Algorithms Lab 6

(Divide-and-conquer, CLRS 4)

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Review

Divide-and-conquer examples covered in class:

- Karatsuba integer multiplication
- Strassen's algorithm for matrix multiplication
- Maximum partial subarray

Review the lecture notes and textbook.

Homework problems

1. You are given a sorted array of numbers where every value except one appears exactly twice; the remaining value appears only once. Design an efficient algorithm for finding which value appears only once.

   Example: Here are some example inputs to the problem:

   $$1, 1, 2, 2, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8$$

   $$10, 10, 17, 17, 18, 18, 19, 19, 21, 21, 23$$

   $$1, 3, 3, 5, 5, 7, 7, 8, 8, 9, 9, 10, 10$$

2. The skyline problem/the upper envelope problem: In this problem we design a divide-and-conquer algorithm for computing the skyline of a set of n buildings.

   A *building* B_i is represented as a triplet $(\mathbf{L_i}, H_i, \mathbf{R_i})$ where $\mathbf{L_i}$ and $\mathbf{R_i}$ denote the left and right x coordinates of the building, and H_i denotes the height of the building (note that the x coordinates are drawn boldfaced.)

   A *skyline* of a set of n buildings is a list of x coordinates and the heights connecting them arranged in order from left to right (note that the list is of length at most $4n$).

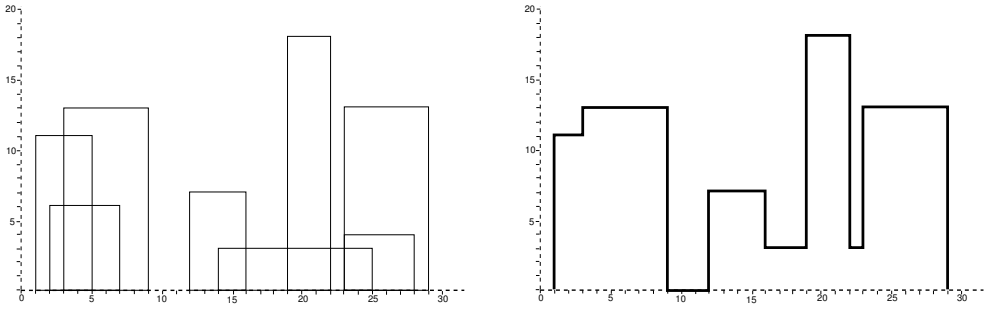
Example: The skyline of the buildings

$$\{(\mathbf{3}, 13, \mathbf{9}), (\mathbf{1}, 11, \mathbf{5}), (\mathbf{12}, 7, \mathbf{16}), (\mathbf{14}, 3, \mathbf{25}), (\mathbf{19}, 18, \mathbf{22}), (\mathbf{2}, 6, \mathbf{7}), (\mathbf{23}, 13, \mathbf{29}), (\mathbf{23}, 4, \mathbf{28})\}$$

is

$$\{(\mathbf{1}, 11), (\mathbf{3}, 13), (\mathbf{9}, 0), (\mathbf{12}, 7), (\mathbf{16}, 3), (\mathbf{19}, 18), (\mathbf{22}, 3), (\mathbf{23}, 13), (\mathbf{29}, 0)\}$$

(note that the x coordinates in a skyline are sorted).



buildings skyline

(a) Let the size of a skyline be the number of elements (tuples) in its list.
Describe an algorithm for combining a skyline A of size n_1 and a skyline B of size n_2 into one skyline S of size $O(n_1 + n_2)$. Your algorithm should run in time $O(n_1 + n_2)$.

(b) Describe an $O(n \log n)$ algorithm for finding the skyline of n buildings.

3. (CLRS 2-4) Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A.

(a) List the inversions of the array $< 2, 3, 8, 6, 1 >$.

(b) What array with elements from the set $\{1, 2, ..., n\}$ has the most inversions? How many does it have?

(c) Give an algorihm that determines the number of inversions in an array in $O(n^2)$ time.

(d) Give an algorihm that determines the number of inversions in an array in $O(n \lg n)$ time worst-case (Hint: modify merge sort).