Algorithms Lab 5
(Selection and D&C)

1 Homework problems

1. Implement the selection algorithm that runs in linear time. Feel free to adjust the code to use a local index i or a global index i.

2. Let A be a list of n (not necessarily distinct) integers. Describe an $O(n)$-algorithm to test whether any item occurs more than $\lceil n/2 \rceil$ times in A. Your algorithm should use $O(1)$ additional space. A general solution should not make any additional assumptions about the integers.

3. (GT C-4.23, CLRS 9.3-7) Given an unordered sequence S of n elements (for simplicity, assume items are integers or real numbers), describe an efficient method for finding the $\lceil \sqrt{n} \rceil$ elements that are closest to the median of S. What is the running time of your method? Try for linear time.

4. (adapted form GT C-4.27, CLRS 9.3-6) Given an unsorted sequence S of n elements, and an integer k, we want to find $O(k)$ elements that have rank $\lceil n/k \rceil$, $2\lceil n/k \rceil$, $3\lceil n/k \rceil$, and so on.

   (a) Describe the "naive" algorithm that works by repeated selection, and analyze its running time function of n and k.

   (b) Describe an improved algorithm that runs in $O(n \lg k)$ time.

5. (CLRS 9-3.9) Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells.The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (either north or south), as shown in textbook CLRS figure 9.2. Given the x- and y-coordinates of the wells, show how the professor should pick the optimal location of the main pipeline, which would be the one that minimizes the total length of the spurs. Show how to determine the optimal location in linear time. Hint: Assume professor Olay is a computer science major and she loves algorithms!

6. Suppose we are given an array $A[1..n]$ with the special property that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a *local minimum* if it is less or equal to both its neighbors, or more formally, if $A[x-1] \geq A[x]$ and $A[x \ \leq A[x+1]$. For example. there are six local minima in the following array:

$$A = [9, 7, 7, 2, 1, 3, 7, 5, 4, 7, 3, 3, 4, 8, 6, 9]$$

.

We can obviously find a a local minimum in $O(n)$ time by scanning through the array. Describe and analyze an algorithm that finds a local minimum in $O(\lg n)$ time. (*Hint: with the given boundary conditions, the array must have at least one local minimum. Why?*)

7. The skyline problem/the upper envelope problem: In this problem we design a divide-and-conquer algorithm for computing the skyline of a set of n buildings.

   A *building* B_i is represented as a triplet $(\mathbf{L_i}, H_i, \mathbf{R_i})$ where $\mathbf{L_i}$ and $\mathbf{R_i}$ denote the left and right x coordinates of the building, and H_i denotes the height of the building (note that the x coordinates are drawn boldfaced.)

   A *skyline* of a set of n buildings is a list of x coordinates and the heights connecting them arranged in order from left to right (note that the list is of length at most $4n$).

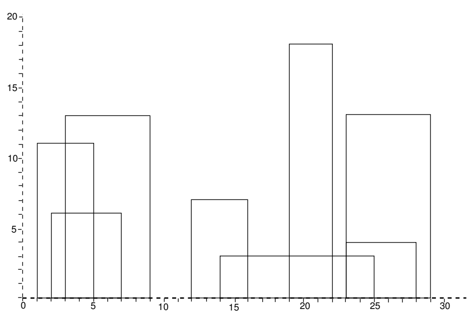
   Example: The skyline of the buildings

   $$\{(\mathbf{3}, 13, \mathbf{9}), (\mathbf{1}, 11, \mathbf{5}), (\mathbf{12}, 7, \mathbf{16}), (\mathbf{14}, 3, \mathbf{25}), (\mathbf{19}, 18, \mathbf{22}), (\mathbf{2}, 6, \mathbf{7}), (\mathbf{23}, 13, \mathbf{29}), (\mathbf{23}, 4, \mathbf{28})\}$$

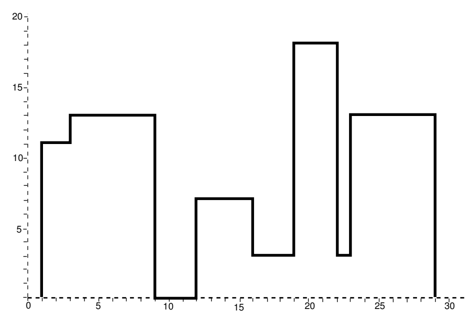
   is

   $$\{(\mathbf{1}, 11), (\mathbf{3}, 13), (\mathbf{9}, 0), (\mathbf{12}, 7), (\mathbf{16}, 3), (\mathbf{19}, 18), (\mathbf{22}, 3), (\mathbf{23}, 13), (\mathbf{29}, 0)\}$$

   (note that the x coordinates in a skyline are sorted).



buildings



skyline

   (a) Let the size of a skyline be the number of elements (tuples) in its list.
   Describe an algorithm for combining a skyline A of size n_1 and a skyline B of size n_2 into one skyline S of size $O(n_1 + n_2)$. Your algorithm should run in time $O(n_1 + n_2)$.

   (b) Describe an $O(n \log n)$ algorithm for finding the skyline of n buildings.

8. (CLRS 2-4) Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A.

   a. List the inversions of the array $< 2, 3, 8, 6, 1 >$.

   b. What array with elements from the set $\{1, 2, ..., n\}$ has the most inversions? How many does it have?

   c. Give an algorihm that determines the number of inversions in an array in $O(n^2)$ time.

   d. Give an algorihm that determines the number of inversions in an array in $O(n \lg n)$ time worst-case (Hint: modify merge sort).