CPS 231 Homework 1

Insertion sort and Growth of Functions

CLRS Chapter 1, 2 and 3, Appendix A

Write and justify your answers on this sheet in the space provided.[1]

Exercises (suggested)

1. (CLRS 2.1-2) How do you modify the $INSERTION - SORT$ procedure to sort into non-increasing instead of non-decreasing order?
2. (CLRS 2.2-4) How can we modify almost any algorithm to have a good best-case running time?

Problems (mandatory)

1. (CLRS 1.2-2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \log n$ steps. For which values of n does insertion sort beat merge sort?

2. (CLRS 1-1) For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithms to solve the problem takes $f(n)$ microseconds.

	1 second	1 minute	1 day	1 month
n				
n^2				
2^n				

[1] Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Write up the solutions on your own.

3. (CLRS 3.1-3) Explain why the statement 'The running time of algorithm A is at least $O(n^2)$' is content free.

4. (part of CLRS 3-3) Order the following expressions by their asymptotic growth and **justify your answer.**

$$2^n, n!, (\log n)!, n^3, e^n, 2^{\log_2 n}, n \log n, 2^{2^n}, n^{\log \log n}$$

5. (CLRS A.1-1) Find a simple formula for $\sum_{k=1}^{n}(2k-1)$.

6. Prove by induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

7. **(CLRS 3-2) Indicate, for each pair of expressions (A,B) in the table below, whether A is $O()$, $o()$, $\Omega(), \omega()$ or $\Theta()$ of B. Assume $k \geq 1, \epsilon > 0$ and $c > 1$ are constants. Your answer should be in the form of "yes" or "no" wrtten in each box.**

A	B	O	o	Ω	ω	Θ
$\lg^k n$	n^ϵ					
n^k	c^n					
$\sqrt{n}$	$n^{\sin n}$					
2^n	$2^{n/2}$					
$n^{\lg c}$	$c^{\lg n}$					
$\lg(n!)$	$\lg(n^n)$					

8. **(part of CLRS 3-4) Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove the following:**

   (a) $f(n) = O(g(n)) \Longleftarrow g(n) = O(f(n))$

   (b) $f(n) + g(n) = \Theta(min(f(n), g(n)))$