Algorithms Lab 5

The in-lab problems are to be solved during the lab time. Work with your team, but write your solutions individually. You do not need to turn in your answers, only discuss them with me. Ask me for feedback.

The homework problem set is due in one week. Work with your team, but write your solutions individually. List the people with whom you discussed the problems clearly on the first page.

1 In lab exercises

- (CLRS 9.3-1) In the algorithm SELECT, the input elements are divided into groups of 5. Will the SELECT algorithm work in linear time if they are divided into groups of 7? Argue that SELECT does not run in linear time of groups of 3 are used.
- (CLRS 9.3-3) Show how QuickSort can be made to run in $O(n \lg n)$ time in the worst case, assuming that all elements are distinct.
- (CLRS 9.3-5) Suppose that you have a "black-box" worst-case linear time median subroutine. Give a simple, linear time algorithm for SELECT (i).
- (GT C-4.12) Show that any comparison-based sorting algorithm can be made to be stable, without affecting the asymptotic running time of the algorithm. (Hint: Change the way elements are compared to each other.)
- (GT C-4.15) Let $S_1, S_2, ...S_k$ be k different sequences whose elements have integer keys in the range $[0, N-1]$, for some parameter $N \geq 2$. Describe an algorithm running in $O(n+N)$ time for sorting all the sequences (not as a union), where n denotes the total size of all sequences.

2 Homework problems

1. Let A be a list of n (not necessarily distinct) integers. Describe an $O(n)$-algorithm to test whether any item occurs more than $\lceil n/2 \rceil$ times in A. Your algorithm should use $O(1)$ additional space.
2. (GT C-4.23, CLRS 9.3-7) Given an unordered sequence S of n comparable elements, describe an efficient method for finding the $\lceil \sqrt{n} \rceil$ items whose rank in an ordered version of S is closest to that of the median. What is the running time of your method?
3. (GT C-4.27, CLRS 9.3-6) Given an unsorted sequence S of n elements, and an integer k, give an $O(n \lg k)$ expected time algorithm for finding the $O(k)$ elements that have rank $\lceil n/k \rceil$, $2\lceil n/k \rceil$, $3\lceil n/k \rceil$, and so on.

4. (CLRS 9-3.9) Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells.The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (eitehr north or south), as shown in textbook CLRS figure 9.2. Given the x- and y-coordinates of the wells, show how the professor should pick the optimal location of the main pipeline, which would be teh one that minimizes the total length of the spurs. Show how to determine the optimal location in linear time. Hint: Assume professor Olay is a computer science professor and her specialty is algorithms!