

CSci 231 Homework 2

Recurrences

CLRS Chapter 4

*Write and justify your answers on this sheet in the space provided.*¹

1. Solve the recurrence: $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n-1) + n(n-1) & \text{if } n \geq 2 \end{cases}$

Hint: use $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

¹Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Write up the solutions on your own.

2. (CLRS 4.2-2) Argue that the solution to the recurrence

$$T(n) = T(n/3) + T(2n/3) + n$$

is $\Omega(n \log n)$ by appealing to a recursion tree.

Solve (that is, give asymptotic upper and lower bounds for) each of the following recurrences using both methods:

- (a) by iteration (show your work; Do NOT use the Master's method);
- (b) by substitution (it is enough if you prove only the upper bound).

Assume $T(n)$ is constant for $n \leq c$, where you can choose c as you need it.

3. $T(n) = T(n - 1) + n$
4. $T(n) = T(\sqrt{n}) + 1$
5. $T(n) = 2T(n/2) + n/\lg n$
6. $T(n) = T(n - 1) + 1/n$
7. $T(n) = 2T(n/4) + \sqrt{n}$
8. $T(n) = 7T(n/2) + n^3$
9. $T(n) = 7T(n/2) + n^2$
10. $T(n) = 5T(n/5) + n/\log n$