Applications of graph traversal

(CLRS 22.2, 22.3)

Undirected graphs:

Concepts:

- paths, cycles
- connectivity
- \bullet shortest paths
- \bullet trees
- spanning trees/ spanning forest

Basic problems on undirected graphs:

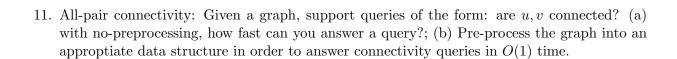
Briefly describe algorithms to answer the following questions, and analyze the complexity of your algorithm. Assume the graph is given as an adjacency list.

1. Is G connected?

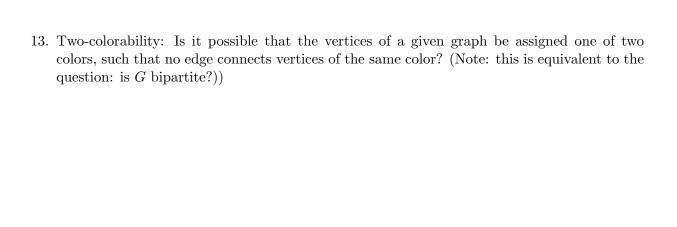
2. How many CCs are in G?

3.	Compute the connected components of G , label each vertex with the id of its CC
4	Civen two vertices are they in the same CC?
4.	Given two vertices, are they in the same CC?
5.	Given two vertice u, v , find a path between u and v
6.	Does G contain a cycle?

7.	Compute a spanning tree (forest) for G .
8.	Is G a tree?
9.	Assume G is a connected undirected graph; given any two vertices u, v , find the shortest path between them.
10.	Assume G is a connected undirected graph with vertices $v_1, v_2,, v_n$; describe how to compute a 2D-array $d[1n][1n]$ such that $d[i][j]$ represents the length (number of edges) of the shortest path from v_i to v_j .



12. All-pair shortest paths: Given a graph, support queries of the form: find the shortest path from u to v. (a) no -preprecessing; (b) Describe how a graph can be pre-processed in order to answer shortest path queries in O(1) time.



Directed graphs (digraphs)

Concepts:

- Reachability
- Directed paths and directed cycles
- Strongly connected components (SCC)
- Directly acyclic graphs (DAGs) and topological ordering
- Transitive closure (TC)

Basic problems on directed graphs:

Briefly describe algorithms to answer the following questions, and analyze the complexity of your algorithm. Assume the graph is given as an adjacency list.

1. Find all verties reachable from a given vertex u.

2. Given a vertex u, compute all vertices v such that u is reachable from v.

3. Given two vertices u, v, is there a (directed) path from u to v? If so, find such a path.

4.	Given two vertices u, v , is there a (directed) path from u to v ? If so, find such a <i>shortest</i> such path.
5.	Does G have a directed cycle?
6.	Is G a DAG? (ie is G acyclic?)

7.	All-pair reachability: Given a graph, support queries of the form: given u, v , is v reachable from u ? (a) no pre-precessing; (b) with pre-precessing, in $O(1)$ time per query;
8.	Are two vertices u, v in the same SCC?
9.	Compute the SCCs of G (label each vertex with the id of its SCC).