CSci 231 Homework 5

Greedy and Dynamic Programming

Write each problem on a separate page. You will be graded on the clarity of the solution as well as correctness and completeness.

1. A game-board consists of a row of n fields, each consisting of two numbers. The first number can be any positive integer, while the second is 1, 2, or 3. An example of a board with $n = 6$ could be the following:

17	2	100	87	33	14
1	2	3	1	1	1

The object of the game is to jump from the first to the last field in the row. The top number of a field is the cost of visiting that field. The bottom number is the maximal number of fields one is allowed to jump to the right from the field. The cost of a game is the sum of the costs of the visited fields.

Let the board be represented in a two-dimensional array $B[n, 2]$. The following recursive procedure (when called with argument 1) computes the cost of the cheapest game:

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Cheap(i)

     IF i>n THEN return 0
     x=B[i,1]+Cheap(i+1)
     y=B[i,1]+Cheap(i+2)
     z=B[i,1]+Cheap(i+3)
     IF B[i,2]=1 THEN return x
     IF B[i,2]=2 THEN return min(x,y)
     IF B[i,2]=3 THEN return min(x,y,z)

END Cheap
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(a) Analyze the asymptotic running time of the procedure.

(b) Describe and analyze a more efficient algorithm for finding the cheapest game.

2. Suppose you are in charge of planning a party for Bowdoin College. The college has a hierarchical structure, which forms a tree rooted at President Mills. On the very last level are the faculty, grouped by department. (I have such a chart in my office, come by if you want to take a look). Each faculty has "underneath" all students taking a class with him/her that particular semester. Assume that every person is listed at the highest possible position in the tree and there are no double affiliations (everybody has one and only one supervisor in this hierarchy and no student is in more than one class).

   You have access to a secret database which ranks each faculty/staff/student with a conviviality rating (a real number, which can be negative if the person is really difficult or boring). In order to make the party fun for everybody, President Mills does not want both a faculty/staff/student and his or her immediate "supervisor" to attend.

   You are given a tree that describes the strucure of Bowdoin College. Each node also holds a name and a conviviality ranking. Describe an algorithm to make up a guest list that maximizes the sum of the conviviality rankings of the guests. Analyze the running time of your algorithm.

3. *(text adapted from B. Thom, Harvey Mudd College)* You've decided to become a ski bum, and hooked up with Sugarloaf Ski Resort. They'll let you ski for free all winter, in exchange for helping their ski rental shop with an algorithm to assign skis to skiers.

   Ideally, each skier should obtain a pair of skis whose heigth matches his or her own height exactly. Unfortunately, this is generally not possible. We define the disparity between a skier and his/her skis as the absolute value of the difference between teh height of the skier and the height of the skis. The objective is to find an assignment of skis to skiers that minimizes the sum of the disparities.

   (a) First, let's assume that there are n skiers and m skis. Consider the following algorithm: consider all possible assignments of skis to skiers; for each one, compute the sum of the disparities; select the one that minimizes the total disparity. How much time would this algorithm take on a 1GHz computer, if there were 50 skiers and 50 skis?

   (b) One day, while waiting for the lift, you make an interesting discovery: if we have a short person and a tall person, it would never be better to give to the shorter person aa taller pair of skis than were given to the taller person. Prove that this is always true.

   (c) Describe a greedy algorithm to assign the skis to skiers, and argue why this algorithm is correct. What is the time complexity of your algorithm?

   (d) Your next task is to design an efficient dynamic programming algorithm for the more general case where there are m skiers and n pairs of skis ($n \geq m$).

Here is some notation that may help you. Sort the skiers and skis by increasing height. Let h_i denote the height of the ith skier in sorted order, and s_j denote the height of the jth pair of skis in sorted order. Let OPT(i, j) be the optimal cost (disparity) for matching the first i skiers with skis from the set $\{1, 2, ..., j\}$. The solution we seek is simply OPT(m, n). Define OPT(i, j) recursively.

(e) Now describe a dynamic programming algorithm for the problem. What is the running time of your algorithm? Explain.

(f) Briefly describe how your algorithm can be modified to allow you to find an actual optimal assignment (rather than just the cost) of skis to skiers. How does this affect the running time of your algorithm?

(g) Illustrate your algorithm by explicitly filling out the table OPT(i, j) for the following sample data: ski heights 1, 2, 5, 7, 13, 21. Skier heights 3,4, 7, 11, 18.

4. Problem 6.1 in textbook

5. Problem 6.2 in textbook