

## Week 3: Lab

COLLABORATION LEVEL 0 (NO RESTRICTIONS). OPEN NOTES.

1. Consider the linear-time merge algorithm discussed in the notes and a possible implementation below. How many element comparisons will the standard merge function take to merge the following left and right lists ?

$left = [1, 3, 4, 5, 6, 7, 8]$ ,  $right = [1, 5, 9, 11, 12, 16]$

**Merge(left, right)**

```
result = []
i=0
j=0
while i < len(left) and j < len(right):
    if left[i] < right[j]:
        result.append(left[i])
        i = i+1
    else:
        result.append(right[j])
        j = j+1

# add any left overs
while i < len(left):
    result.append(left[i])
    i = i+1
while j < len(right):
    result.append(right[j])
    j = j+1

return result
```

- A 8
- B 9
- C 10
- D 13

Find a  $\Theta()$  bound for the following recurrences using iteration. Assume  $T(1) = 1$ .

*What we expect: show  $O(1)$  steps of your iteration, with the general formula, the recursion depth, and the final  $\Theta()$  bound for  $T(n)$ . **Do not write your answers on this page.** Use a separate page for each problem and show your work.*

2.  $T(n) = T(n/2) + 1$

3.  $T(n) = T(n/3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ).

4.  $T(n) = T(n/10) + 1$  (assume  $T(i) = 1$  for  $i < 10$ ).

5.  $T(n) = T(2n/3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ).

6.  $T(n) = T(n - 1) + 1$

7.  $T(n) = T(n - 2) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ).

8.  $T(n) = T(n - 3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2, 3$ ).

9.  $T(n) = T(n/2) + n$

10.  $T(n) = T(n/3) + n$  (assume  $T(i) = 1$  for  $i < 3$ ).

11.  $T(n) = 3T(n/3) + \Theta(n)$  (assume  $T(i) = 1$  for  $i < 3$ ).

12.  $T(n) = 5T(n/5) + \Theta(n)$  (assume  $T(i) = 1$  for  $i < 5$ ).

13.  $T(n) = T(n - 1) + n$

14.  $T(n) = T(n - 2) + \Theta(n)$  (assume  $T(i) = 1$  for  $i = 1, 2$ ).

15.  $T(n) = T(n - 1) + 2n - 3$ , with  $(T(1) = 1)$

16.  $T(n) = T(\sqrt{n}) + 1$
17.  $T(n) = 7T(n/2) + n^3$
18.  $T(n) = 7T(n/2) + n^2$
19.  $T(n) = 4T(n/3) + 2n - 1$ , with  $(T(1) = T(2) = 1$
20.  $T(n) = 3T(n/2) + n^2$ , with  $(T(1) = 1$
21.  $T(n) = 2T(n - 1) + \Theta(1)$
22. (challenge)  $T(n) = T(n/3) + T(2n/3) + \Theta(n)$  (only guess the solution)
23. (challenge)  $T(n) = T(n/3) + T(n/4) + \Theta(n)$  (only guess the solution)
24. (challenge)  $T(n) = T(n/2) + T(n/4) + T(n/10) + \Theta(n)$  (only guess the solution)
25. Based on all examples seen so far, list recurrences that solve to:
  - (a)  $\Theta(\lg n)$
  - (b)  $\Theta(n)$
  - (c)  $\Theta(n \lg n)$
  - (d)  $\Theta(n^2)$
  - (e) exponential

For each category, enumerate all recurrences seen so far that fall into that category, and add at last one *new* one.

## Partial Answers

1. B (9 comparisons)
2.  $T(n) = T(n/2) + 1 : \Theta(\lg n)$
3.  $T(n) = T(n/3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ):  $\Theta(\lg n)$
4.  $T(n) = T(n/10) + 1$  (assume  $T(i) = 1$  for  $i < 10$ ):  $\Theta(\lg n)$
5.  $T(n) = T(2n/3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ):  $\Theta(\lg n)$
6.  $T(n) = T(n - 1) + 1: \Theta(n)$
7.  $T(n) = T(n - 2) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2$ ):  $\Theta(n)$
8.  $T(n) = T(n - 3) + 1$  (assume  $T(i) = 1$  for  $i = 1, 2, 3$ ):  $\Theta(n)$
9.  $T(n) = T(n/2) + n: \Theta(n)$
10.  $T(n) = T(n/3) + n$  (assume  $T(i) = 1$  for  $i < 3$ ):  $\Theta(n)$
11.  $T(n) = 3T(n/3) + \Theta(n)$  (assume  $T(i) = 1$  for  $i < 3$ ):  $\Theta(n \lg n)$
12.  $T(n) = 5T(n/5) + \Theta(n)$  (assume  $T(i) = 1$  for  $i < 5$ ):  $\Theta(n \lg n)$
13.  $T(n) = T(n - 1) + n: \Theta(n^2)$
14.  $T(n) = T(n - 2) + \Theta(n)$  (assume  $T(i) = 1$  for  $i = 1, 2$ ):  $\Theta(n^2)$
15.  $T(n) = T(n - 1) + 2n - 3$ , with  $(T(1) = 1): \Theta(n^2)$
16.  $T(n) = T(\sqrt{n}) + 1: \Theta(\lg \lg n)$
17.  $T(n) = 7T(n/2) + n^3: \Theta(n^3)$
18.  $T(n) = 7T(n/2) + n^2: T(n) = \Theta(n^{\lg 7})$
19.  $T(n) = 4T(n/3) + 2n - 1$ , with  $(T(1) = T(2) = 1): T(n) = \Theta(n^{\log_3 4})$
20.  $T(n) = 3T(n/2) + n^2$ , with  $(T(1) = 1): T(n) = \Theta(n^2)$
21.  $T(n) = 2T(n - 1) + \Theta(1): T(n) = \Theta(2^n)$  Note: For exponential recurrences we are usually happy with just a lower bound.
22. (challenge)  $T(n) = T(n/3) + T(2n/3) + \Theta(n): n/a$
23. (challenge)  $T(n) = T(n/3) + T(n/4) + \Theta(n) : n/a$
24. (challenge)  $T(n) = T(n/2) + T(n/4) + T(n/10) + \Theta(n) : n/a$