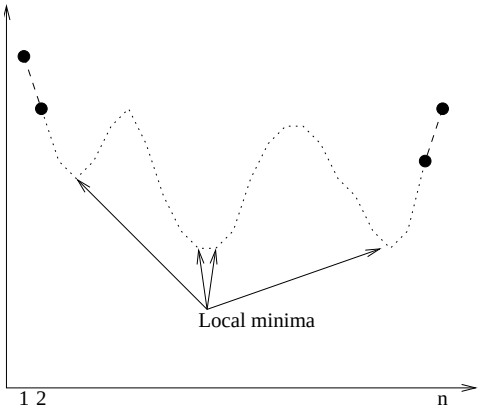
CSci 231 Homework 2

1. Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists.

2. Given a set of n numbers, we wish to find the i largest in sorted order using a comparison-based algorithm. Find the algorithm that implements each of the following methods with the best asymptotic worst-case running time, and analyze the running times of the algorithms on terms of n and i.

   (a) Sort the numbers, and list the i largest.
   (b) Build a max-priority queue from the numbers, and call EXTRACT-MAX i times.
   (c) Use a SELECT algorithm to find the ith largest number, partition around that number, and sort the i largest numbers.

3. Consider an array A of length n for which we know that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that $A[x]$ is a *local minimum* if $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. Note that A must have at least one local minimum.



   We can obviously find a local minimum in $O(n)$ time by scanning through A. Describe an $O(\log n)$ algorithm for finding a local minimum.

4. Describe an $O(n)$ algorithm that, given a set S of n distinct numbers and a positive integer $k \leq n$, determines the k numbers in S that are closest (in value) to the median of S.

5. Let A be a list of n (not necessarily distinct) integers. Describe an $O(n)$-algorithm to test whether any item occurs more than $\lceil n/2 \rceil$ times in A. Your algorithm should use $O(1)$ additional space.

6. Give an $O(n \lg k)$ algorithm to find the $k-1$ elements in a set that partition the set into (approx.) k equal-sized sets $A_1, A_2, \ldots A_k$ such that all elements in A_i are smaller than all elements in A_{i+1}. Assume k is a power of 2.

7. Show how to sort n integers in the range 1 to n^2 in $O(n)$ time.