CPS 130 Homework 1 - Solutions

1. (CLRS 1.2-2) Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \log n$ steps. For which values of n does insertion sort beat merge sort?

   Solution: We want to find n such that $8n^2 \leq 64n \lg n$:

   $$\begin{aligned} 8n^2 \leq 64n \lg n &\Rightarrow n \leq 8 \lg n \\ &\Rightarrow \frac{1}{8} \leq \frac{\lg n}{n}, \end{aligned}$$

   which is satisfied for $n \leq 43$.

2. (CLRS 1-1) For each function $f(n)$ and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithms to solve the problem takes $f(n)$ microseconds.

   Solution:

	1 second	1 minute	1 day	1 month
n	10^6	60×10^6	864×10^8	26×10^{11}
n^2	10^3	7745	29×10^4	16×10^5
2^n	20	26	36	41

3. (CLRS 2.1-2) How do you modify the $INSERTION - SORT$ procedure to sort into non-increasing instead of non-decreasing order?

   Solution: Modify the **while** loop test as follows: **while** $i > 0$ and $A[i] < key$