CPS 130 Homework 7 - Solutions

1. (CLRS 8.3-1) Illustrate the operation of Radix-Sort on the following list of English words: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

   Solution:

COW		SEA		TAB		BAR
DOG		TEA		BAR		BIG
SEA		MOB		EAR		BOX
RUG		TAB		TAR		COW
ROW		DOG		SEA		DIG
MOB		RUG		TEA		DOG
BOX		DIG		DIG		EAR
TAB	$\Rightarrow$	BIG	$\Rightarrow$	BIG	$\Rightarrow$	FOX
BAR		BAR		MOB		MOB
EAR		EAR		DOG		NOW
TAR		TAR		COW		ROW
DIG		COW		ROW		RUG
BIG		ROW		NOW		SEA
TEA		NOW		BOX		TAB
NOW		BOX		FOX		TAR
FOX		FOX		RUG		TEA

2. (CLRS 8.3-2) Which of the following sorting algorithms are stable: Insertion-Sort, Merge-Sort, Quicksort? Give a simple scheme that makes any sorting algorithm stable. How much additional time and space does your scheme entail?

   Solution: Insertion-Sort is stable (page 3 CLRS), Merge-Sort is stable (page 12 CLRS), and Quicksort (page 154 CLRS) is not stable.

   There are many solutions to the second part. One idea is to add to each key the position in the initial array and to sort using the additional secondary key. This requires $O(n)$ additional space and has the same time requirement.

3. (CLRS 8.3-4) Show how to sort n integers in the range 1 to n^2 in $O(n)$ time.

   Solution: Use Radix-Sort in base n. Since the numbers are in base n, the range of digits is 1 to n so $k = n$. The number of passes needed is 3 since $n^2 = 100_n$ so $d = 3$. The running time of Radix-Sort is $\Theta(dn + dk) = \Theta(3n + 3n) = \Theta(n) \in O(n)$.

4. (CLRS 8.4-1) Illustrate the operation of Bucket-Sort on the array

$$A = [.79, .13, .16, .64, .39, .20, .89, .53, .71, .42].$$

   Solution:

$$
\begin{array}{c|lllll}
0 & / & & & & \\
1 & \rightarrow & .13 & \rightarrow & .16 & / \\
2 & \rightarrow & .20 & / & & \\
3 & \rightarrow & .39 & / & & \\
4 & \rightarrow & .42 & / & & \\
5 & \rightarrow & .53 & / & & \\
6 & \rightarrow & .64 & / & & \\
7 & \rightarrow & .71 & \rightarrow & .79 & / \\
8 & \rightarrow & .89 & / & & \\
9 & / & & & &
\end{array}
$$