CSci 231 Homework 5

Heaps

CLRS Chapter 6

Write and justify your answers on this sheet in the space provided. Collaboration is allowed and encouraged, if it is constructive and helps you study better. Remember, exams will be individual. Write up the solutions on your own and list the names of the collaborators.

1. (CLRS 6.1-1) What are the minimum and maximum number of elements in a heap of height h? Note: the height of a heap is the number of edges on the longest root-to-leaf path.

2. (CLRS 6.1-4) Where in a min-heap might the largest element reside, assuming that all elements are distinct?

3. (CLRS 6.1-5) Is an array that is in sorted order a min-heap?

4. (CLRS 6.2-4) What is the effect of calling MIN-HEAPIFY(A, i) for $i > size[A]/2$?

5. (CLRS 6.5-3) Write pseudocode for the procedures HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY and HEAP-INSERT that implement a min-priority queue with a min-heap.

6. *(CLRS 6.5-8) Give an $O(n \lg k)$-time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (Hint: use a min-heap for k-way merging.)*

7. *(CLRS 9-1) Given a set of n numbers, we wish to find the i largest in sorted order using a comparison-based algorithm. Find the algorithm that implements each of the following methods with the best asymptotic worst-case running time, and analyze the running times of the algorithms on terms of n and i.*

   (a) Sort the numbers, and list the i largest.

   (b) Build a max-priority queue from the numbers, and call EXTRACT-MAX i times.

   (c) Use a SELECT algorithm to find the ith largest number, partition around that number, and sort the i largest numbers.