Algorithms Lab 3

The in-lab problems are to be solved during the lab time. Work with your team, but write your solutions individually. You do not need to turn in your answers, only discuss them with me. Ask me for feedback on how you write your answers.

The homework problem set is due in one week. Work with your team, but write your solutions individually. List the people with whom you discussed the problems.

1 In lab exercises

Finish the in-class exercises on heaps.

2 Homework problems

1. (GT C-2.31) Develop an algorithm that computes the kth smallest element in a set of n distinct integers in $O(n + k \lg n)$ time.

2. (CLRS 6.5-9) Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (*Hint: use a min-heap for k-way merging.*)

3. (CLRS 9-1) Given a set of n numbers, we wish to find the i largest in sorted order using a comparison-based algorithm. Find the algorithm that implements each of the following methods with the best asymptotic worst-case running time, and analyze the running times of the algorithms on terms of n and i.

   (a) Sort the numbers, and list the i largest.

   (b) Build a max-priority queue from the numbers, and call EXTRACT-MAX i times.

4. (CLRS 7-3) Professors Dewey, Cheatham, and Howe have proposed the following "elegant" sorting algorithm:

   STOOGE-SORT(A, i, j)
   if $A[i] > A[j]$
       then exchange $A[i] \leftrightarrow A[j]$
   if $i + 1 \geq j$
       then return
   $k \leftarrow \lfloor (j - i + 1)/3 \rfloor$
   STOOGE-SORT$(A, i, j - k)$
   STOOGE-SORT$(A, i + k, j)$
   STOOGE-SORT$(A, i, j - k)$

a. Argue that STOOGE-SORT$(A, 1, length[A])$ correctly sorts the input array $A[1..n]$, where $n = length[A]$. If you know how to do induction, do a proof by induction. Otherwise, just provide a clear argument.

b. Give a recurrence for the worst-case running time of STOOGE-SORT and a tight asymptotic (Θ-notation) bound on the worst-case running time.

c. Compare the worst-case running time of STOOGE-SORT with that of insertion sort, merge sort, and heapsort. Do the professors deserve tenure?