csci 210: Data Structures

Maps and Hash Tables

Summary

- Topics
  - the Map ADT
  - hash tables and hashing

Map ADT

- A Map is an abstract data structure (ADT)
  - it stores key-value (k,v) pairs
  - there cannot be duplicate keys

- Maps are useful in situations where a key can be viewed as a unique identifier for the object
  - the key is used to decide where to store the object in the structure.
- Maps are sometimes called associative arrays

Map ADT

- size()
- isEmpty()
- get(k): ← this can be viewed as searching for key k
  - if M contains an entry with key k, return it; else return null
- put(k,v): ← this can be viewed as inserting key k
  - if M does not have an entry with key k, add entry (k,v) and return null
  - else replace existing value of entry with v and return the old value
- remove(k): ← this can be viewed as deleting key k
  - remove entry (k,*) from M

Java.util.Map

- check out the interface
- additional handy methods
  - putAll
  - entrySet
  - containsValue
  - containsKey

Map example

(k,v) key=integer, value=letter

M={}

- put(5,A) — M={(5,A)}
- put(7,B) — M={(5,A), (7,B)}
- put(2,C) — M={(5,A), (7,B), (2,C)}
- put(8,D) — M={(5,A), (7,B), (2,C), (8,D)}
- put(2,E) — M={(5,A), (7,B), (2,E), (8,D)}
- get(7) — return B
- get(4) — return null
- get(2) — return E
- remove(5) — M={(7,B), (2,E), (8,D)}
- remove(2) — M={(7,B), (8,D)}
- get(2) — return null

Map example

(k,v) key=string, value=string

- put(“Dan”, <Dan’s favorite tune>)
- put(“John”, <John’s favorite song>)
- put<Helen, <Helen’s favorite song>)
- ...
- get (“Dan”)
- get (“Helen”)

Example

- Let's say you want to implement a language dictionary. That is, you want to store words and their definition. You want to insert words to the dictionary, and retrieve the definition given a word.

- Options:
  - vector
  - linked list
  - binary search tree
  - map

- The map will store (word, definition of word) pairs.
- key = word
  - note: words are unique
- value = definition of word
- get(word)
  - returns the definition if the word is in dictionary
  - returns null if the word is not in dictionary

Class-work

- Write a program that reads from the user the name of a text file, counts the word frequencies of all words in the file, and outputs a list of words and their frequency.
    - e.g. text file: article, poem, science, etc

- Questions:
    - Think in terms of a Map data structure that associates keys to values.
    - What will be your <key-value> pairs?

    - Sketch the main loop of your program.

Map Implementations

- Arrays (Vector, ArrayList)
- Linked-list
- Binary search trees
- Hash tables

A LinkedList implementation of Maps

- store the (k,v) pairs in a doubly linked list

- get(k)
  - hop through the list until find the element with key k

- put(k,v)
  - Node x = get(k)
  - if (x != null)
    - replace the value in x with v
  - else create a new node(k,v) and add it at the front

- remove(k)
  - Node x = get(k)
  - if (x == null) return null
  - else remove node x from the list
  - Note: why doubly-linked? need to delete at an arbitrary position

- Analysis:
  - assume a map with n elements

Map Implementations

- Linked-list:
  - get/search, put/insert, remove/delete: O(n)
- Binary search trees <--------- we'll talk about this later
  - search, insert, delete: O(n) if not balanced
  - O(lg n) if balanced BST
- Hash tables:
  - we'll see that (under some assumptions) search, insert, delete: O(1)

Hashing

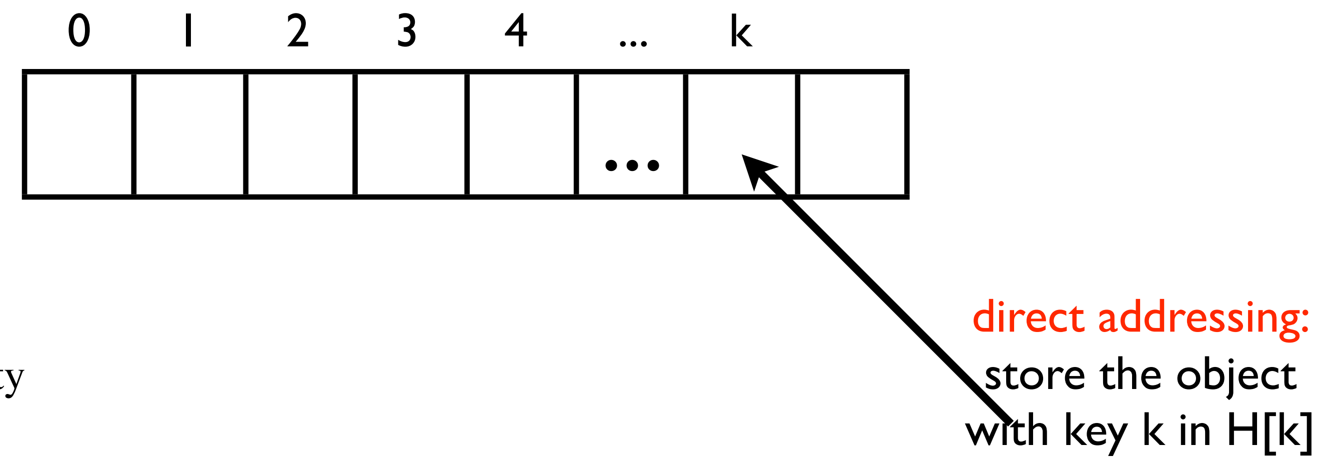
- A completely different approach to searching from the comparison-based methods (binary search, binary search trees)

  - hashing tries to reference an element in a table directly based on its key (rather than navigating through a dictionary data structure comparing the search key with the elements)

  - hashing transforms a key into a table address

Hashing

- If the keys were integers in the range 0 to 99
- The simplest idea:
  - store keys in an array H[0..99]



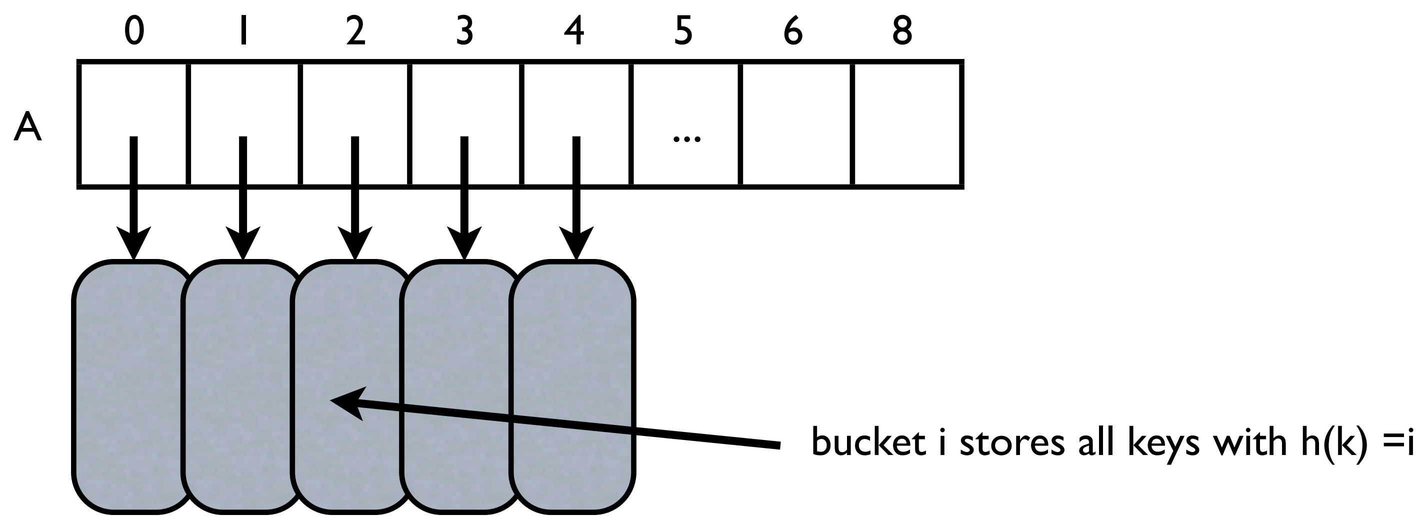
  - H initially empty
  - put(k, value)
    - store <k, value> in H[k]
  - get(k)
    - check if H[K] is empty

Issues:

- This works if keys are integers in a small range
- Space may be wasted is H not full

Hashing

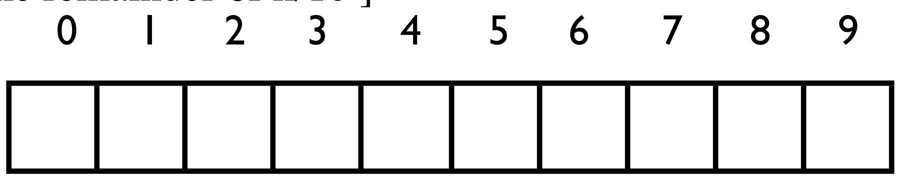
- Hashing has 2 components
  - the hash table: an array A of size N
    - Can think of each entry as a bucket (a bucket array)
  - a hash function: maps each key to a bucket
    - h is a function : {all possible keys} ----> {0, 1, 2, ..., N-1}
    - key k is stored in bucket h(k)



- The size of the table N and the hash function are decided by the user
-

Example

- keys: integers
- chose N = 10
- chose h(k) = k % 10
  - [k % 10 is the remainder of k/10]



- add (2,*), (13,*), (15,*), (88,*), (2345,*), (100,*)

- Collision: two keys that hash to the same value
  - e.g. 15, 2345 hash to slot 5

- Note: if we were using direct addressing: N = 2^32. Unfeasible.

Hashing

The user needs to chose N and the hash function

- h : {universe of all possible keys} ----> {0,1,2,...,N-1}
- The keys need not be integers
  - e.g. strings: define a hash function that maps strings to integers
- The universe of all possible keys need not be small
  - e.g. strings

Hashing

The user needs to chose N and the hash function

- h : {universe of all possible keys} ----> {0,1,2,...,N-1}
- Hashing is an example of space-time trade-off:
  - if there were no memory(space) limitation, simply store a huge table
    - O(1) search/insert/delete and a lot of space
  - if there were no time limitation, use a linked list and search sequentially
    - O(n) search/insert/delete and O(n) space
- Hashing: use a reasonable amount of memory and strike a balance space-time
  - adjust hash table size
- Under some assumptions, hashing supports insert, delete and search in in O(1) time and O(n) space

Hashing

- Notation:
  - U = the universe of keys
    - e.g. U = set of all integers
  - |U| = the size of the universe of keys
    - e.g. |U| = 2^32
  - N = hash table size
  - n = number of entries
    - note: n may be unknown beforehand

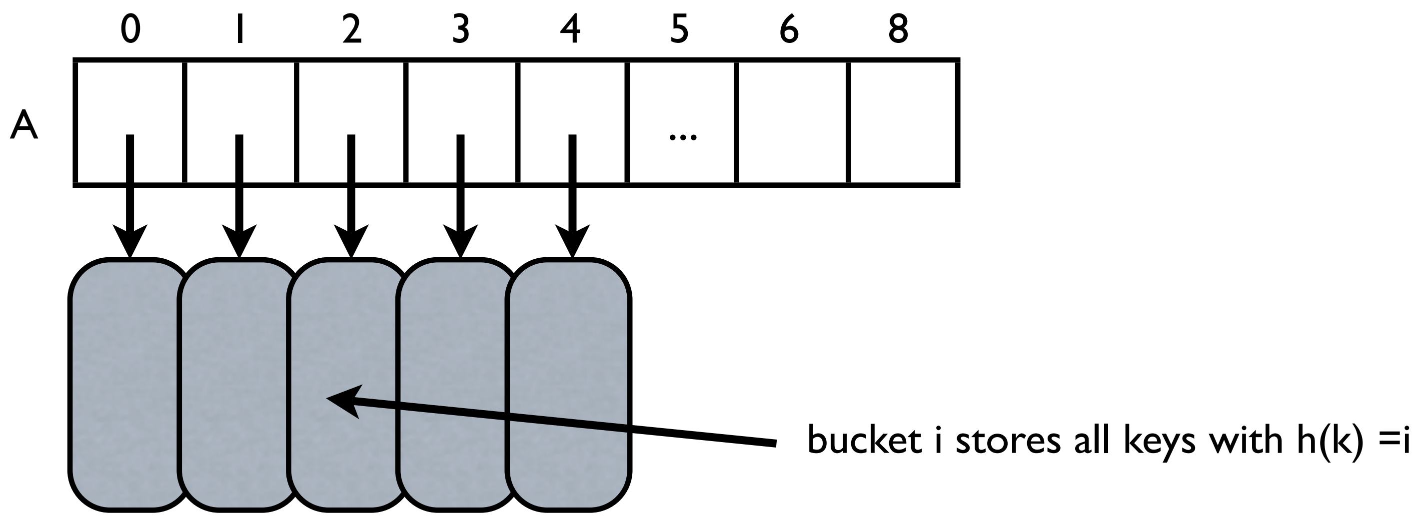
Collisions

- Collision: two keys that hash to the same value
- Collision handling: Decide how to handle when two kets hash to the same address
- Note: if $n > N$ there must be collisions

- Collision with chaining
  - bucket arrays
- Collision with probing
  - linear probing
  - quadratic probing
  - double hashing

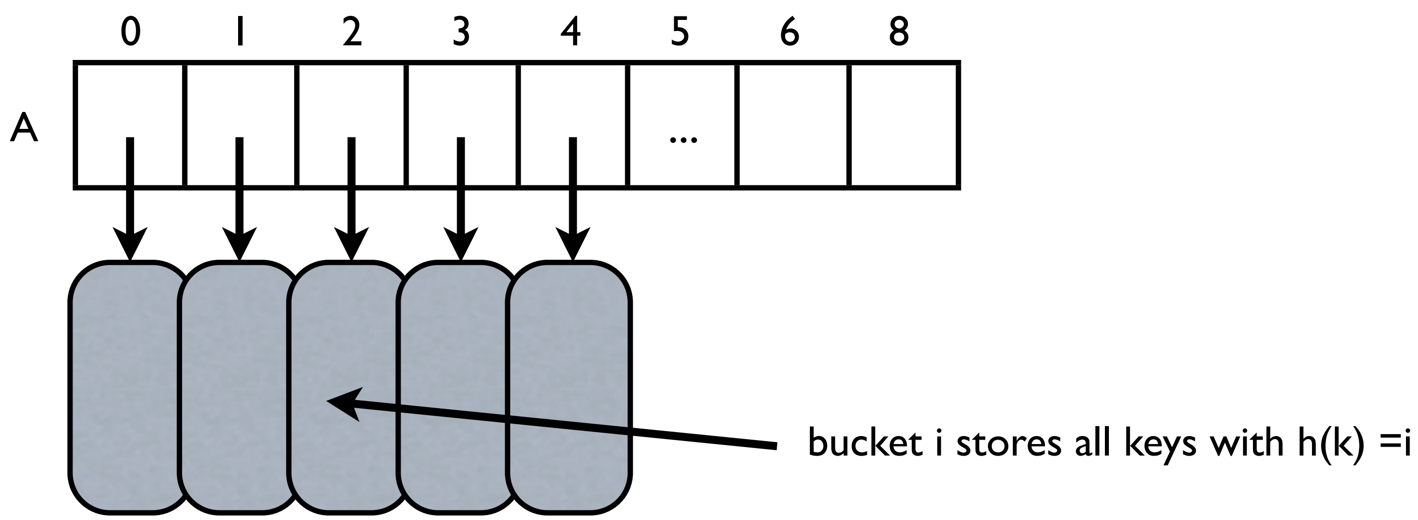
Collisions with chaining

- Store all elements that hash to the same entry in a linked list (array/vector)



- Can chose to store the lists in sorted order or not
- Insert(k)
  - insert k in the linked list of h(k)
- Search(k)
  - search in the linked list of h(k)
- Delete(k)
  - find and delete k from the linked list of h(k)

Collisions with chaining



- Pros:
  - can handle arbitrary number of collisions as there is no cap on the list size
  - don't need to guess n ahead: if N is smaller than n, the elements will be chained
- Cons: space waste
  - use additional space in addition to the hash table
  - if N is too large compared to n, part of the hash table may be empty
- Choosing N: space-time tradeoff
- Rule of thumb:
  - chose N as 1/5 to 1/10 of the number of keys that we expect in the table, so that keys are expected to have about 10 elements each. Keep lists unsorted.

Collisions with probing

- Idea: do not use extra space, use only the hash table
- Idea: when inserting key k, if slot h(k) is full, then try some other slots in the table until finding one that is empty
  - the set of slots tried for key k is called the probing sequence of k
- Linear probing:
  - if slot h(k) is full, try next, try next, ...
  - probing sequence: h(k), h(k) + 1, h(k) + 2, ...
  - insert(k)
  - search(k)
  - delete(k)
  - Example: N = 10, h(k) = k % 10, collisions with linear probing
  - insert 1, 7, 4, 13, 23, 25, 25

Linear probing

- Notation: alpha = n/N (load factor of the hash table)

- In general performance of probing degrades inversely proportional with the load of the hash
  - for a sparse table (small alpha) we expect most searches to find an empty position within a few probes
  - for a nearly full table (alpha close to 1) a search could require a large number of probess

- It is known that: Under certain randomness assumption it can be shown that the average number of probes examined when searching for key k in a hash table with linear probing is 1/2 (1 + 1/(1 - alpha))
  - alpha = 0: 1 probe
  - alpha = 1/2: 1.5 probes (half-full)
  - alpha= 2/3: 2 probes (2/3 full)
  - alpha = 9/10: 5.5 probes

- Collisions with probing: cannot insert more than N items in the table
  - need to guess n ahead
  - if at any point n is > N, need to re-allocate a new hash table, and re-hash everything. Expensive!

Linear probing

- Pros:
  - space efficiency

- Cons:
  - need to guess n correctly and set N > n
  - if alpha gets large ==> high penalty
    - the table is resized and and all objects re-inserted into the new table

- Rule of thumb: good performance with probing if alpha stays less than 2/3.

Double hashing

- Empirically linear hashing introduces a phenomenon called clustering:
    - insertion of one key can increase the time for other keys with other hash values
    - groups of keys clustered together in the table

- Double hashing:
    - instead of examining every successive position, use a second hash function to get a fixed increment
    - probing sequence: h1(k), h1(k) + h2(k), h1(k) + 2h2(k), h1(k) + 3h2(k),...

- Chose h2 so that it never evaluates to 0 for any key
    - would give an infinite loop on first collision
- Rule of thumb:
    - chose h2(k) relatively prime to N

- Performance:
    - double hashing and linear hashing have the same performance for sparse tables
    - empirically double hashing eliminates clustering
    - we can allow the table to become more full with double hashing than with linear hashing before performance degrades

Hashing

- Choosing h and N
    - Goal: distribute the keys evenly throughout the hashtable
    - n is usually unknown
    - If n > N, then the best one can hope for is that each bucket has O(n/N) elements
        - need a good hash function
        - search, insert, delete in O(n/N) time
    - If n <= N, then the best one can hope for is that each bucket has O(1) elements
        - need a good hash function
        - search, insert, delete in O(1) time
    - If N is large==> less collisions and easier for the hash function to perform well
    - Best: if you can guess n beforehand, chose N order of n
        - no space waste

Hash functions

- How to define a good hash function?

- An ideal has function approximates a random function: for each input element, every output should be in some sense equally likely
    - This is called “universal hashing”

- In general impossible to guarantee

- Every hash function has a worst-case scenario where all elements map to the same entry

- Hashing = transforming a key to an integer
- There exists a set of good heuristics

Hashing strategies

- Summing components
  - let the binary representation of key k = $<x_0,x_1,x_2,\ldots,x_{k-1}>$
  - use all bits of k when computing the hash code of k
  - sum the high-order bits with the low-order bits
    - (int) $<x_0,x_1,x_2,.x_{31}>$ + (int)$<x_{32},.,x_{k-1}>$
  - e.g. String s;
    - sum the integer representation of each character
    - (int)s[0] + (int)s[1] + (int) s[2] + ...

Hashing strategies

- summation is not a good choice for strings/character arrays
- e.g. s1 = “temp10” and s2 = “temp01” collide
- e.g. “stop”, “tops”, “pots”, “spot” collide

- Polynomial hash codes
  - $k = <x_0, x_1, x_2, \ldots, x_{k-1}>$
  - take into consideration the position of x[i]

  - chose a number a >0 (a !=1)
  - $h(k) = x_0 a^{k-1} + x_1 a^{k-2} + \ldots + x_{k-2} a + x_{k-1}$

  - experimentally, a = 33, 37, 39, 41 are good choices when working with English words
  - produce less than 7 collision for 50,000 words!!!
  - Java hashCode for Strings uses one of these constants

Hashing strategies

- Need to take into account the size of the table
- Modular hashing
  - h(k) = i mod N
    - If take N to be a prime number, this helps the spread out the hashed values
  - If N is not prime, there is a higher likelihood that patterns in the distribution of the input keys will be repeated in the distribution of the hash values
  - e.g. keys = {200, 205, 210, 215, 220, ... 600}
  - N = 100
    - each hash code will collide with 3 others
  - N = 101
    - no collisions

Hashing strategies

- Combine modular and multiplicative:
  - h(k) = a k % N
  - chose a = random value in [0,1]
  - advantage: the value of N is not critical and need not be prime

  - empirically:
    - a popular choice is a = 0.618033 (the golden ratio)
    - chose N = power of 2

Hashing strategies

- Universal hashing

  - chose N prime
  - chose p a prime number larger than N
  - chose a, b at random from {0,1,...p-1}

  - $h(k) = ((a\,k + b) \bmod p) \bmod N$

  - This gets very close to throwing the keys into the hash table randomly (two keys collide with probability 1/N), and thus leads to as few collisions as possible, on the average.

- Many other variations of these have been studied, particularly has functions that can be implemented with efficient machine instructions such as shifting

Java.util.Hashtable

- This class implements a hash table, which maps keys to values. Any non-`null` object can be used as a key or as a value.

- `java.lang.Object`
- `  java.util.Dictionary`
- **`      java.util.Hashtable`**

- **`implements Map`**

- [check out Java docs]
- implements a Map with linear probing; uses .75 as maximal load factor, and rehashes every time the table gets fuller

- Example

```
//create a hashtable of <key=string, value=number> pairs
Hashtable numbers = new Hashtable();
numbers.put("one", new Integer(1));
numbers.put("two", new Integer(2));
numbers.put("three", new Integer(3));

 //retrieve a string
Integer n = (Integer)numbers.get("two");
if (n != null) {
    System.out.println("two = " + n);
}
```

Hash functions in Java

- The generic Object class comes with a default hashCode() method that maps an Object to an integer
  - int hashCode()
- Inherited by every Object
- The default hashCode() returns the address of the Object's location in memory
  - too generic
  - poor choice for most situations
- Typically you want to override it
- e.g. class String
  - overrides Strng.hashCode() with a hash function that works well on Strings

Perspective

- Best hashing method depends on application

- Probing is the method of choice if n can be guessed
    - Linear probing is fastest if table is sparse
    - Double hashing makes most efficient use of memory as it allows the table to become more full, but requires extra time to to compute a second hash function
    - rule of thumb: load factor < .66
- Chaining is easiest to implement and does not need guessing n
    - rule of thumb: load factor < .9 for O(1) performance, but not vital

- Hashing can provide better performance than binary search trees if the keys are sufficiently random so that a good hash function can be developed
    - when hashing works, better use hashing than BST
- However
    - Hashing does not guarantee worst-case performance
    - Binary search trees support a wider range of operations

Exercises

- What is the worst-case running time for inserting n key-value pairs into an initially empty map that is implemented with a list?
- Describe how to use a map to implement the basic ops in a dictionary ADT, assuming that the user does not attempt to insert entries with the same key
- Describe how an ordered list implemented as a doubly linked list could be used to implement the map ADT.
- Draw the 11-entry hash that results from using the hash function h(i) = (2i+5) mod 11 to hash keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5.
  - (a) Assume collisions are handled by chaining.
  - (b) Assume collisions are handled by linear probing.
  - (c) Assume collisions are handled with double hashing, with the secondary hash function h'(k) = 7 - (k mod 7).
- Show the result of rehashing this table in a table of size 19, using teh new hasah function h(k) = 2k mod 19.
- Think of a reason that you would not use a hash table to implement a dictionary.