

Algorithms Lab 5

(Selection, CLRS 9)

Laura Toma, csci2200, Bowdoin College

Topics covered since previous lab: selection.

Homework problems (COLLABORATION ¹LEVEL:1)

1. Let A be a list of n (not necessarily distinct) integers. Describe an $O(n)$ -algorithm to test whether any item occurs more than $\lceil n/2 \rceil$ times in A . Your algorithm should use $O(1)$ additional space. A general solution should not make any additional assumptions about the integers.
2. (GT C-4.23, CLRS 9.3-7) Given an unordered sequence S of n elements (for simplicity, assume items are integers or real numbers), describe an efficient method for finding the $\lceil \sqrt{n} \rceil$ elements whose values are closest to (the value of) the median of S . What is the running time of your method? Try for linear time.
3. (CLRS 9-3.9) Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (either north or south), as shown in textbook CLRS figure 9.2. Given the x - and y -coordinates of the wells, show how the professor should pick the optimal location of the main pipeline, which would be the one that minimizes the total length of the spurs. Show how to determine the optimal location in linear time. Hint: Assume professor Olay is a computer science major and she loves algorithms!
4. Suppose we are given an array $A[1..n]$ with the special property that $A[1] \geq A[2]$ and $A[n-1] \leq A[n]$. We say that an element $A[x]$ is a *local minimum* if it is less or equal to both its neighbors, or more formally, if $A[x-1] \geq A[x]$ and $A[x] \leq A[x+1]$. For example, there are six local minima in the following array:

$$A = [9, 7, 7, 2, 1, 3, 7, 5, 4, 7, 3, 3, 4, 8, 6, 9]$$

We can obviously find a local minimum in $O(n)$ time by scanning through the array. Describe and analyze an algorithm that finds a local minimum in $O(\lg n)$ time. (*Hint: with the given boundary conditions, the array must have at least one local minimum. Why?*)

¹Collaboration level 1: verbal collaboration without solution sharing. You are allowed and encouraged to discuss ideas with other class members, but the communication should be verbal and additionally it can include diagrams on board. No one is allowed to take notes during the discussion (being able to recreate the solution later from memory is proof that you actually understood it). Communication cannot include sharing pseudocode for the problem. Check complete guidelines at: <https://turing.bowdoin.edu/dept/collab.php>