

Worksheet: Heaps

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1. (CLRS 6.1-1) What are the minimum and maximum number of elements in a heap of height h ? Note: the height of a heap is the number of edges on the longest root-to-leaf path.
2. (CLRS 6.1-2) Show that an n -element heap has height $\Theta(\lg n)$ (more precisely, $\lceil \lg n \rceil$).
3. (CLRS 6.1-3) Where in a min-heap might the largest element reside, assuming that all elements are distinct?
4. (CLRS 6.1-5) Is an array that is in sorted order a min-heap?
5. (CLRS 6.1-7) Argue that the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1, \dots, \lfloor n/2 \rfloor + 2, \dots, n$.
6. What is the effect of calling $\text{HEAPIFY}(A, i)$ for $i > \text{size}[A]/2$?. Here i is the index of the node where HEAPIFY is called.
7. (CLRS 6.5-2) Illustrate the operation of $\text{HEAP-INSERT}(A, 7)$ on the heap (note: this is a min-heap):

$$A = \{2, 5, 10, 6, 8, 100, 11, 9, 15, 9, 10, 200, 101\}$$

8. (CLRS 6.5-3) Write pseudocode for the procedure HEAP-INSERT on a min-heap.
9. (CLRS 6.2-1) Illustrate the operation of $\text{HEAPIFY}(A, 1)$ on

$$A = \{(20, 5, 10, 6, 8, 100, 11, 9, 15, 9, 10, 200, 101, 12)\}$$

10. (CLRS 6.3-1) Illustrate the operation of BUILD-MAX-HEAP on the array

$$A = \{5, 3, 17, 10, 84, 19, 6, 22, 9\}$$

11. (CLRS 6.4-1) Illustrate the operation of Heapsort on the array

$$A = \{5, 13, 2, 25, 7, 17, 20, 8, 4\}$$

12. (CLRS 6.4-3) What is the running time of Heapsort on an array of length n that is already sorted in increasing order? What about decreasing order?
13. How would you implement a function that searches for a given element in a heap, and how long would it take in the worst case?
14. (GT C-2.31) Develop an algorithm that computes the k th smallest element in a set of n distinct integers in $O(n + k \lg n)$ time.