

# CPS 130 Homework 17

## Amortized Analysis

due Thu June 13th

*Write and justify your answers in the space provided.*<sup>1</sup>

1. In this problem we consider two stacks A and B manipulated using the following operations ( $n$  denotes the size of A and  $m$  the size of B):
  - *PushA*( $x$ ): Push element  $x$  on stack A.
  - *PushB*( $x$ ): Push element  $x$  on stack B.
  - *MultiPopA*( $k$ ): Pop  $\min\{k, n\}$  elements from A.
  - *MultiPopB*( $k$ ): Pop  $\min\{k, m\}$  elements from B.
  - *Transfer*( $k$ ): Repeatedly pop an element from A and push it on B, until either  $k$  elements have been moved or A is empty.

Assume that A and B are implemented using doubly-linked lists such that *PushA* and *PushB*, as well as a single pop from A or B, can be performed in  $O(1)$  time worst-case.

- (a) What is the worst-case running time of the operations *MultiPopA*, *MultiPopB* and *Transfer*?

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<sup>1</sup>Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Students must write up the solutions on their own.

- (b) Define a potential function  $\Phi(n, m)$  and use it to prove that the operations have amortized running time  $O(1)$ .