Algorithms Lab 11

In lab

1. Prim's algorithm: To implement it, we need to store the edges with one endpoint in T so that we can select the minimum weight edge fast. We'll use a priority queue.

   - The priority queue will store all the vertices that are not in T yet;
   - A vertex v in the PQ has priority equal to the weight of the minimum edge that connects v with a vertex already in T. The other endpoint of this edge is stored in $visit(v)$.
   - Essentially the priority queue stores all edges that *cross* the cut between the vertices in T and the vertices in $V - T$ (If a vertex not in T is connected by several edges to vertices in T, the pq will store only one of these edges, the smallest).
   - Initially T is empty and PQ contains all vertices in G with priority ∞, except an arbitrary vertex v which has priority 0.

   (a) Show how Prim's algorithm run on the example graph in teh notes (or textbook).

   (b) What is the role of checking whether $v \in PQ$?

   (c) How many INSERT operations are performed by the algorithm?

   (d) How many DELETE-MIN operations are performed by the algorithm?

   (e) How many DECREASE-KEY operations are performed by the algorithm?

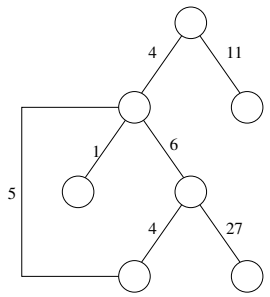
   (f) Assuming the priority is implemented as a heap, what is the complexity of the algorithm?

2. (CLRS 23.1-1) Show that a minimum-weight edge in G belongs to some MST of G.

3. (CLRS 24.2-4) Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How can you take advantage of this in Kruskal's algorithm, and how fast can you make it run? What if the edge weights are integers from 1 to W for some constant W?

4. Is the path between a pair of nodes u and v in a minimum spanning tree (i.e. the path composed of edges in the MST) necessarily the minimum weight path between u and v if we consider all the edges in the graph G? If yes, prove it. If no, provide a counterexample.

5. Consider an undirected weighted graph which is formed by taking a binary tree and adding an edge from *exactly one* of the leaves to another node in the tree. We call such a graph a *loop-tree*. An example of a loop-tree could be the following:



Let n be the number of vertices in a loop-tree and assume that the graph is given in the normal edge-list representation without any extra information. In particular, the representation does not contain information about which vertex is the root.

(a) How many edges are in a graph of n vertices?

(b) How long time would it take Prim's or Kruskal's algorithms to find the minimal spanning tree of a loop-tree?

(c) Describe and analyze a more efficient algorithm for finding the minimal spanning tree of a loop-tree. Remember to argue that the algorithm is correct.