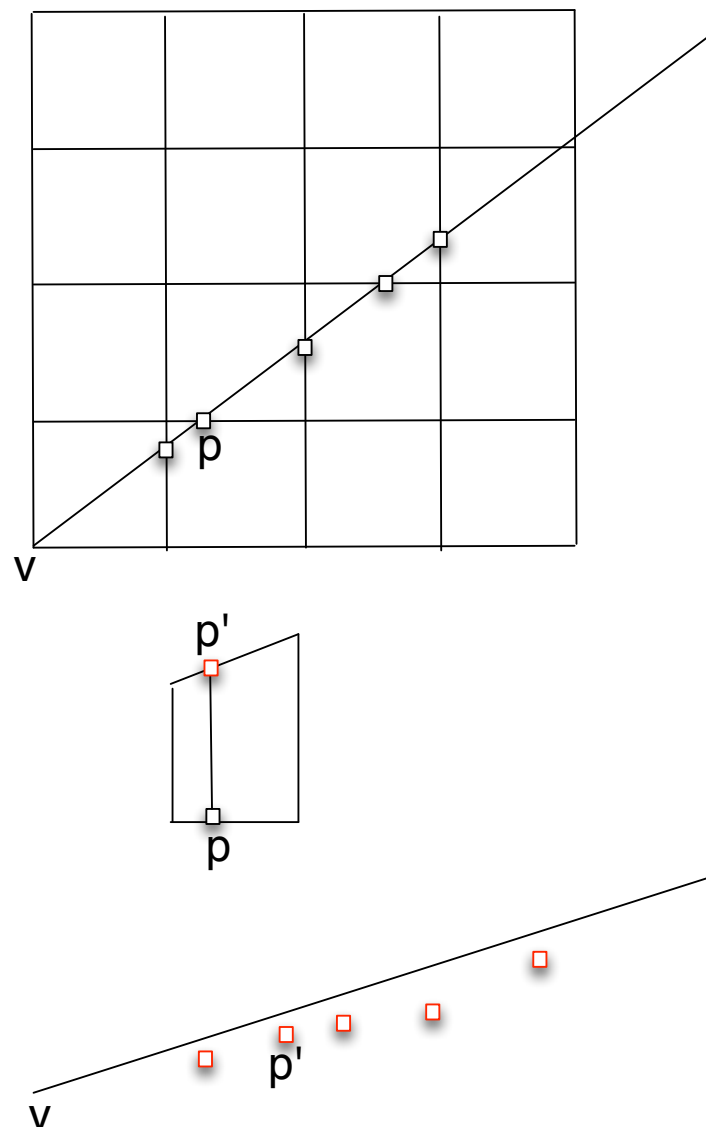
Assume grid of  $n$  points  
( $\sqrt{n} \times \sqrt{n}$ )

Running time:  $O(n\sqrt{n})$



# Basic viewshed algorithm

1. Find the 2D intersections between vp and the grid lines
2. Lift to 3D: find the height of p by linear interpolation
3. Check if slope(vp') is below slope (LOS)



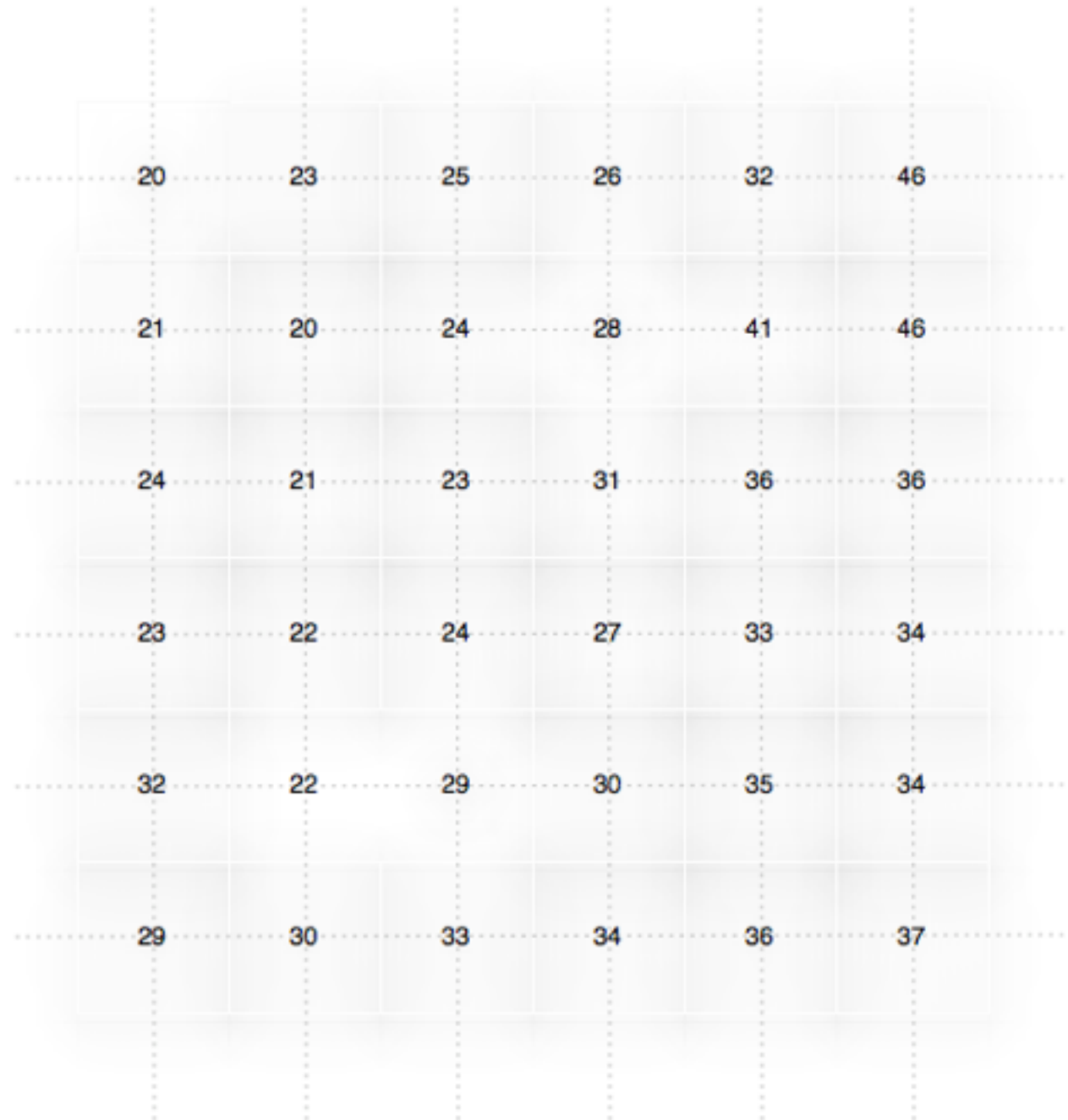
$$y_1 = d_1 \tan a$$
$$y_2 = d_2 \tan a$$

# Viewshed on grids

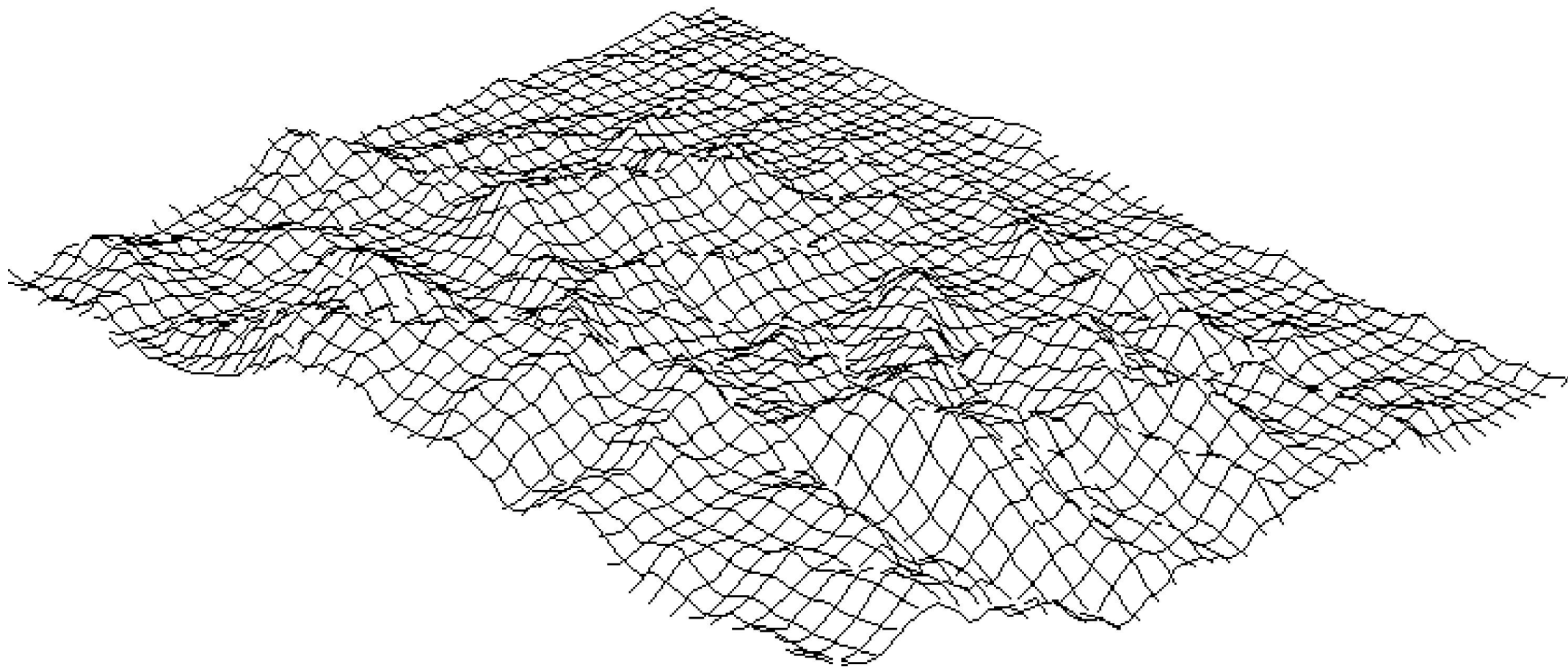
Grid of $n$ points: $\sqrt{n} \times \sqrt{n}$
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- The straightforward  $O(n\sqrt{n})$  algorithm
  - uses linear interpolation
  - “exact” as much as data allows; uses all data available
- Can we do better (faster) without introducing any approximation?
- Van Kreveld[vK'96]
  - nearest neighbor interpolation <— simpler
  - $O(n \lg n)$

# Grids

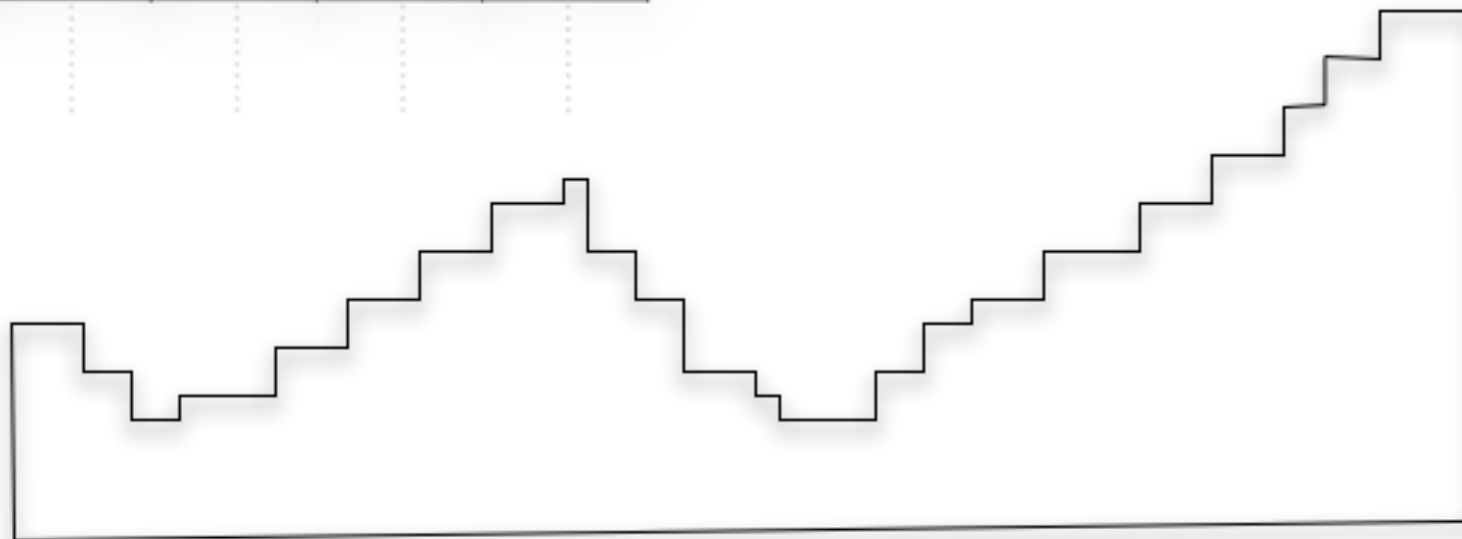
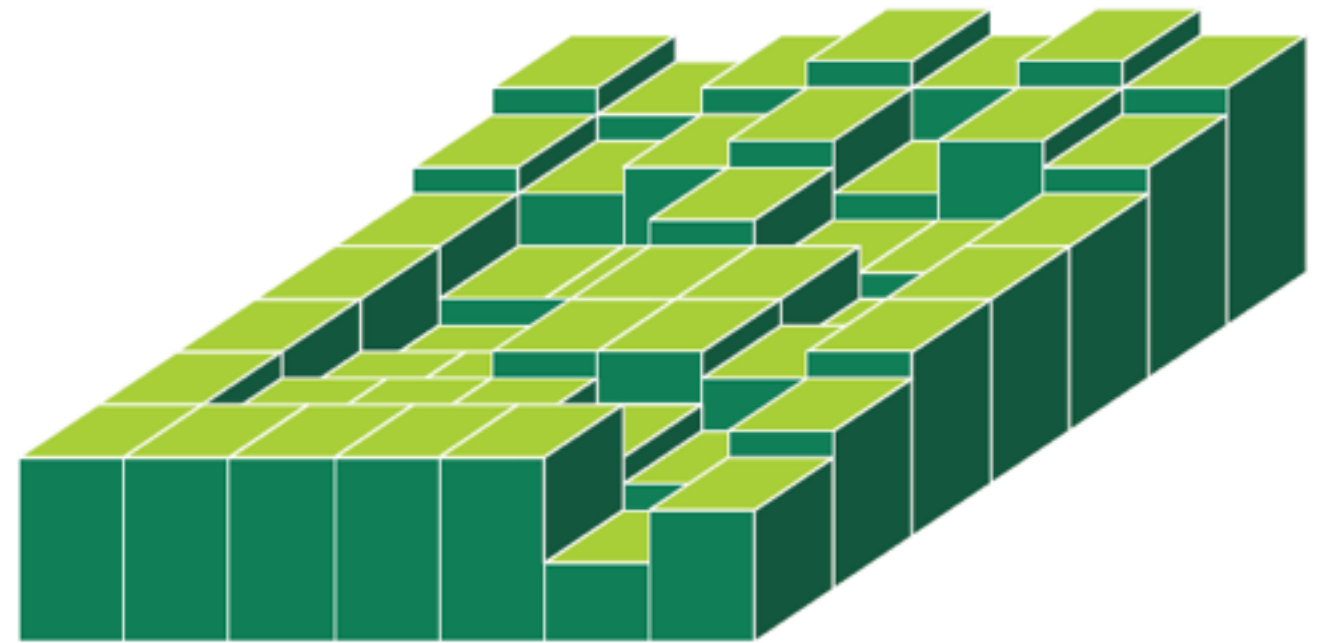


# Grids with linear interpolation



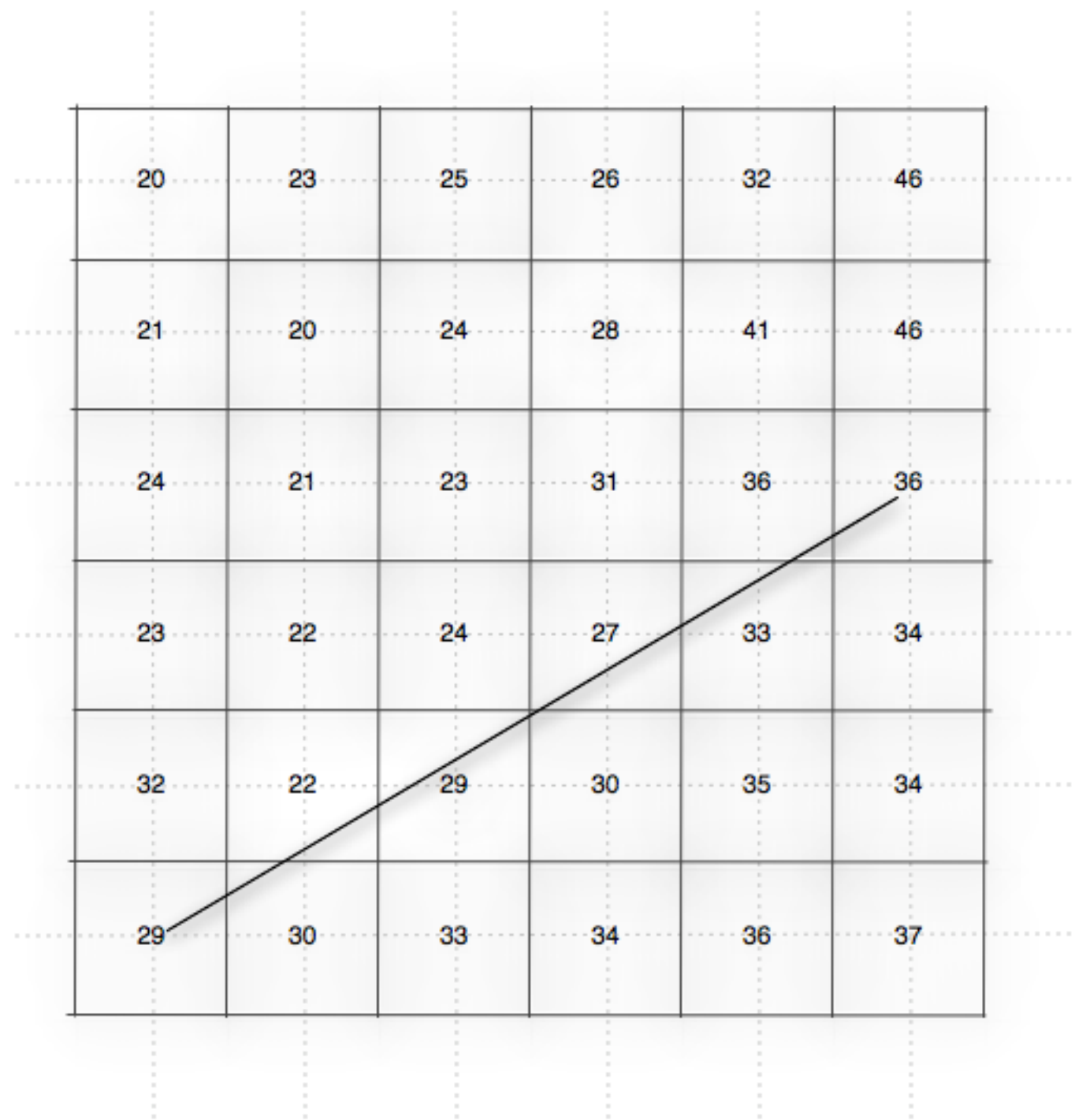
# Grids with nearest neighbor interpolation

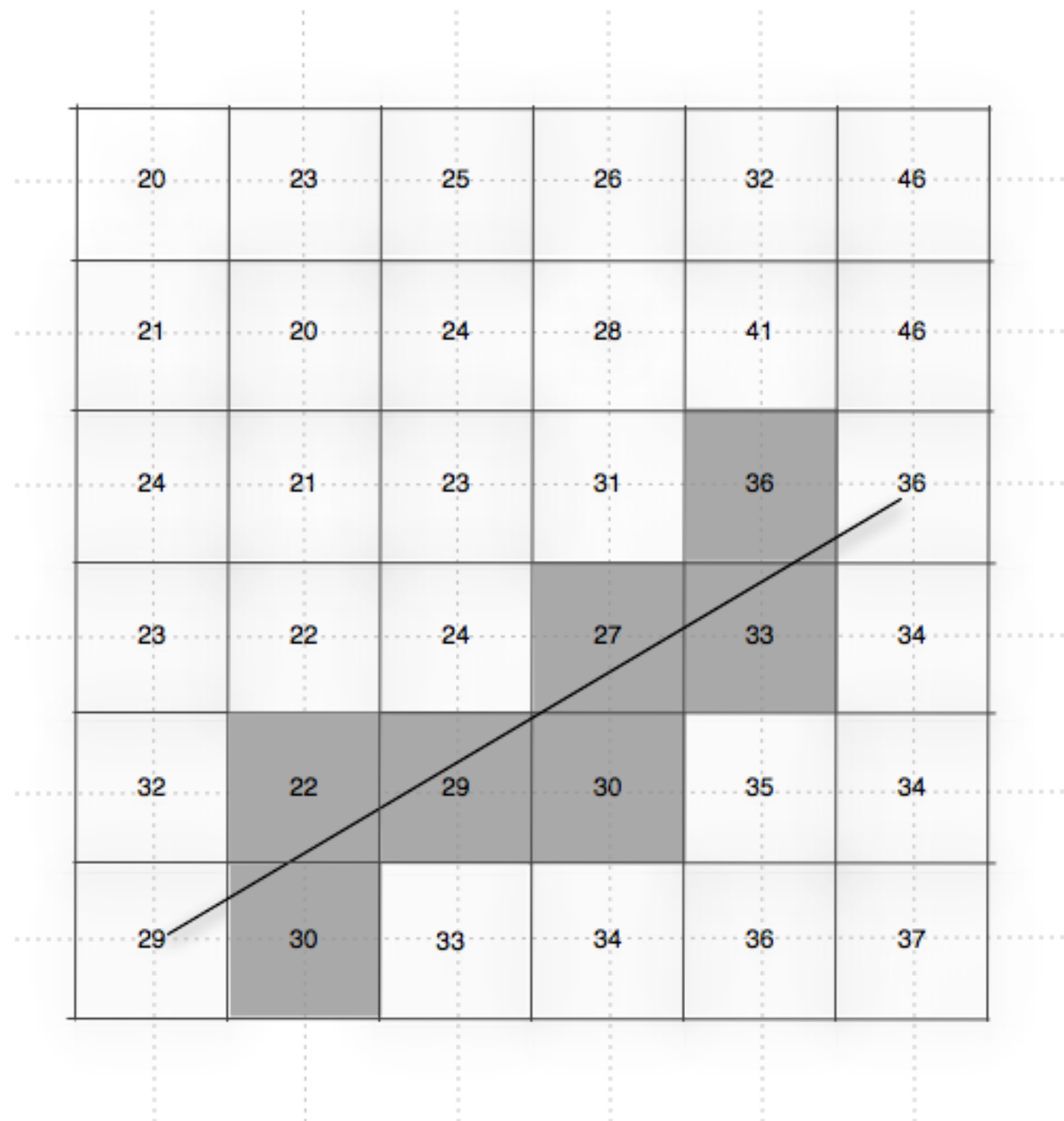
20	23	25	26	32	46
21	20	24	28	41	46
24	21	23	31	36	36
23	22	24	27	33	34
32	22	29	30	35	34
29	30	33	34	36	37

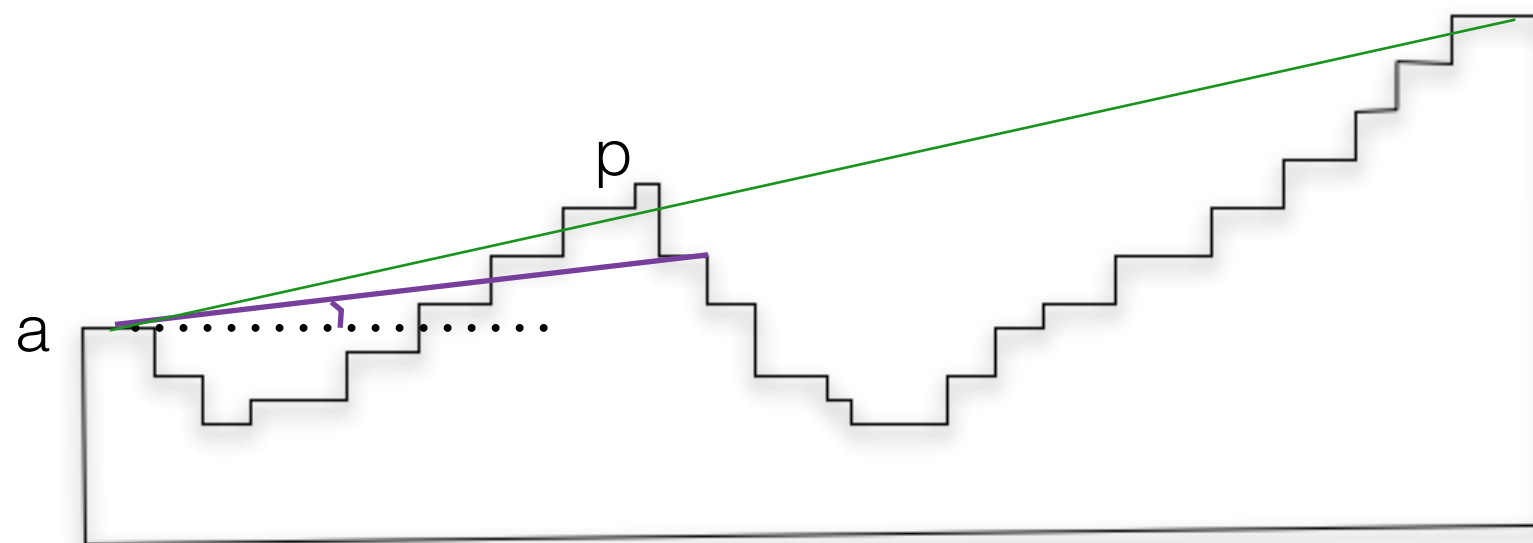


20	23	25	26	32	46
21	20	24	28	41	46
24	21	23	31	36	36
23	22	24	27	33	34
32	22	29	30	35	34
29	30	33	34	36	37

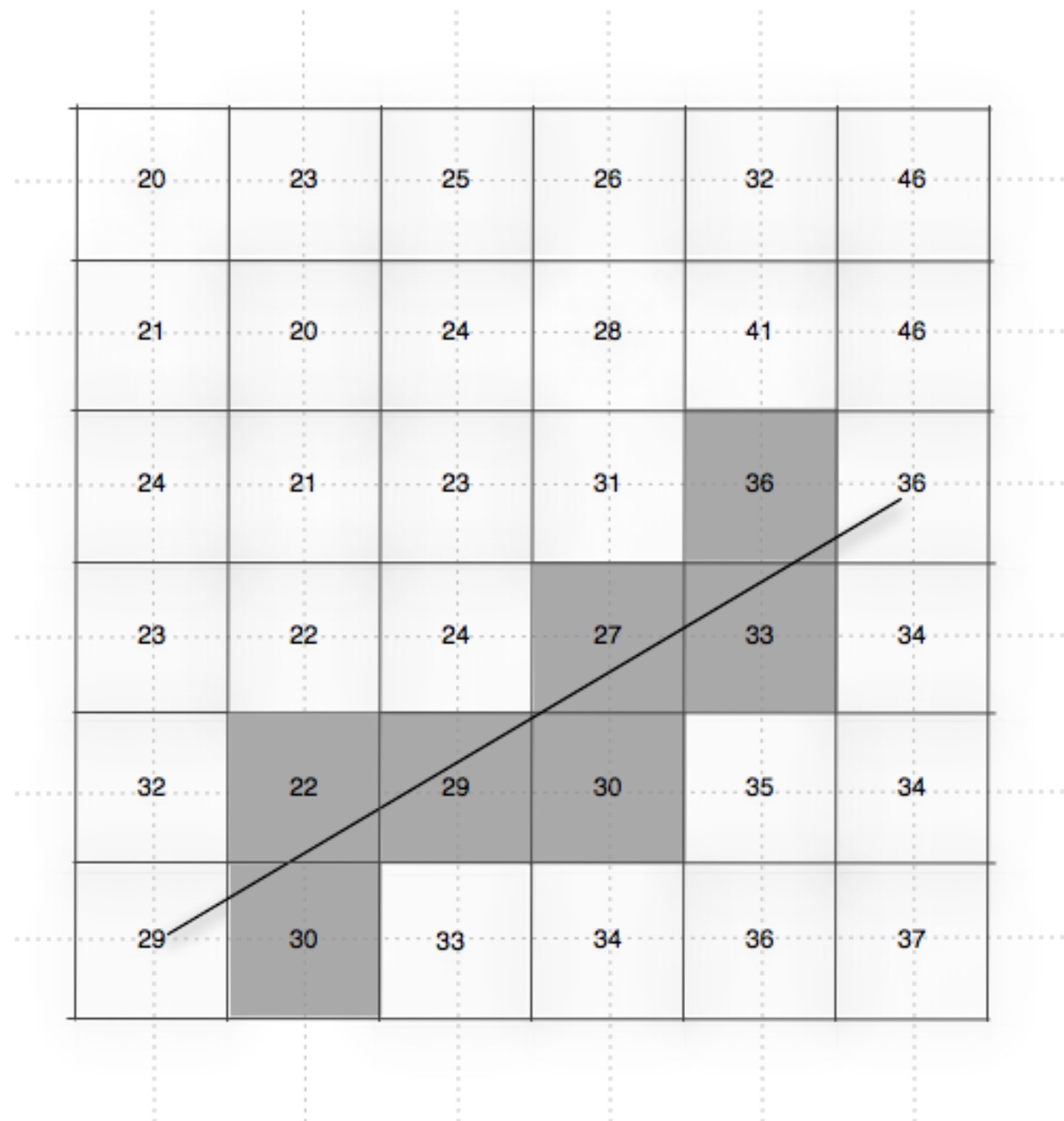


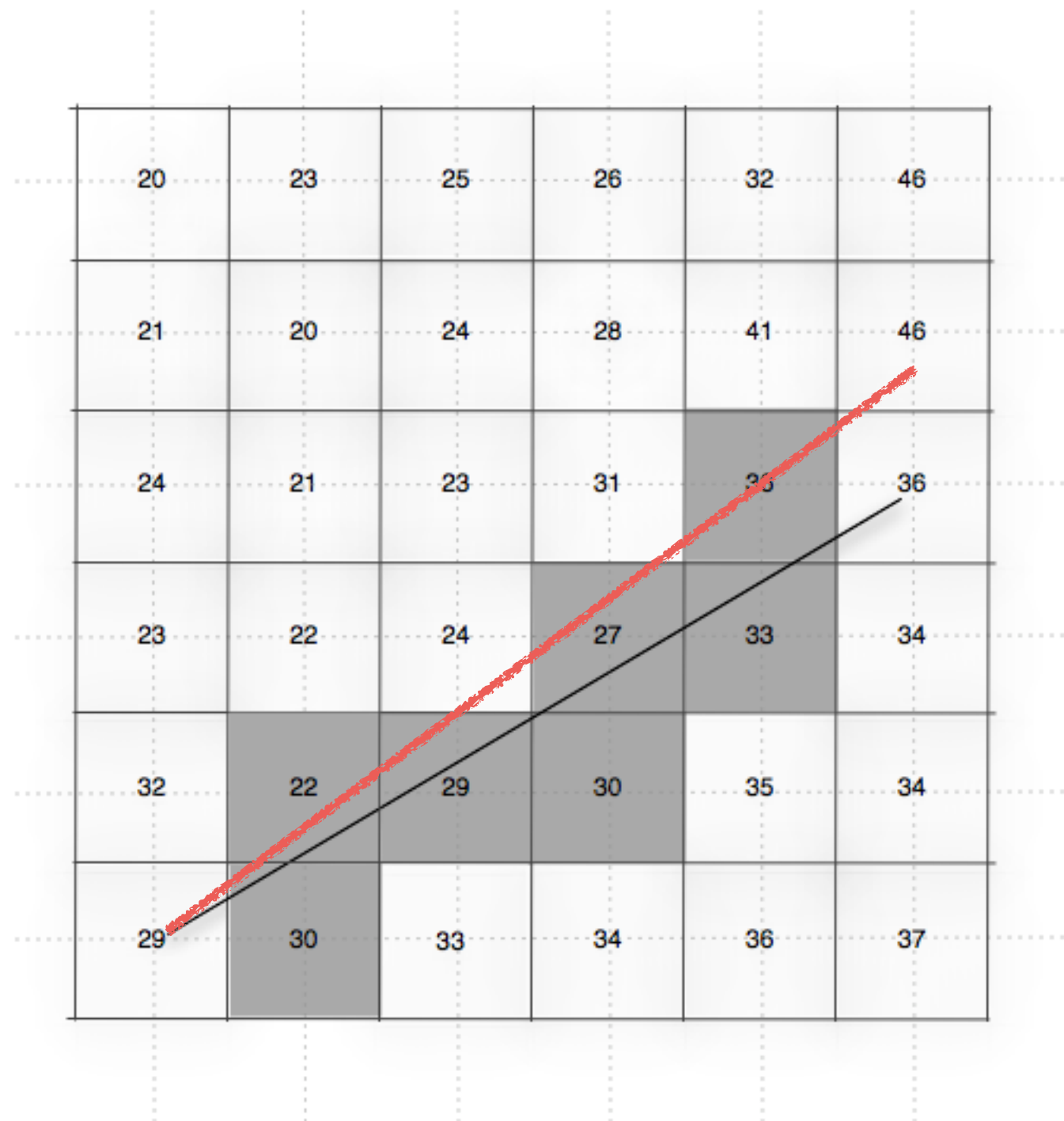


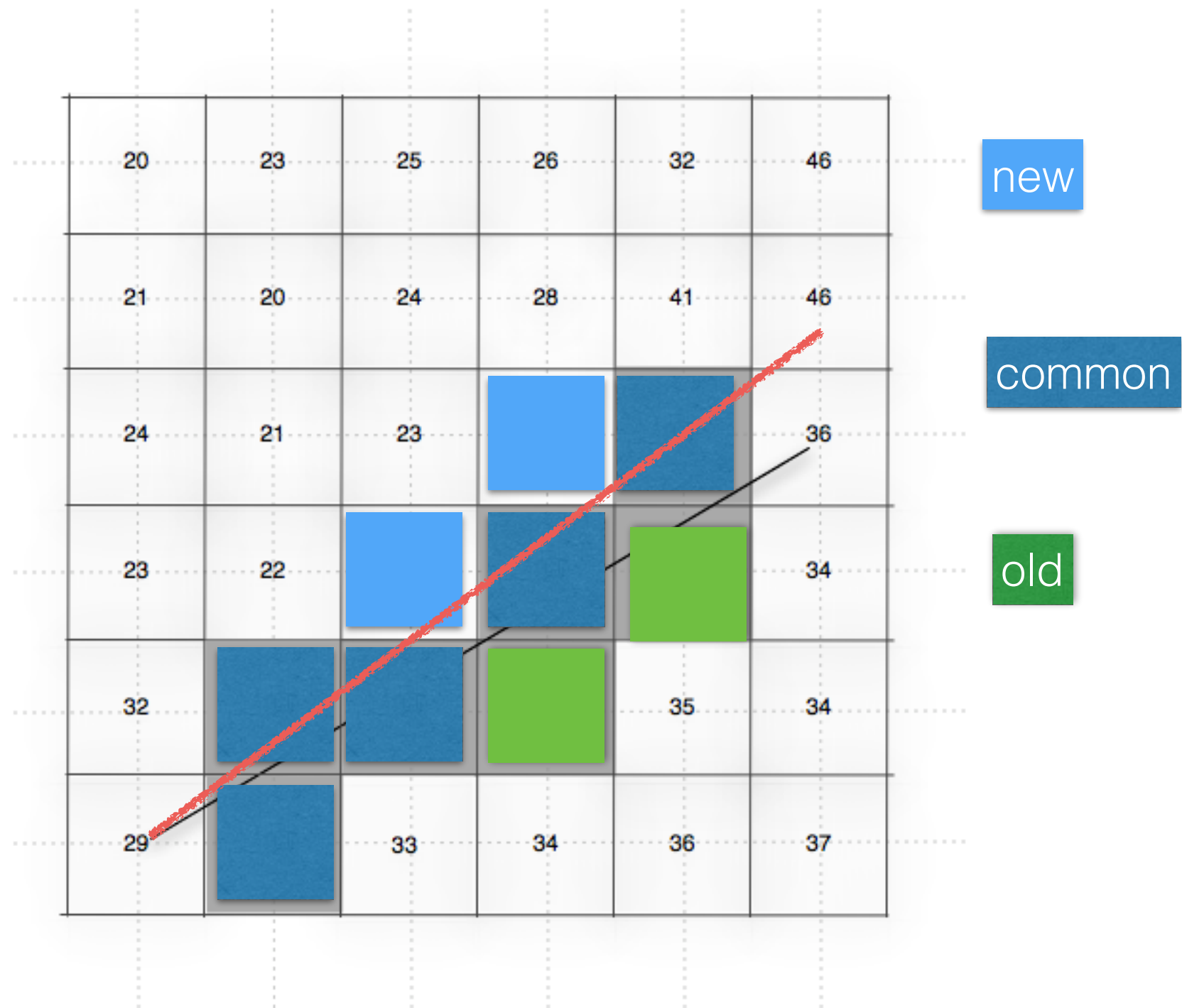


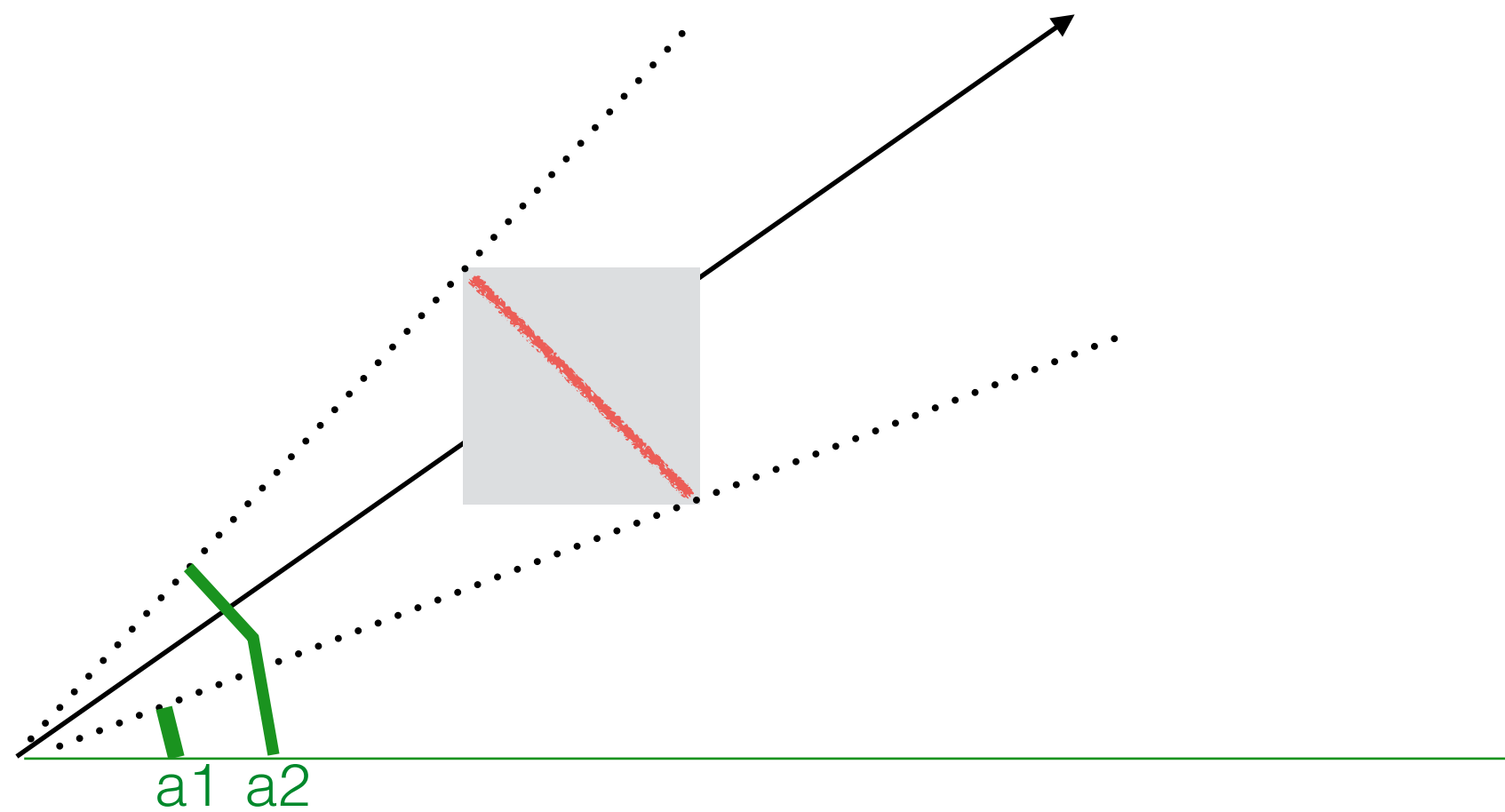


$$\text{vertical slope}(p,a) = (h_p - h_a) / d(a,p)$$



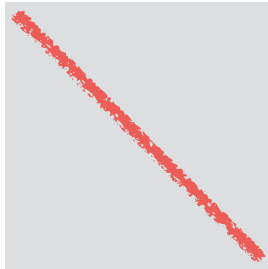




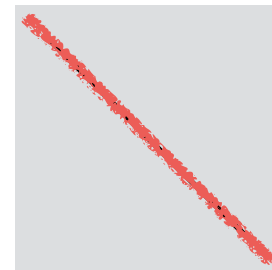
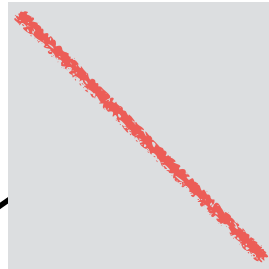




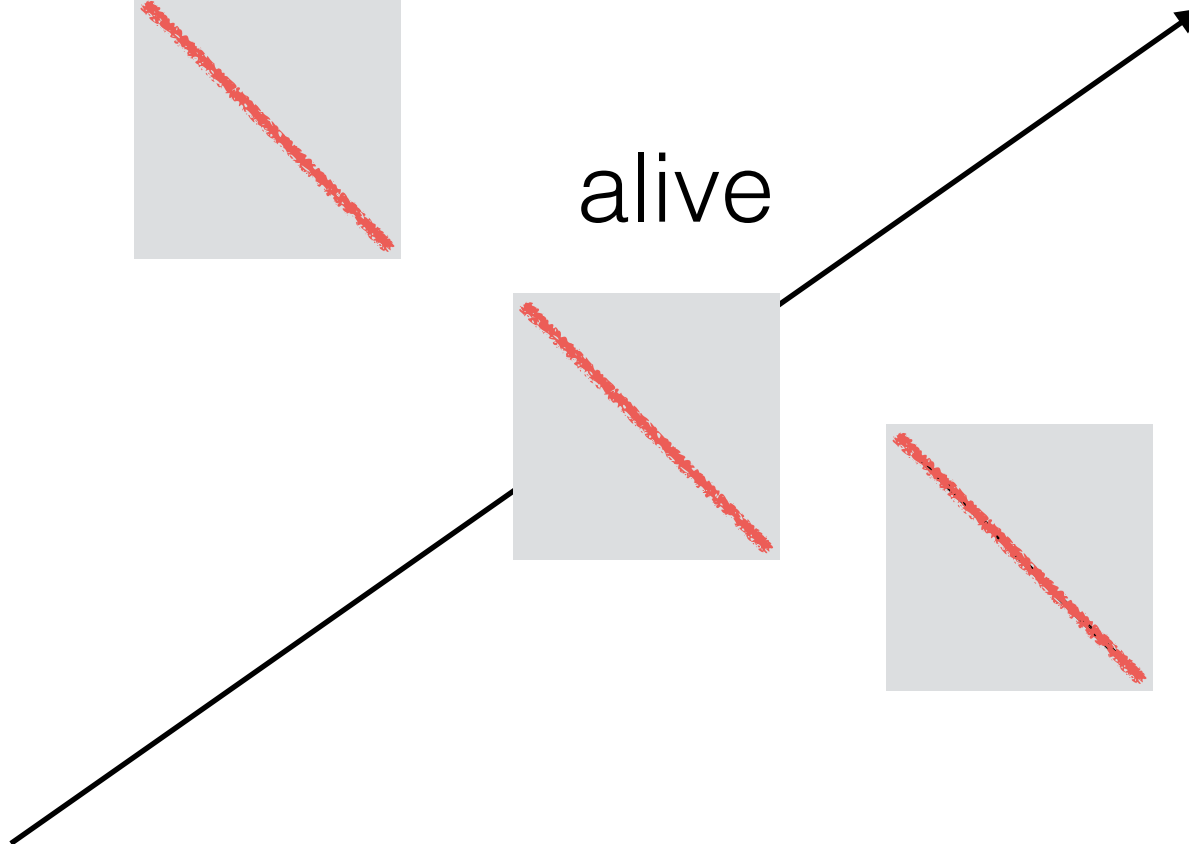
future



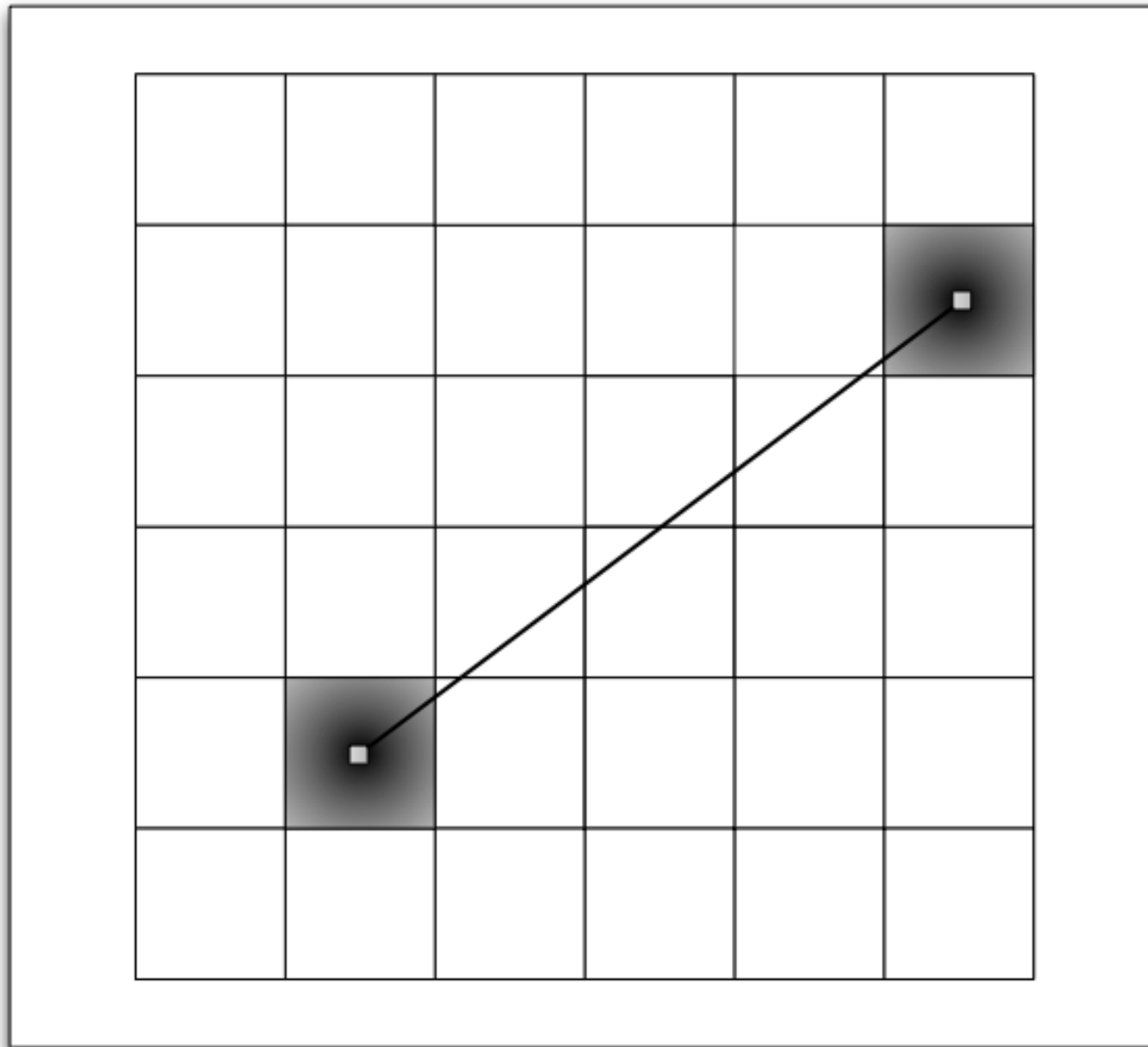
alive



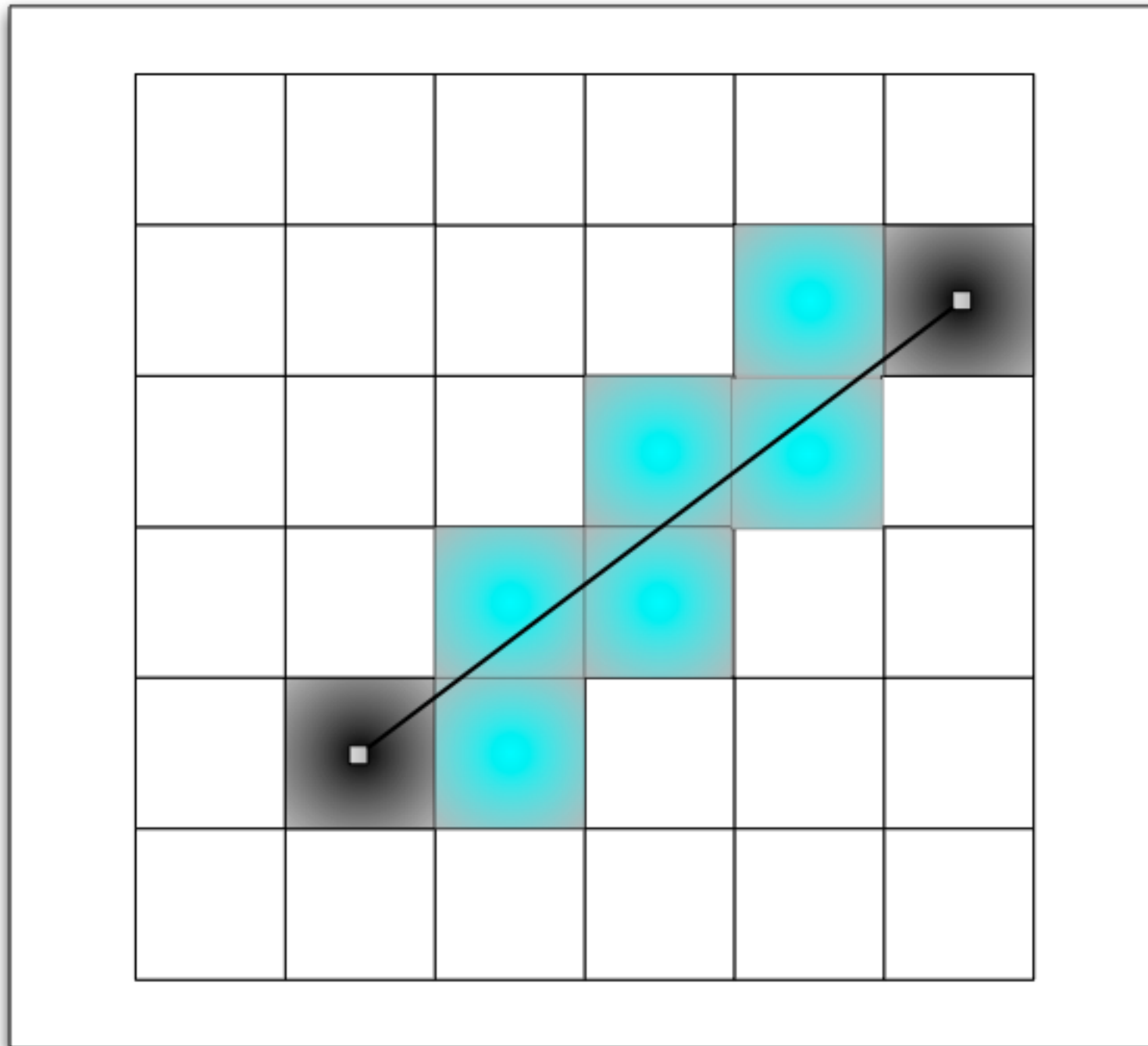
dead



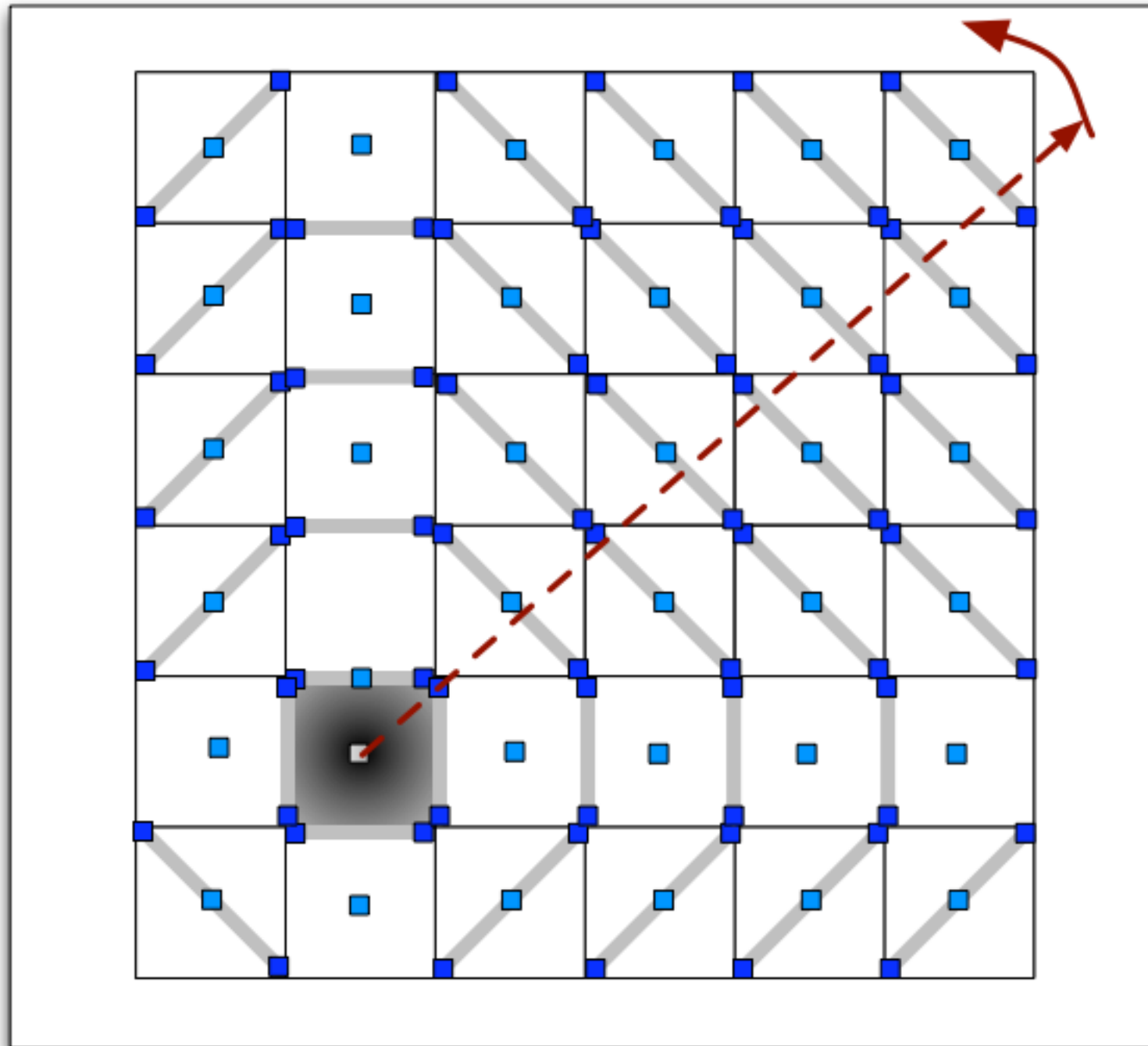
# Van Kreveld's radial sweep algorithm



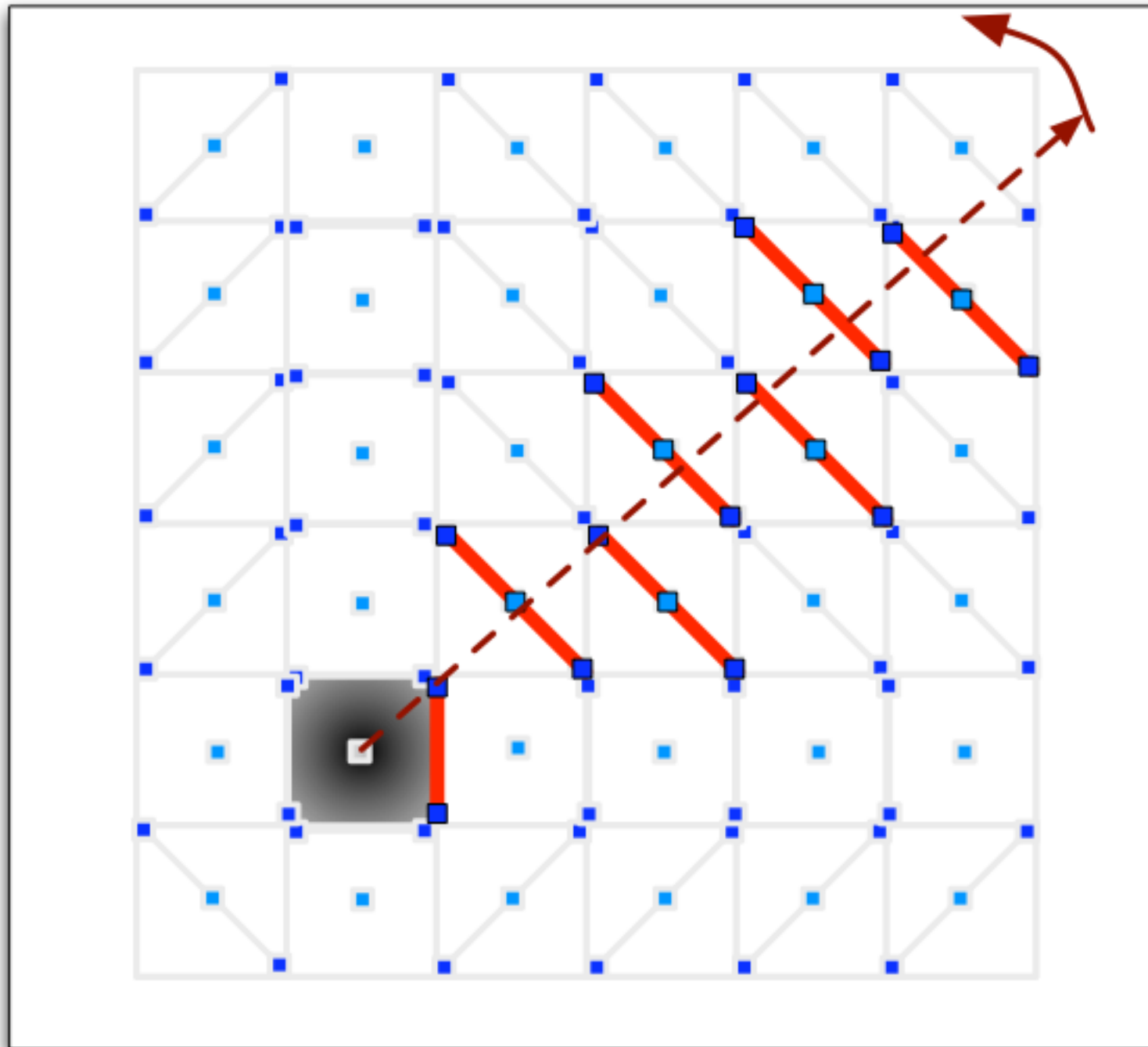
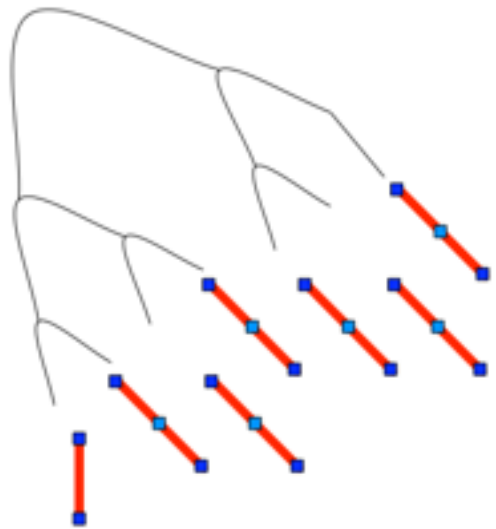
# Van Kreveld's radial sweep algorithm



# Van Kreveld's radial sweep algorithm



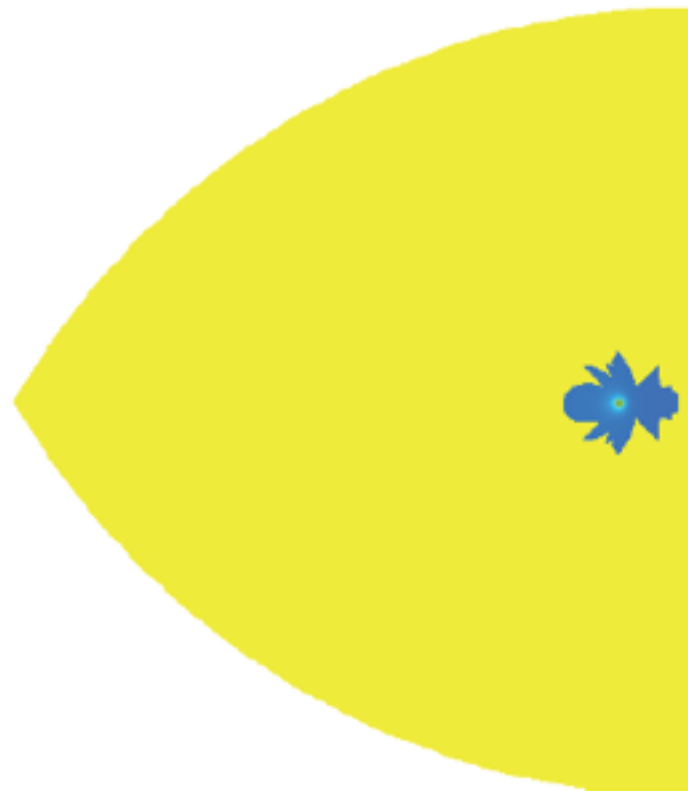
# Van Kreveld's radial sweep algorithm



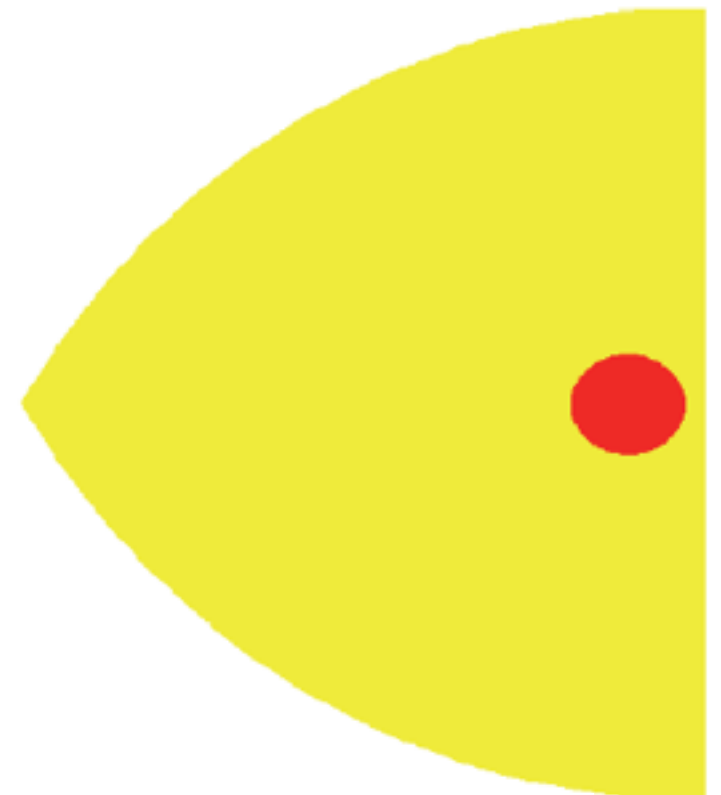
# Accuracy!!



test grid: hemisphere



viewshed with NN interpolation



viewshed with linear  
interpolation

# Computing viewsheds

Grid of $n$ points: $\sqrt{n} \times \sqrt{n}$
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## 1. Straightforward algorithm

- uses linear interpolation
- can be adapted to other interpolations
- $O(n\sqrt{n})$

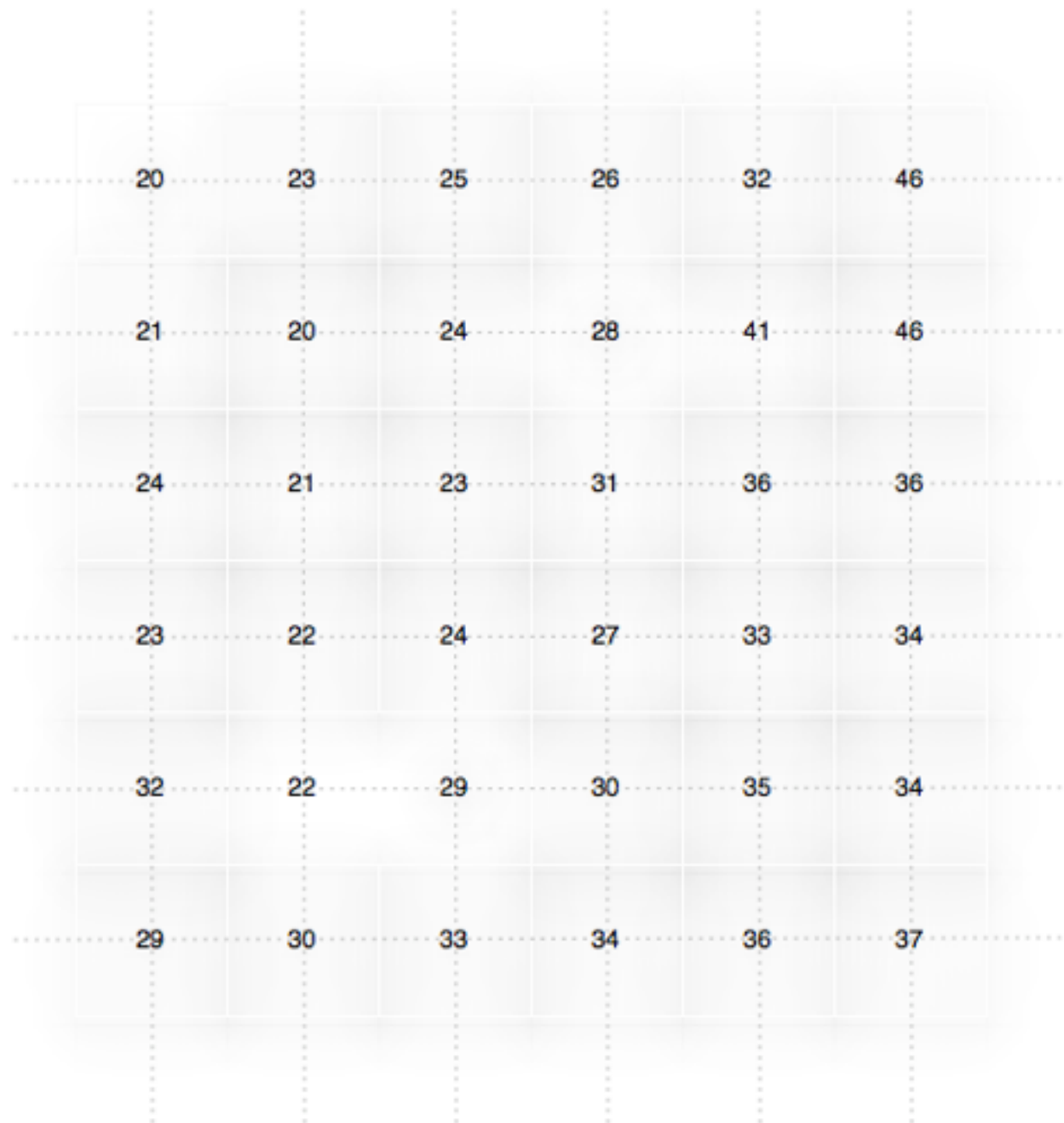
## 2. Radial sweep approach

- uses nearest neighbor interpolation
- crucially exploits that cells are “flat” obstacles
- uses radial sweep + augmented RB tree
- has accuracy issues  $\Rightarrow$  undesirable
- $O(n \lg n)$

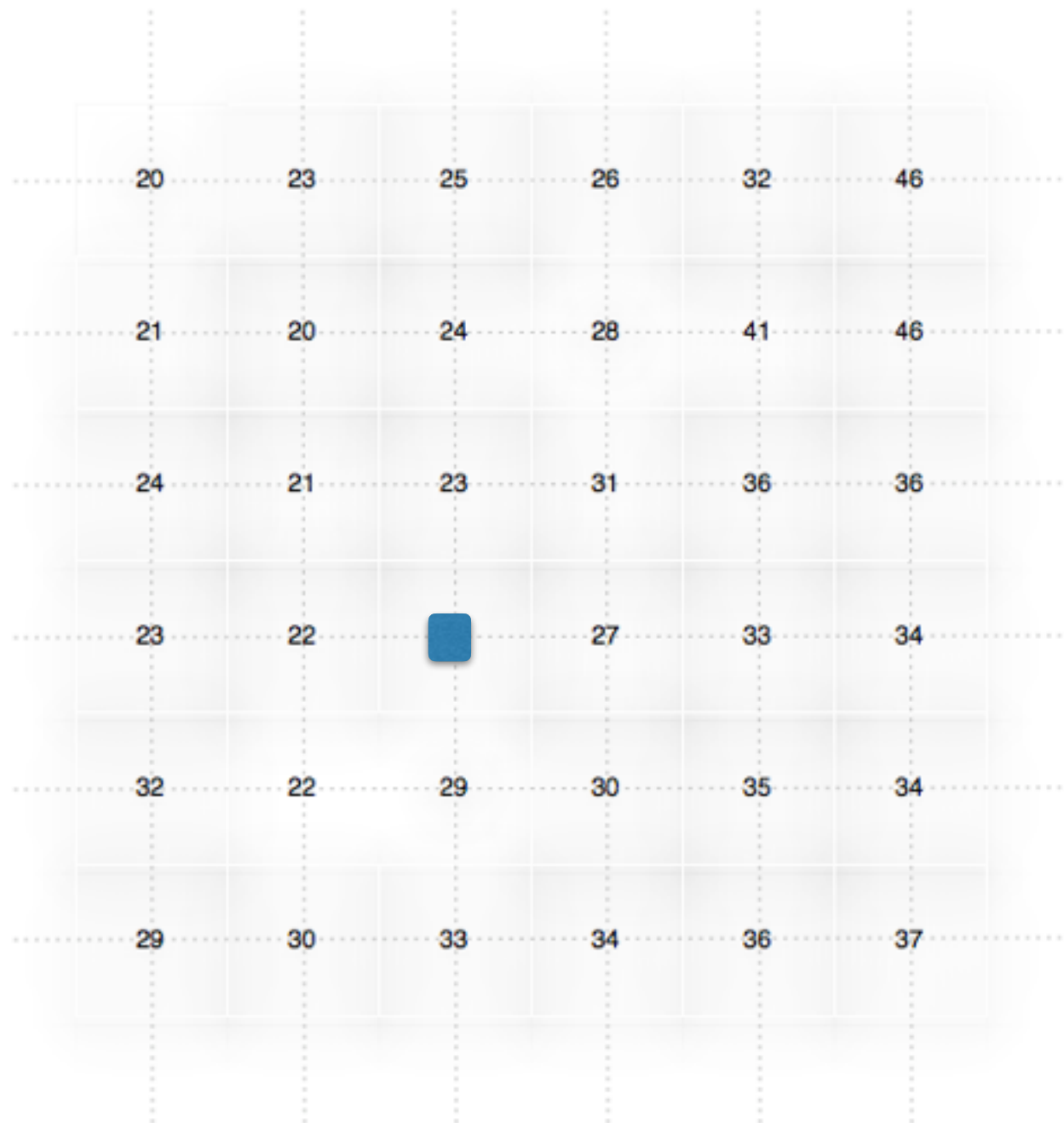
## 3. A different approach: concentric sweep using horizons



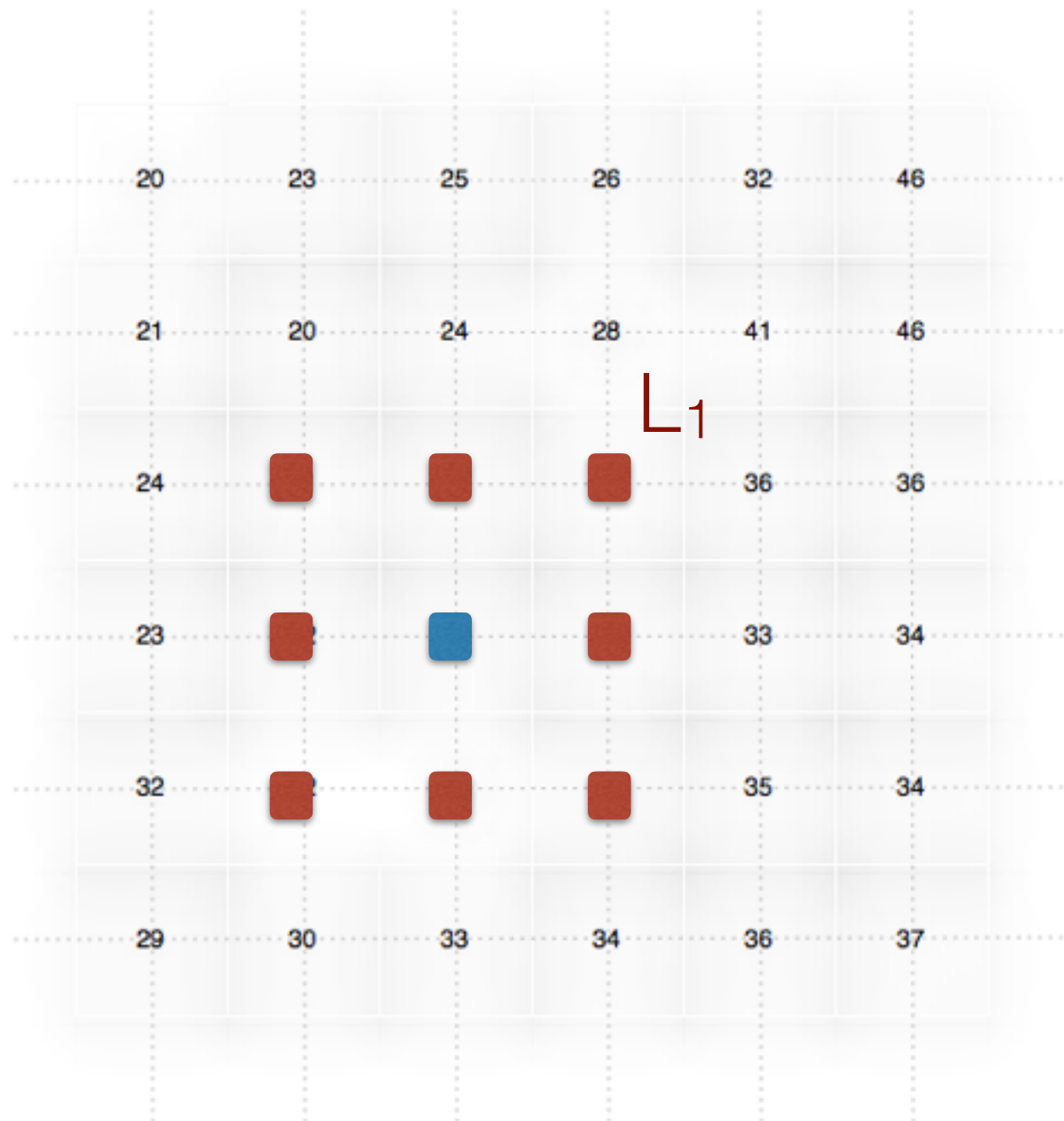
# Viewshed on grids using a concentric sweep



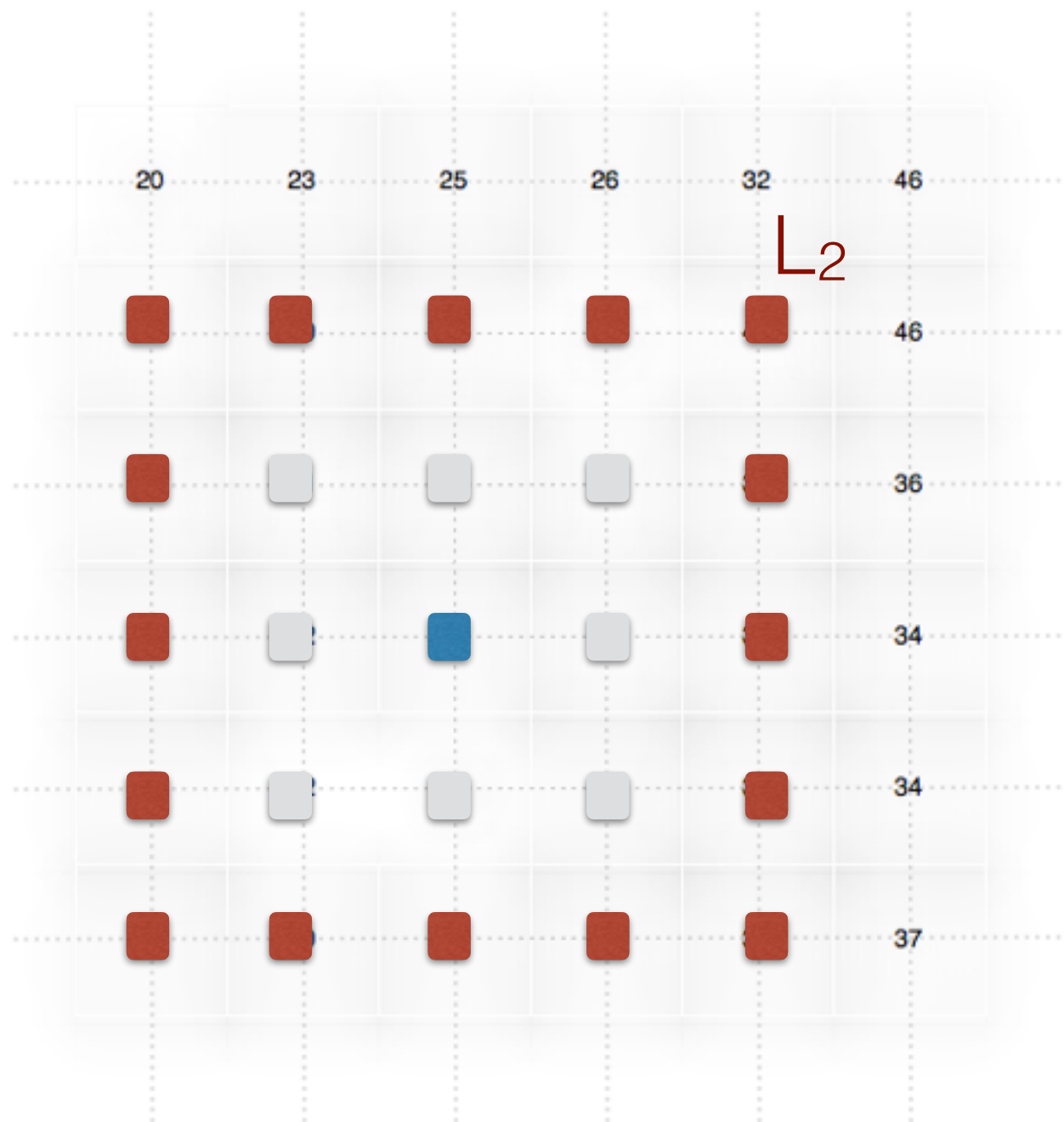
# Viewshed on grids using a concentric sweep



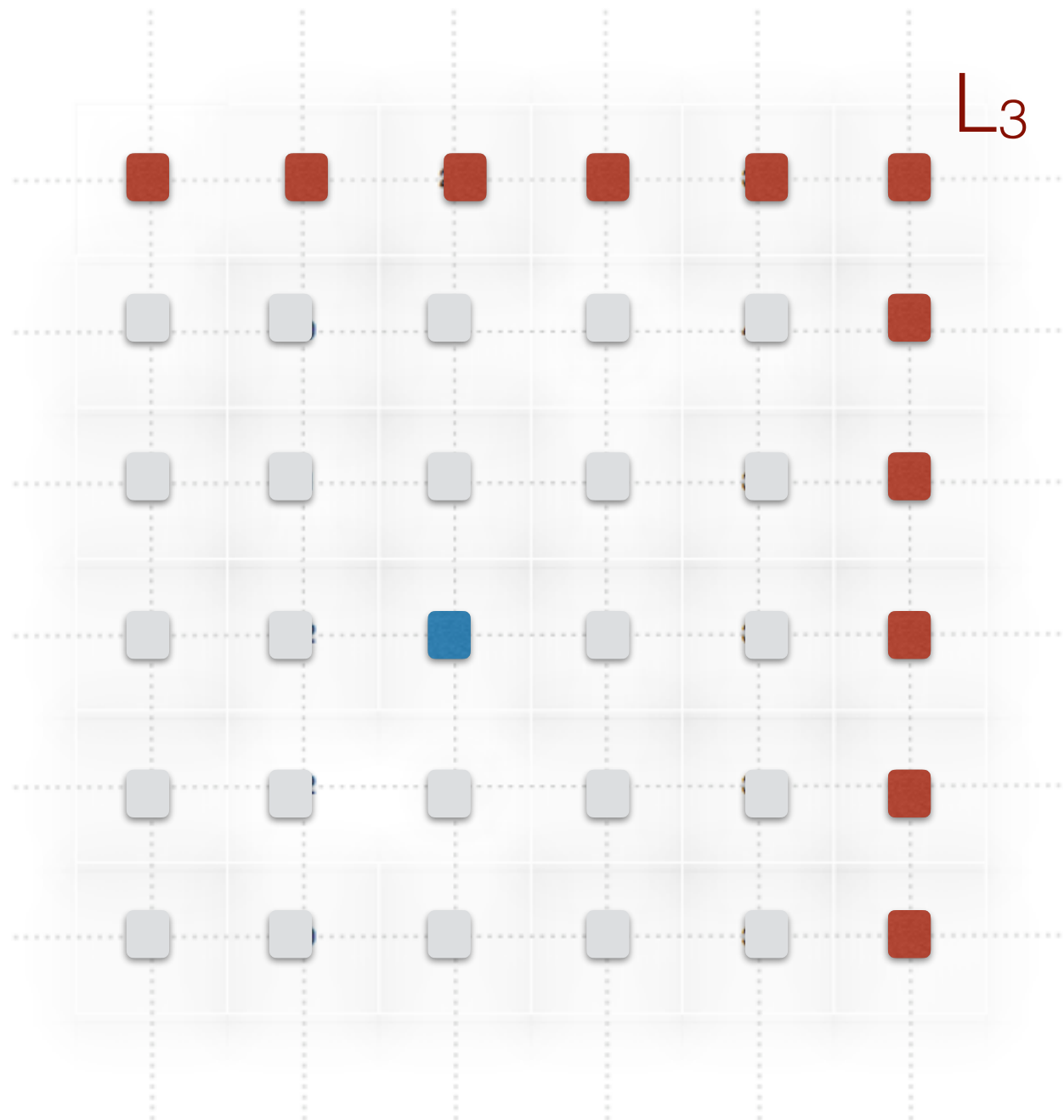
# Viewshed on grids using a concentric sweep



# Viewshed on grids using a concentric sweep



# Viewshed on grids using a concentric sweep



# Horizons

- Merriam Webster:
  - the line where the terrain and the sky seem to meet





# Horizons

- Merriam Webster:
  - the line where the terrain and the sky seem to meet





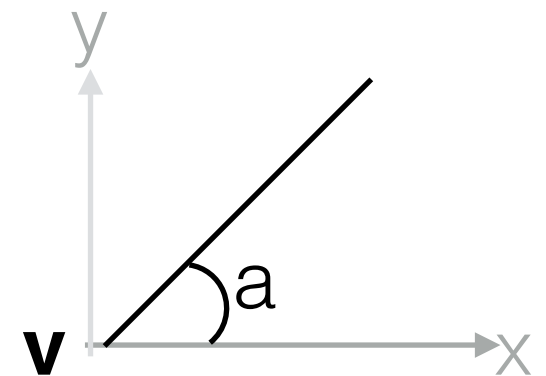
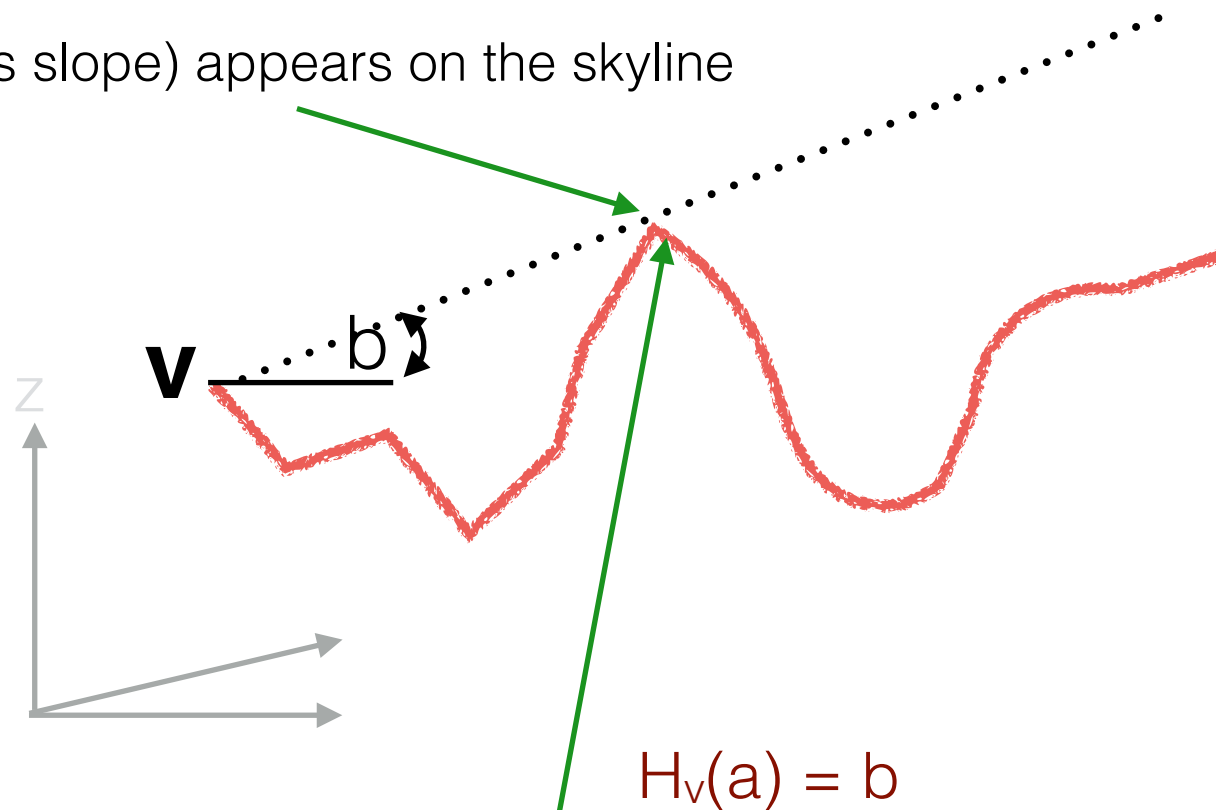
# Horizon

$H_v : [0, 2\pi) \rightarrow \mathbb{R}$

Horizon (with respect to  $v$ ) in direction  $a$ ,  $H_v(a)$

- cut the terrain with a vertical plane through a ray from  $v$  of azimuth  $a$
- $H_v(a)$  is the maximum vertical angle (zenith) of all points intersected by this plane (all the points on  $T$  whose projection on the  $xy$ -plane has azimuth  $a$ )

this point (its slope) appears on the skyline



- Note: a point beyond this point is visible if and only if it's above the horizon

# Horizons

- Merriam Webster:
  - the line where the terrain and the sky seem to meet



# Horizon

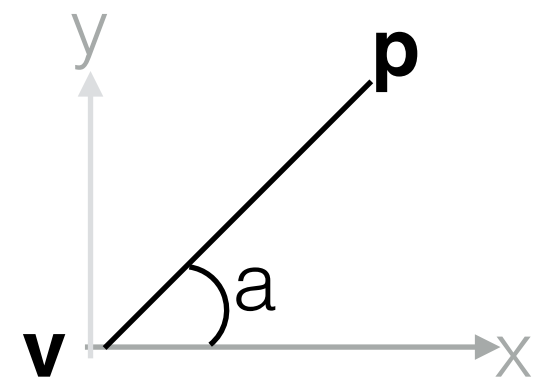
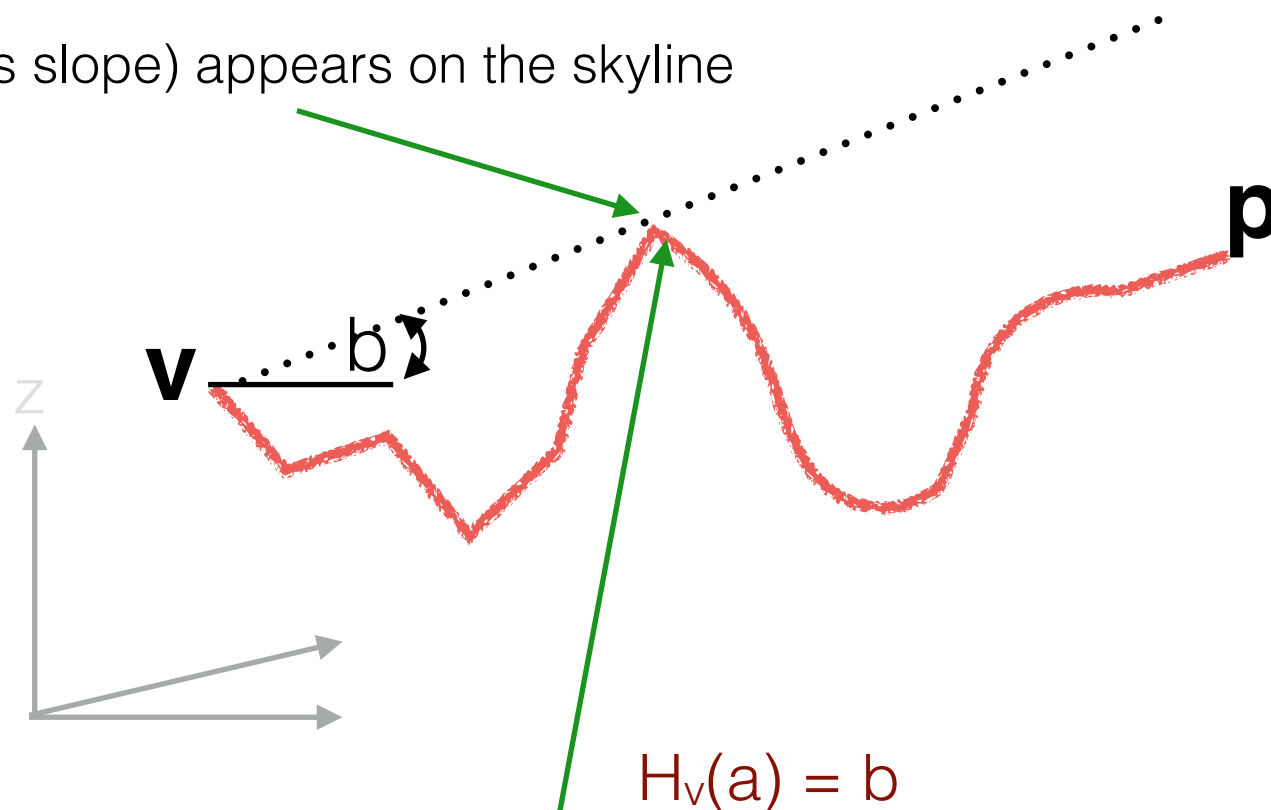
Given point  $p$  on the terrain

Look at the cross-section of the terrain from  $v$  towards  $p$ , and consider only the points between  $v$  and  $p$

Point  $p$  defines the azimuth angle  $\Rightarrow$  We can define a horizon in the same way

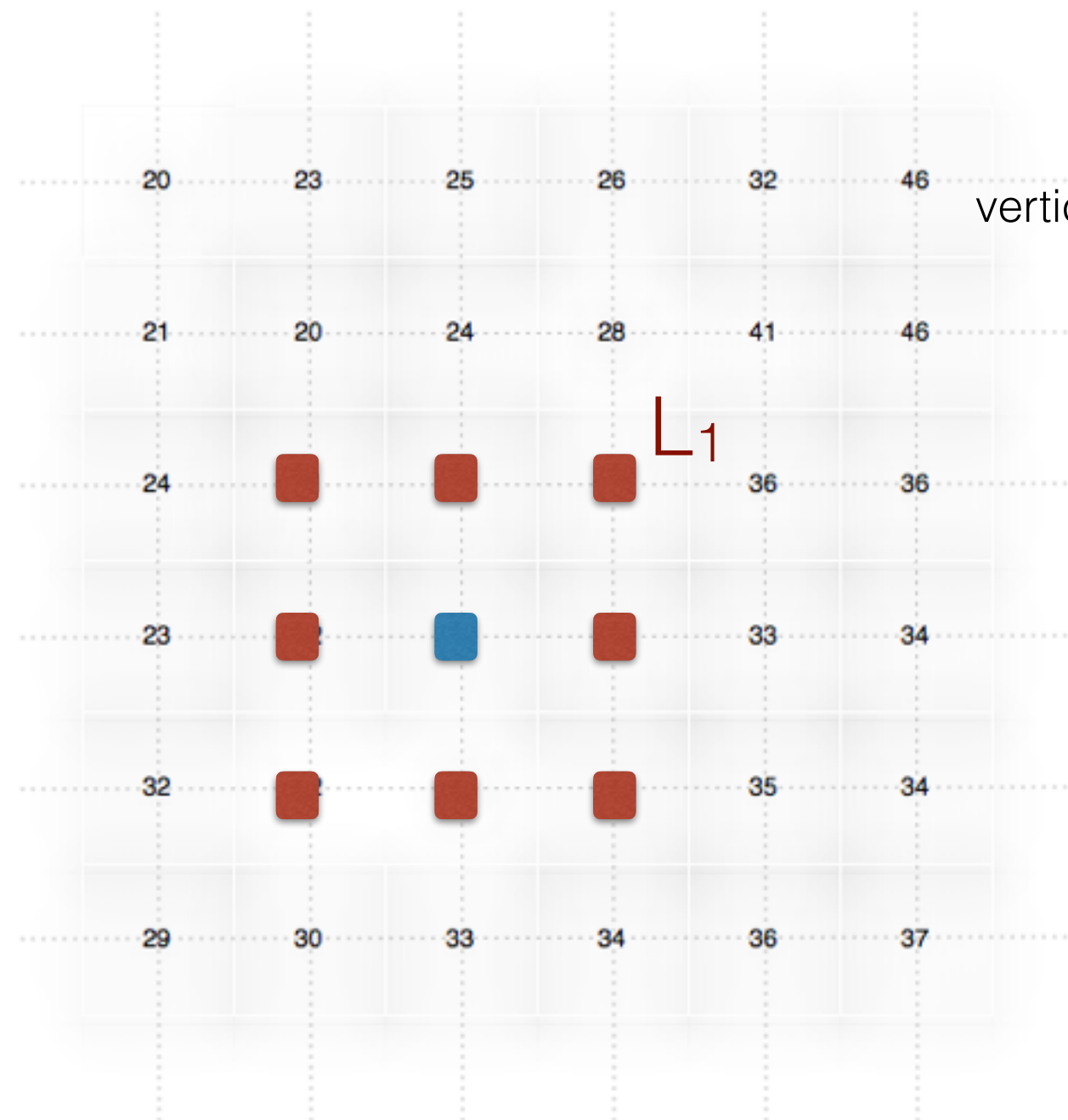
- $H_v(p)$  is the maximum vertical angle (zenith) of all points between  $v$  and  $p$  whose projection on the  $xy$ -plane has azimuth = azimuth( $vp$ )

this point (its slope) appears on the skyline

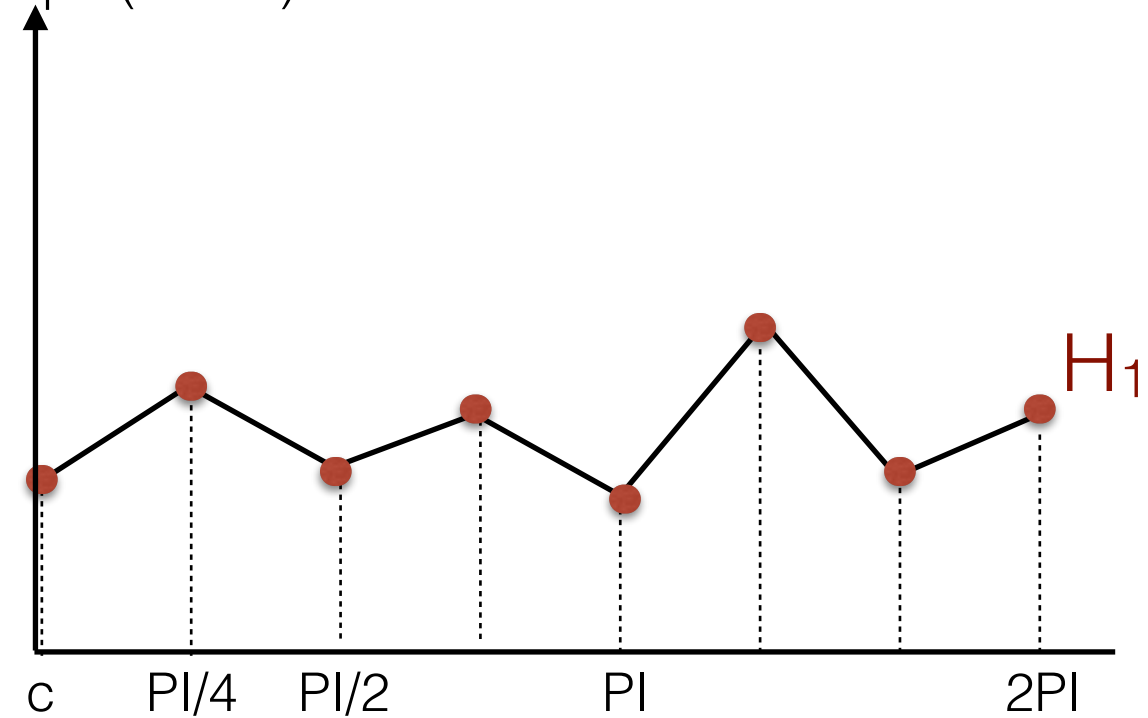


- Note: a point beyond this point is visible if and only if it's above the horizon

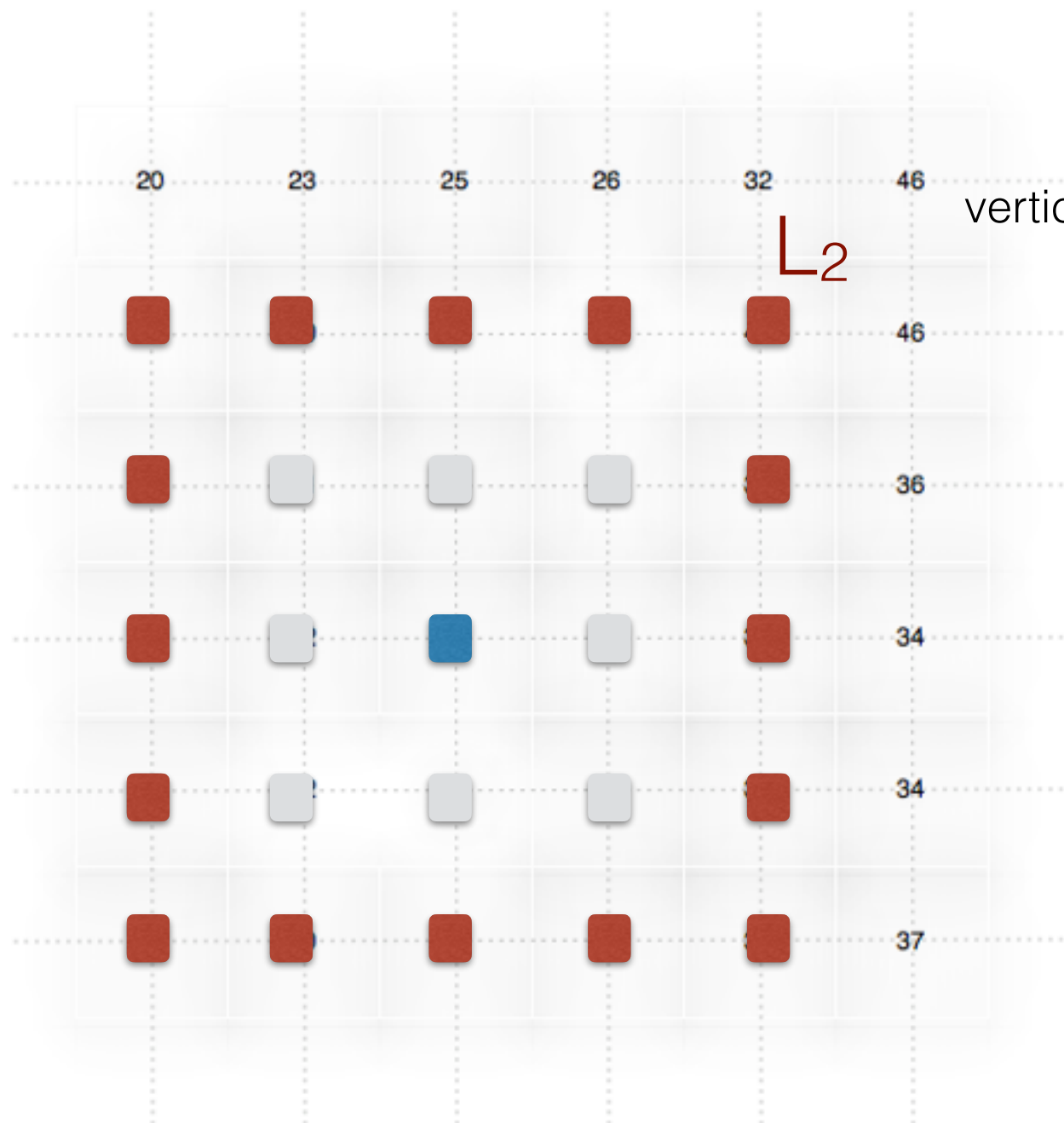
# Viewshed on grids using a concentric sweep



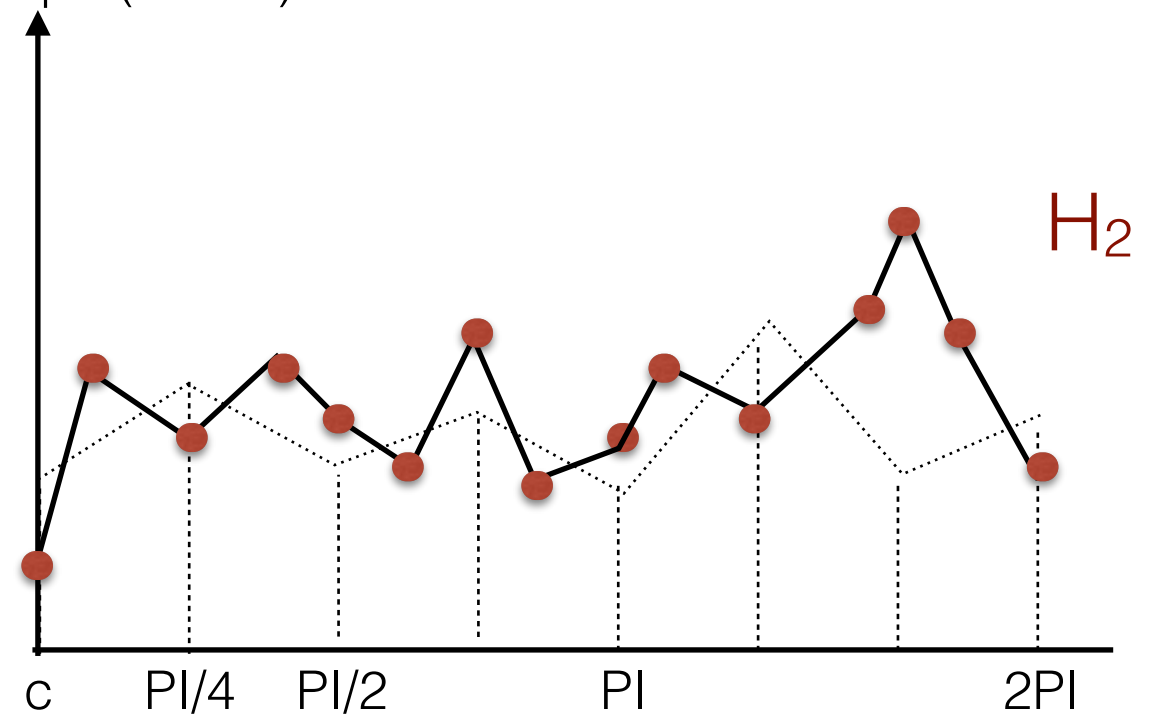
vertical slope (zenith)



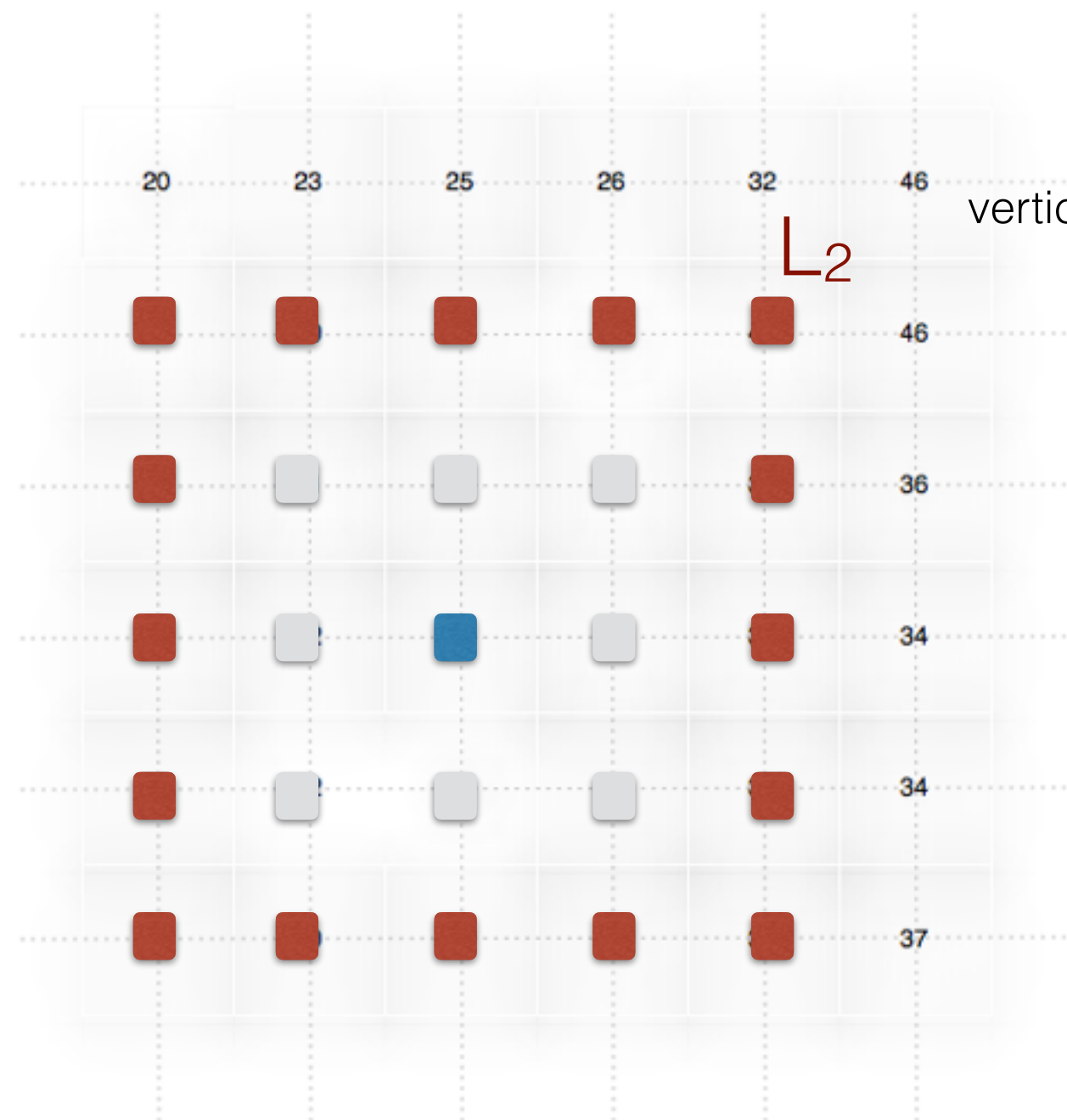
# Viewshed on grids using a concentric sweep



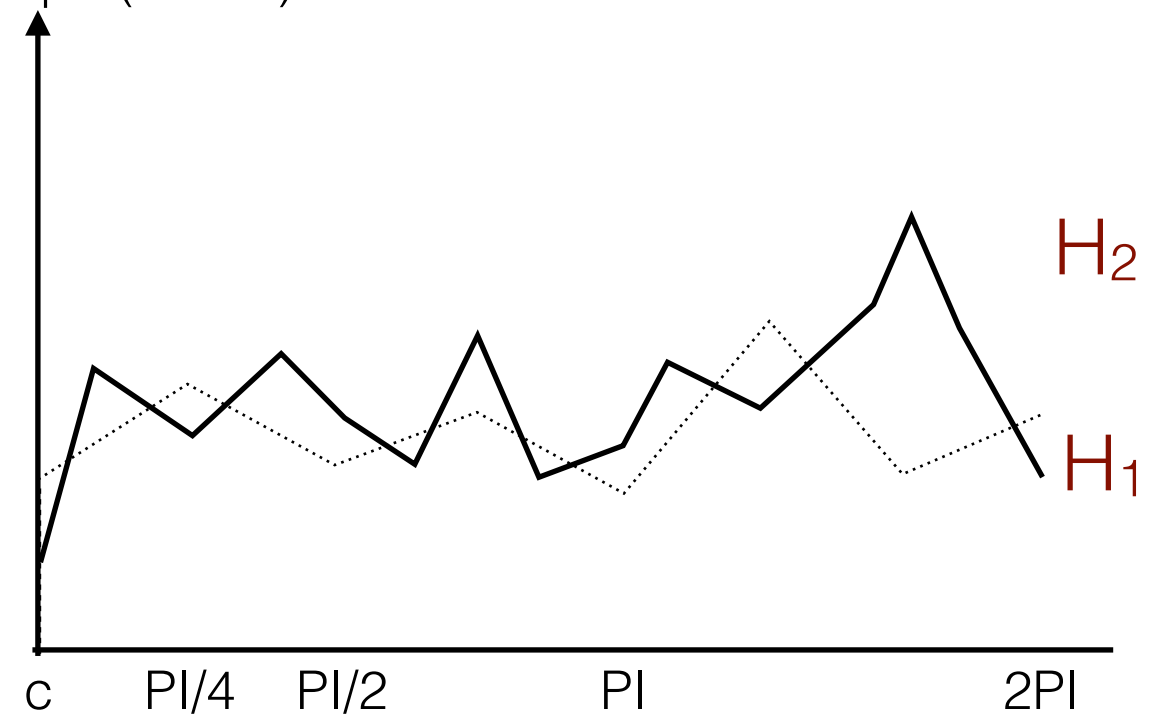
vertical slope (zenith)



# Viewshed on grids using a concentric sweep

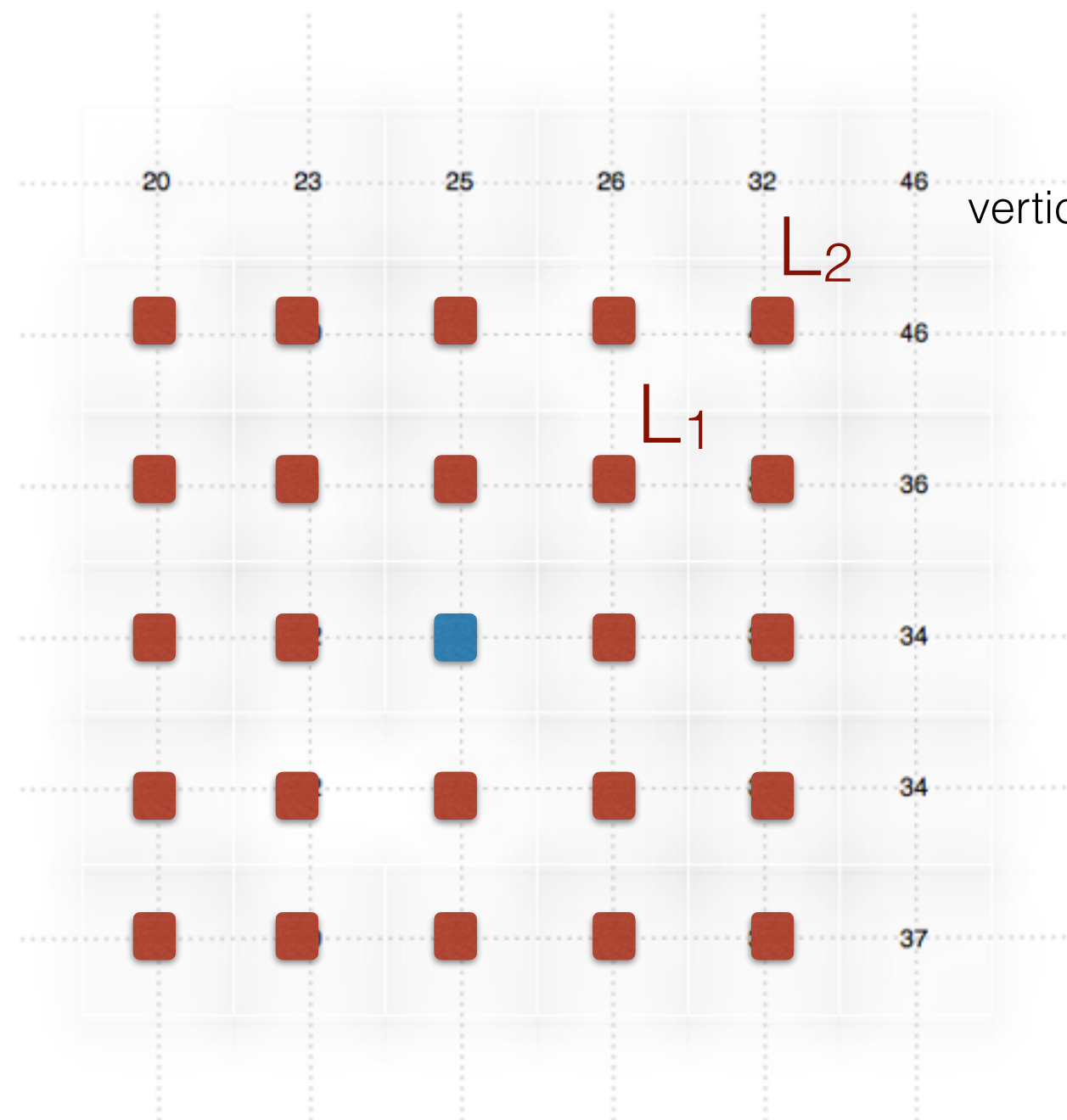


vertical slope (zenith)

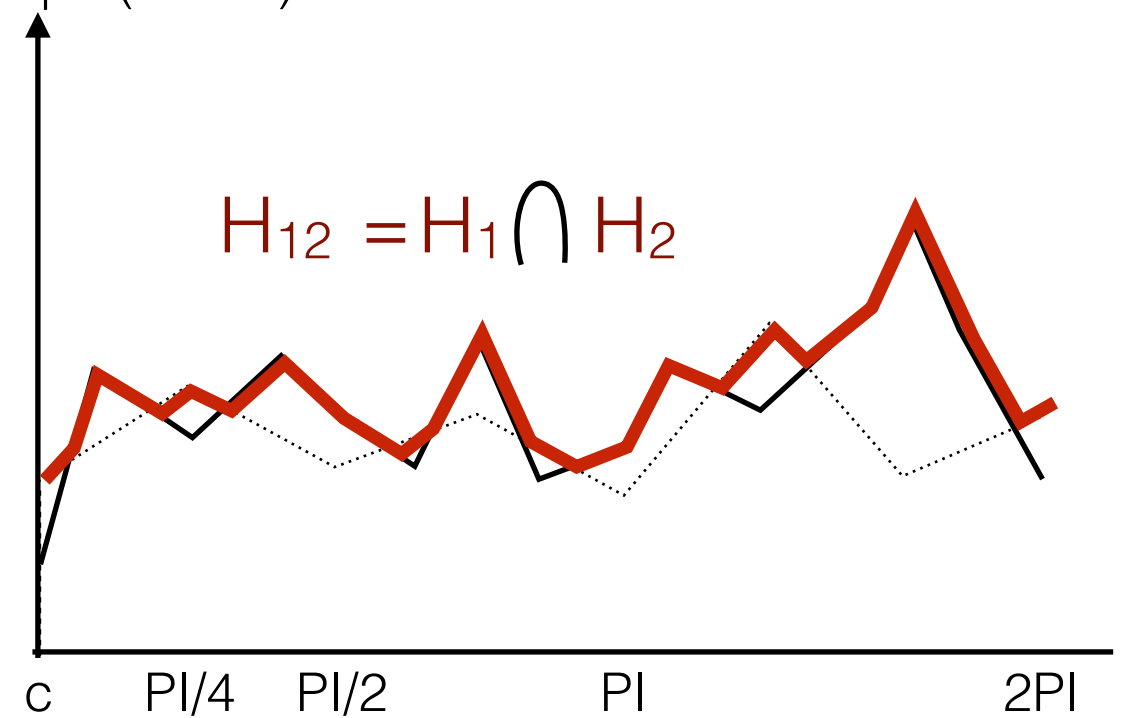




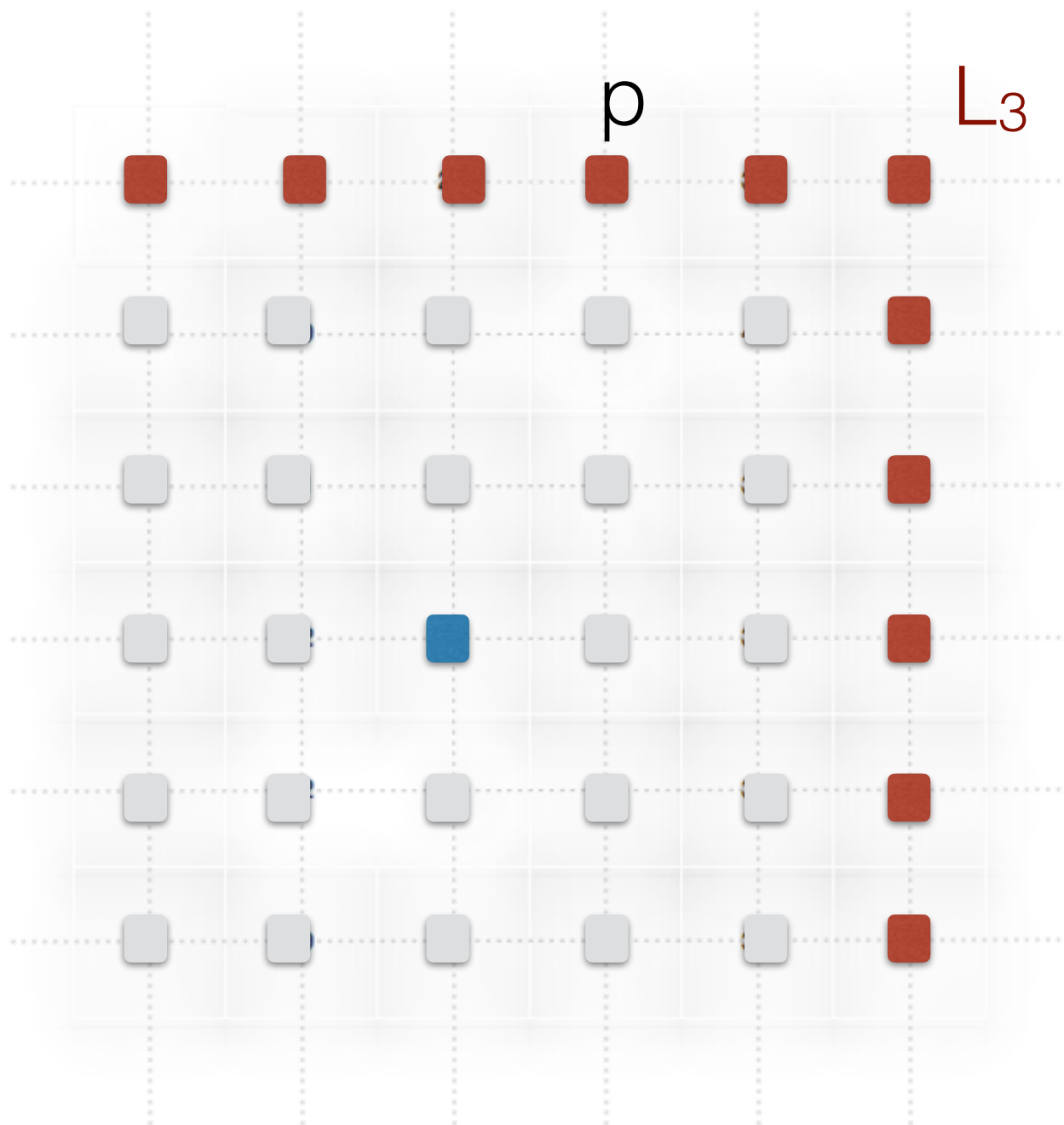
# Viewshed on grids using a concentric sweep



vertical slope (zenith)



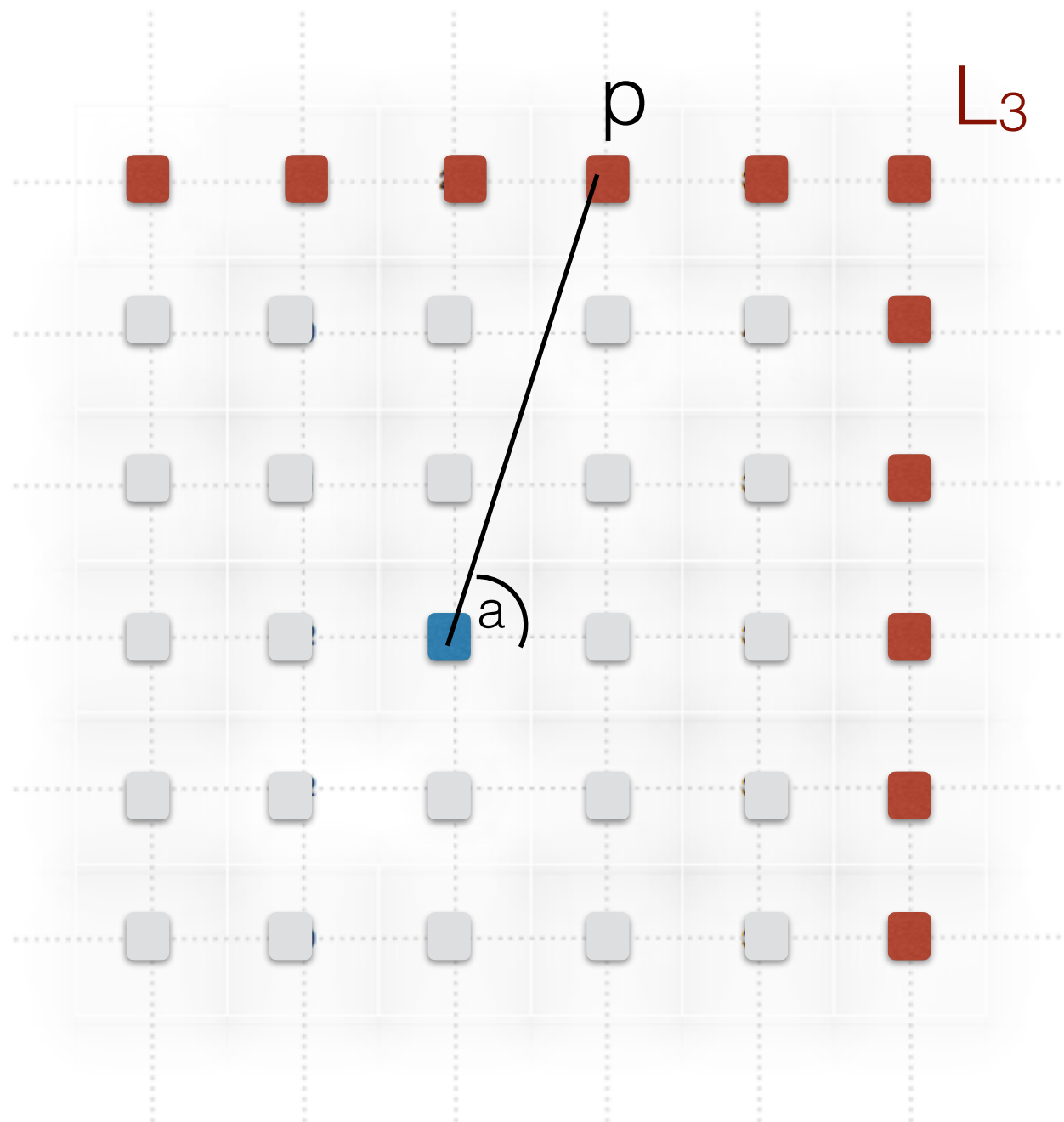
# Viewshed on grids using a concentric sweep



Is point p in L3 visible ?

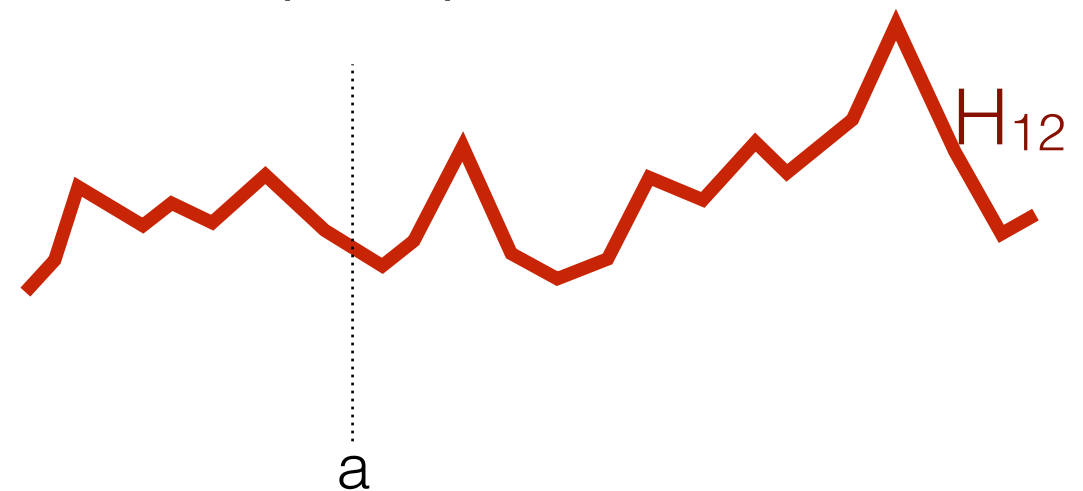


# Viewshed on grids using a concentric sweep



Is point p in L3 visible ?

p is visible if  
 $\text{slope}(vp) < H_{12}(a)$



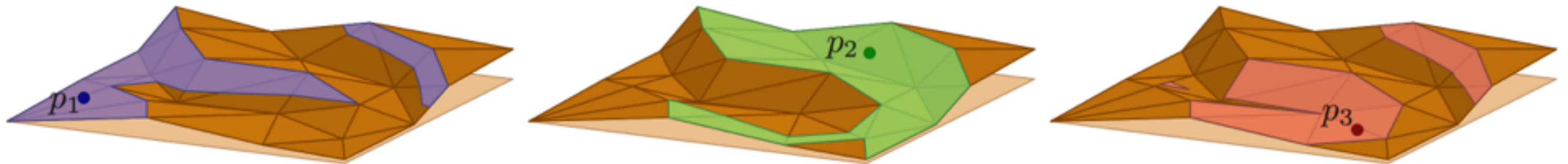
$n$  = nb. of  
cells in the grid

# Computing viewsheds

- Straightforward algorithm:  $O(n \sqrt{n})$  with linear interpolation
- $O(n \lg n)$  with nearest neighbor (NN) interpolation
  - uses radial sweep + augmented RB tree
  - crucially exploits NN
  - loses some accuracy
- A different approach: concentric sweep using horizons
  - can be adapted for linear interpolation or nearest neighbor
  - can be used for triangulated terrains
  - fast in practice because horizons stay very small

# Viewsheds on triangulated terrains

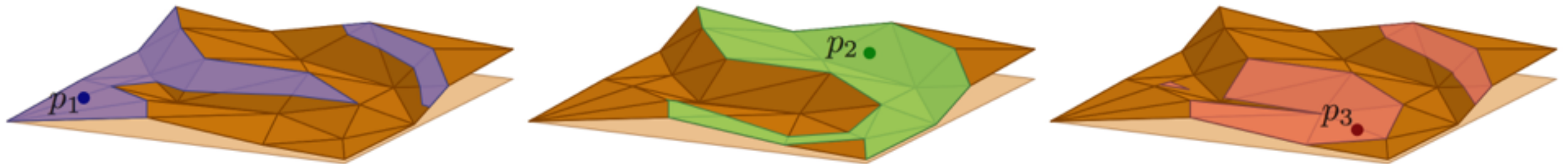
- $\text{viewshed}(p)$  contains all points of the terrain that are visible from  $p$



from: <http://arxiv.org/pdf/1309.4323.pdf>

# Viewsheds on triangulated terrains

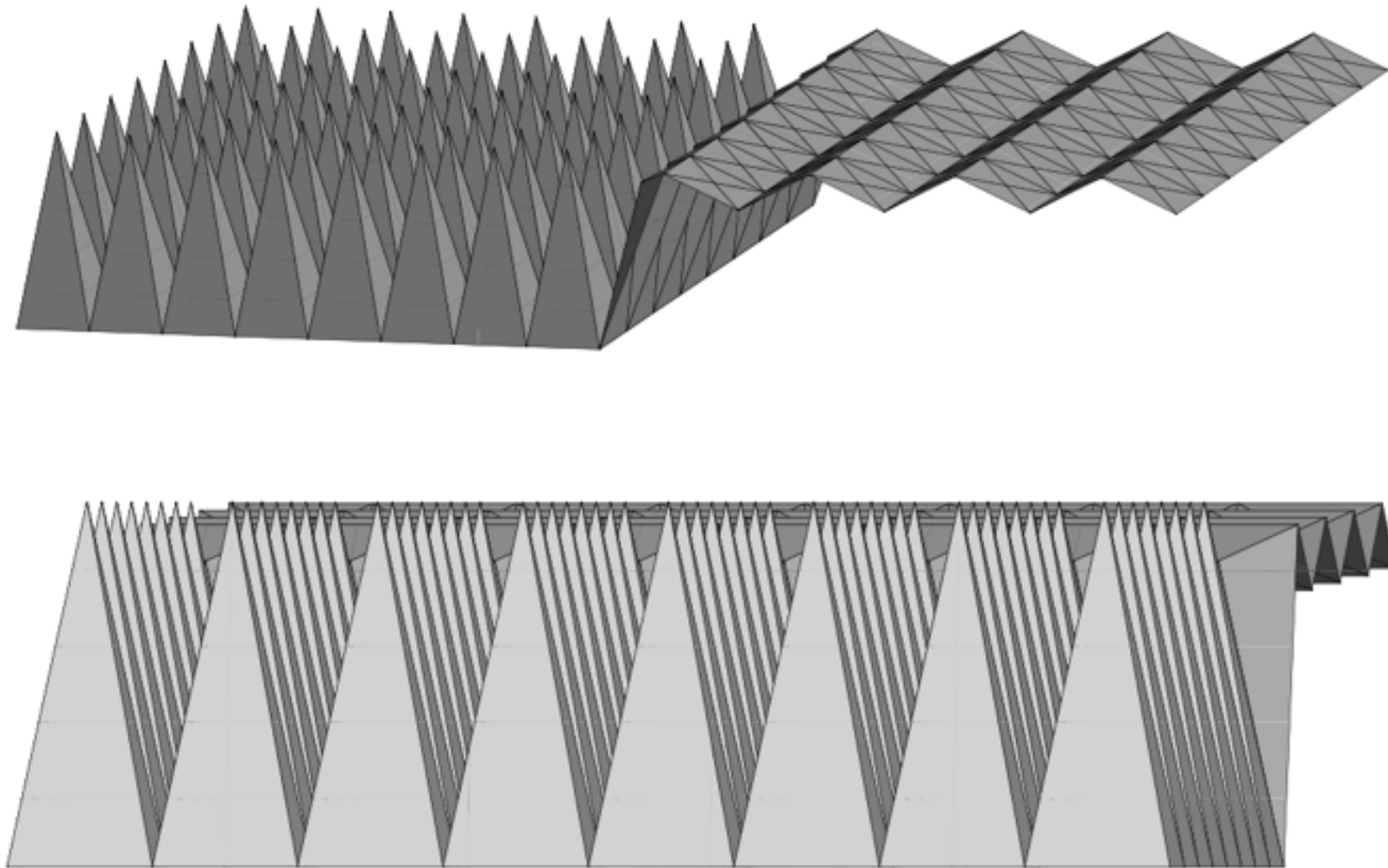
- $\text{viewshed}(p)$  contains all points of the terrain that are visible from  $p$



from: <http://arxiv.org/pdf/1309.4323.pdf>

- $\text{viewshed}(p)$  may intersect a triangle
- $\text{viewshed}(p)$  may intersect a triangle multiple times
- Question: what is the complexity of  $\text{viewshed}(p)$  in the worst case?
- Theorem: The complexity (number of vertices) of a viewshed on a triangulated terrain is  $O(n^2)$ .

from: HH, MdB, KT 2009



**Figure 1:** Two views of the same terrain defined by a regular grid. The second view gives a visibility map of complexity  $\Theta(n\sqrt{n})$ . Note that the terrain can be flattened further without changing the view combinatorially.

# Computing viewsheds on triangulated terrains

- Based on horizon computation
  - similar to grids
  - idea: traverse triangles in order of increasing distance from viewpoint, and update horizon.
  - bootstrap with divide and conquer

# Summary

- Visibility is a fundamental problem
- Straightforward solution is fairly intuitive/simple/fast
  - works well even for very large terrains
- Refinement of straightforward solution exposes elegant ideas
  - radial sweep
  - augmented RB-trees
  - visibility via horizons
    - starting point for all improved solutions
- Accuracy
  - Interpolation is important

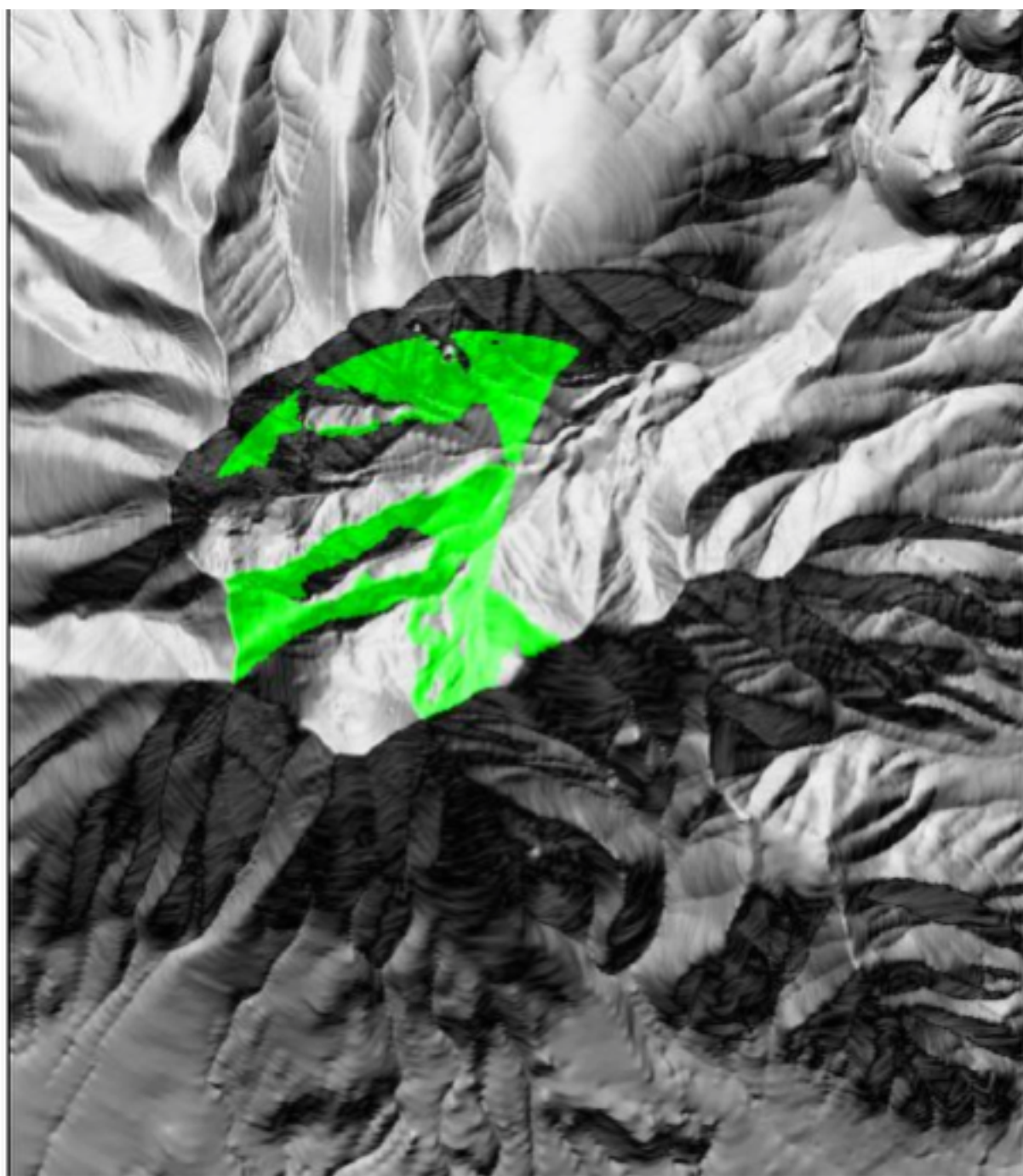
# Visibility on terrains

- Viewsheds are starting point for many other problems
  - Given a set of viewpoints, compute their joint visibility
    - aka multiple-source viewshed
  - Find locations of watch towers so that together they can guard the terrain
  - Find point of maximum/minimum visibility
  - Find viewshed count (VC) grid
    - $VC(i,j) = \text{nb. visible points in viewshed}(i,j)$
  - ...

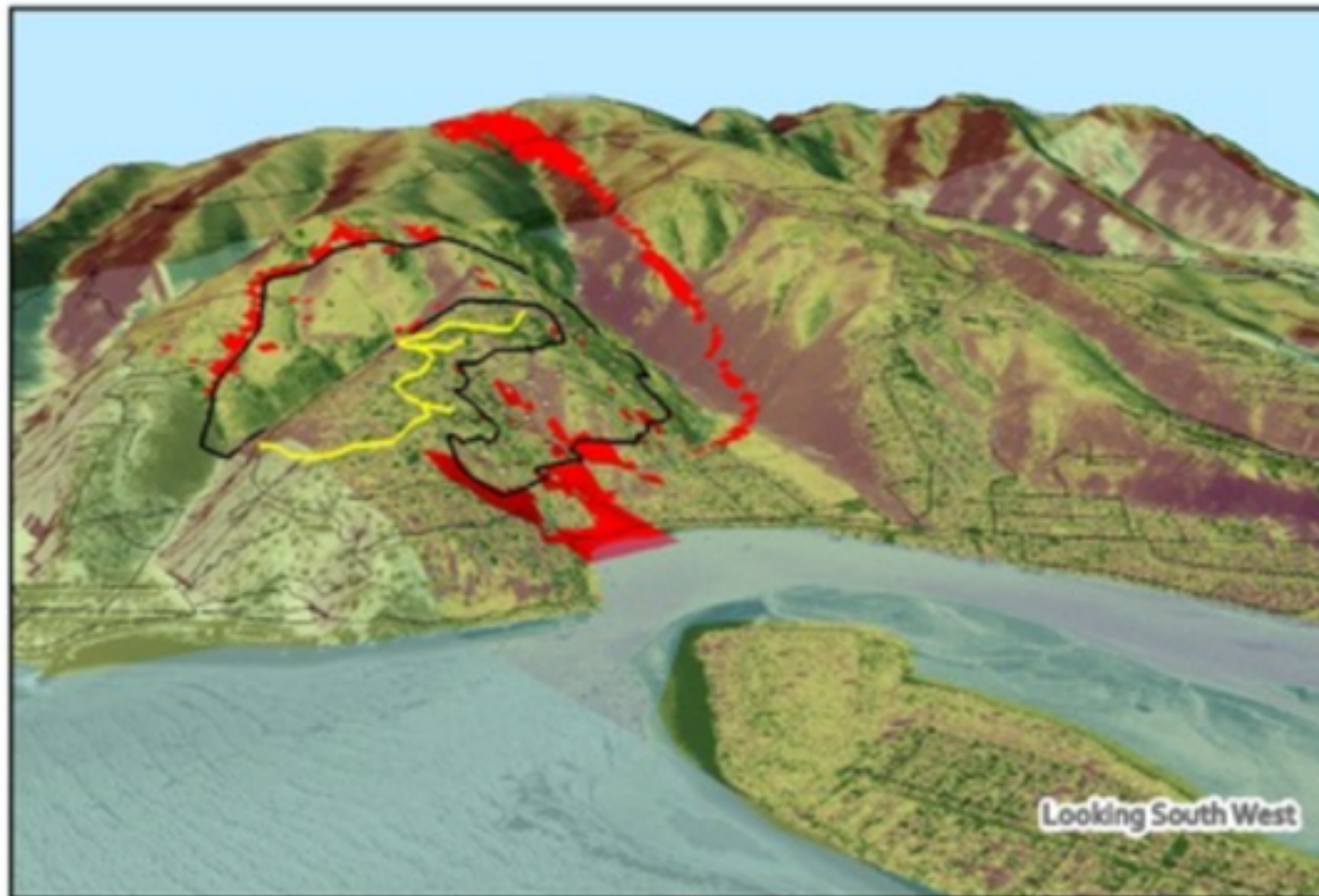


# Viewshed count

- Input: elevation grid  $G$
- Output: VC grid
  - $VC(i,j)$  = size of viewshed( $i,j$ )
- Sketch an algorithm to compute VC and its running time



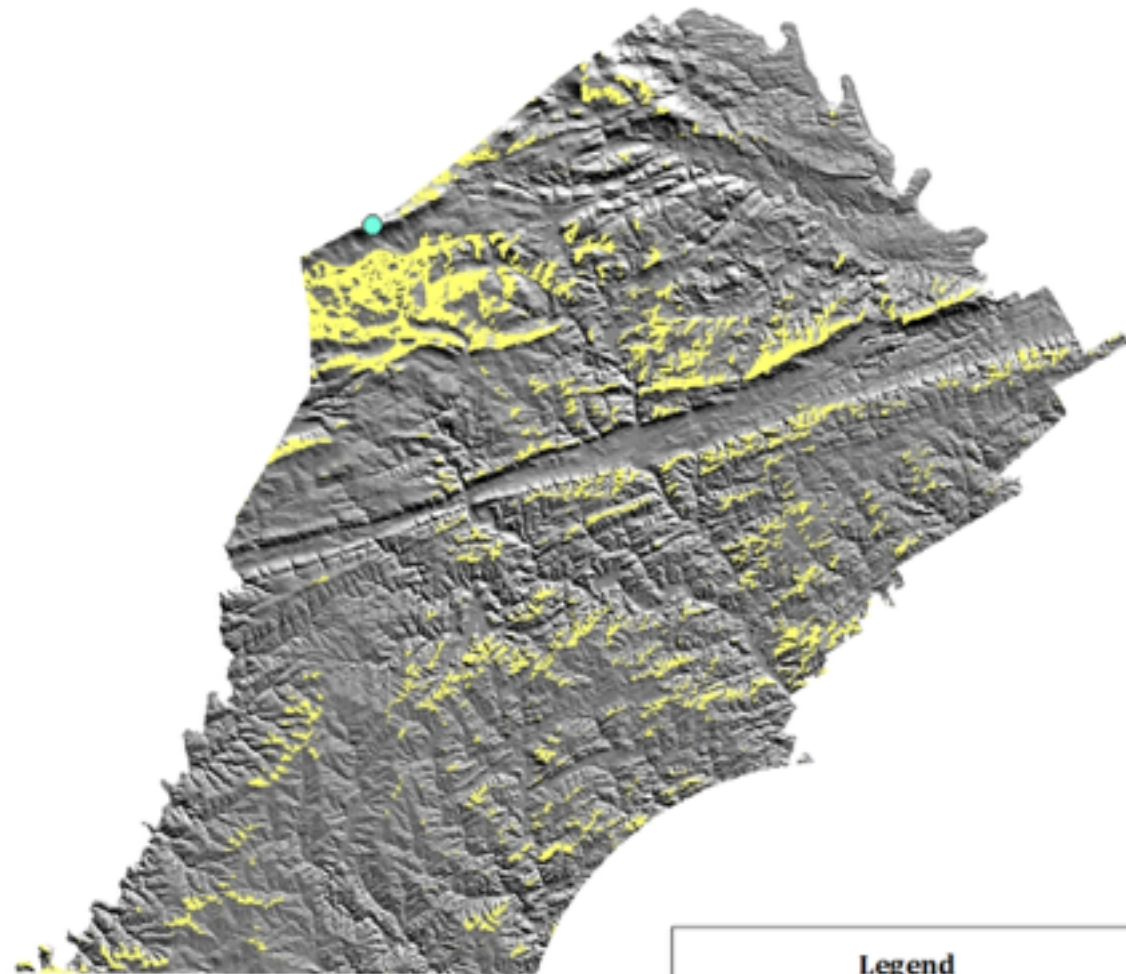
## Visual Impact of Proposed Subdivision Port Hills, Christchurch



The above images present the proposed new subdivision (outlined black) on the Port Hills. The red area shows potential viewshed loss to residence on Hurst Seager Lane, Panorama Road, Revelation Drive and Starwood Lane (highlighted yellow) if the proposed subdivision was to go ahead. The impact is based on potential buildings being up to 10m in height.



## Areas Visible From Welsh Mountain, Highest Point in Chester County, PA



0 3 6 12 Miles

