CPS 130 Homework 13
Augmented Search Trees

Write and justify your answers in the space provided.[1]

1. (CLRS 14.1-5) Given an element x in an n-node order-statistic tree and a natural number i, how can the ith successor of x in the linear order of the tree be determined in $O(\log n)$ time?

[1] Collaboration is allowed, even encouraged, provided that the names of the collaborators are listed along with the solutions. Students must write up the solutions on their own.

2. In this problem we consider a data structure for maintaining a multi-set M. We want to support the following operations:

   - $\mathit{Init}(M)$: create an empty data structure M.
   - $\mathit{Insert}(M, i)$: insert (one copy of) i in M.
   - $\mathit{Remove}(M, i)$: remove (one copy of) i from M.
   - $\mathit{Frequency}(M, i)$: return the number of copies of i in M.
   - $\mathit{Select}(M, k)$: return the k'th element in the sorted order of elements in M.

   If for example M consists of the elements

   $$< 0, 3, 3, 4, 4, 7, 8, 8, 8, 9, 11, 11, 11, 11, 13 >$$

   then $\mathit{Frequency}(M, 4)$ will return 2 and $\mathit{Select}(M, 6)$ will return 7.

   Let $|M|$ and $\|M\|$ denote the number of elements and the number of *different* elements in M, respectively.
   a) Describe an implementation of the data structure such that $\mathit{Init}(M)$ takes $O(1)$ time and all other operations take $O(\log \|M\|)$ time.

   b) Design an algorithm for sorting a list L in $O(|L| \log \|L\|)$ time using this data structure.