CPS 130 Homework 7 - Solutions

1. (CLRS 8.3-1) Illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

   Solution:

$$
\begin{array}{ccccccc}
\text{COW} & & \text{SEA} & & \text{TAB} & & \text{BAR} \\
\text{DOG} & & \text{TEA} & & \text{BAR} & & \text{BIG} \\
\text{SEA} & & \text{MOB} & & \text{EAR} & & \text{BOX} \\
\text{RUG} & & \text{TAB} & & \text{TAR} & & \text{COW} \\
\text{ROW} & & \text{DOG} & & \text{SEA} & & \text{DIG} \\
\text{MOB} & & \text{RUG} & & \text{TEA} & & \text{DOG} \\
\text{BOX} & & \text{DIG} & & \text{DIG} & & \text{EAR} \\
\text{TAB} & \Rightarrow & \text{BIG} & \Rightarrow & \text{BIG} & \Rightarrow & \text{FOX} \\
\text{BAR} & & \text{BAR} & & \text{MOB} & & \text{MOB} \\
\text{EAR} & & \text{EAR} & & \text{DOG} & & \text{NOW} \\
\text{TAR} & & \text{TAR} & & \text{COW} & & \text{ROW} \\
\text{DIG} & & \text{COW} & & \text{ROW} & & \text{RUG} \\
\text{BIG} & & \text{ROW} & & \text{NOW} & & \text{SEA} \\
\text{TEA} & & \text{NOW} & & \text{BOX} & & \text{TAB} \\
\text{NOW} & & \text{BOX} & & \text{FOX} & & \text{TAR} \\
\text{FOX} & & \text{FOX} & & \text{RUG} & & \text{TEA}
\end{array}
$$

2. (CLRS 8.3-2) Which of the following sorting algorithms are stable: INSERTION-SORT, MERGE-SORT, QUICKSORT? Give a simple scheme that makes any sorting algorithm stable. How much additional time and space does your scheme entail?

   Solution: INSERTION-SORT is stable (page 3 CLRS), MERGE-SORT is stable (page 12 CLRS), and QUICKSORT (page 154 CLRS) is not stable.

   There are many solutions to the second part. One idea is to add to each key the position in the initial array and to sort using the additional secondary key. This requires $O(n)$ additional space and has the same time requirement.

3. (CLRS 8.3-4) Show how to sort n integers in the range 1 to n^2 in $O(n)$ time.

   Solution: Use RADIX-SORT in base n. Since the numbers are in base n, the range of digits is 1 to n so $k = n$. The number of passes needed is 3 since $n^2 = 100_n$ so $d = 3$. The running time of RADIX-SORT is $\Theta(dn + dk) = \Theta(3n + 3n) = \Theta(n) \in O(n)$.

4. (CLRS 8.4-1) Illustrate the operation of BUCKET-SORT on the array

$$A = [.79, .13, .16, .64, .39, .20, .89, .53, .71, .42].$$

   Solution:

$$
\begin{array}{c|llllll}
0 & / & & & & & \\
1 & \rightarrow & .13 & \rightarrow & .16 & / \\
2 & \rightarrow & .20 & / & & \\
3 & \rightarrow & .39 & / & & \\
4 & \rightarrow & .42 & / & & \\
5 & \rightarrow & .53 & / & & \\
6 & \rightarrow & .64 & / & & \\
7 & \rightarrow & .71 & \rightarrow & .79 & / \\
8 & \rightarrow & .89 & / & & \\
9 & / & & & & &
\end{array}
$$