csci 210: Data Structures

More Recursion

Summary

- Topics: more recursion
  - Subset sum: finding if there exists a subset of an array that sum up to a given target
  - Permute: finding all permutations of a given string
  - Subset: finding all subsets of a given string

Subset Sum

- Given an array of numbers and a target value, find whether there exists a subset of those numbers that sum up to the target value.

```
//return true if there exists such a subset, and false otherwise
boolean subsetSum (int[] a, int target)
```

- Example:

- Recursive structure:
  - consider the next element in the array
  - try making a sum WITH this element
  - try making a sum WITHOUT this element
  - if neither is possible, return false

Subset Sum

- So: consider the next element, it is either in the solution, or not. Try both ways. If both fail, return false.
- Need to keep track of the partial sum so far. When starting a recursive call, need to know the sum of the current subset. Also need to know the index of the next element to consider.

```
void recSubset(int[] a, int target, int i, int sumSoFar)
```

- The problem asked for a subsetSum function with the following signature:

```
boolean subsetSum (int[] a, int target)
```

- Need a wrapper:

```
boolean subsetSum (int[] a, int target) {
    return recSubset(a, target, 0, 0);
}
```

Subset Sum

```
//i is the index of the next element to consider
//sumSoFar is the sum of elements included in the solution so far.
boolean recSubset(int[] a, int target, int i, int sumSoFar) {
    //basecases
    //we got it
    if (sumSoFar == target) return true;
    //we reached the end and sum is not equal to target
    if (i == a.length) return false;

    //recursive case: try next element both in and out of the sum
    boolean with = recSubset(a, target, i+1, sumSoFar + a[i]);
    boolean without = recSubset(a, target, i+1, sumSoFar);
    return (with || without);
}
```

Subset Sum

- The tree of recursive calls for recSubset([1, 2, 3, 4], target, 0, 0)

Subset Sum

- You may notice that there is no need to keep both target and sumSoFar as arguments in recSubset; instead, use target, and subtract from it when you include an element in a set.
- //i is the index of the next element to consider

```
boolean recSubset(int[] a, int target, int i) {
    //basecases
    //we got it
    if (target==0) return true;
    //we reached the end and sum is not equal to target
    if (i == a.length) return false;

    //recursive case: try next element both in and out of the sum
    boolean with = recSubset(a, target-a[i], i+1);
    boolean without = recSubset(a, target, i+1);
    return (with || without);
}
```

Subset Sum

- Variations
  - How could you change the function so that it prints the elements of the subset that sum to target?
    - store partial subsets in another array
    - or print element at the end of recursive call
  - How could you change the function to report not only if such a subset exists, but to count all such subsets?
  - Alternative strategy: at each step, chose one of the remaining element to be part of the subset and recurse on the remaining part.

Permutations

- Write a function to print all permutations of a given string.
- Example: permute "abc" should print: abc, acb, bca, bac, cab, cba.

void printPerm(String s)

- Recursive structure:
  - Chose a letter from the input, and make this the first letter of the output
  - Recursively permute remaining input
  - chose a, permute "bc": should generate "a" + all permutations of "bc"
  - chose all letters in turn to be first letters
  - chose b, permute "ac": should generate "b" + all permutations of "ac"
  - chose c, permute "ab": should generate "c" + all permutations of "ab"

  - What is the base case?
  - Can you make sure that each permutation is generated precisely once?

Permutations

- So: pick a letter, add it to the solution, recurse on remaining
- When starting a recursive call, we know the list of letters chosen so far; that is, we know the first part of the permutation generated so far.
- Need to keep track of it.

```
//print soFar + all permutations of remaining string
void recPermute(String soFar, String remaining)
```

- The problem asked for a printPermute with a different signature: we need a wrapper

```
//print all permutations of s
void printPerm (String s) {
    recPermute("", s);
}
```

- Why use wrappers? the user does not need to know the internals of the implementation (In this case, that it is recursive).

Permutations

```
//prints soFar+all permutations of remaining
void  recPermute(String soFar, String remaining) {

    //base case
    if (remaining.length() == 0)
        System.out.println(soFar);
    else {
        for (int i=0; i< remaining.length(); i++) {
            String nextSoFar = soFar + remaining[i];
            String nextRemaining = remaining.substring(0,i) +  remaining.substring(i+1);
            recPermute(nextSoFar, nextRemaining)
        }
    }
}
```

Permutations

- The tree of recursive calls for recPermute(“”, “abc”)

Subsets

- Enumerate all subsets of a given string
- Example: subsets of “abc” are a, b, c, ab, ac, bc, abc
  - Order does not matter: “ab” is the same as “ba”

- Recursive structure
  - chose one element from input
  - can either include it in current subset or not
  - recursively form subsets including it
  - recursively form subsets excluding it
  - make sure to generate each set once
  - base case?

Subsets

```
void recSubsets(String soFar, String remaining) {
    if (remaining.length()==0)
        System.out.println(soFar);
    else {
        //add to subset, remove from rest, recurse
        recSubsets(soFar+remaining[0], remaining.substring(1);
        //don't add to subset, remove from rest, recurse
        recSubsets(soFar, remaining.substring(1);
    }
}

void subsets(String s) {
    recSubsets("", s);
}
```

Subsets

- The tree of recursive calls for recSubsets(“”, “abcd”)

Thinking recursively

- Finding the recursive structure of the problem is the hard part
- Common patterns
  - divide in half, solve one half
  - divide in sub-problems, solve each sub-problem recursively, “merge”
  - solve one or several problems of size n-1
  - process first element, recurse on remaining problem

- Recursion
  - functional: function computes and returns result.
    - Example: computing the sum of n numbers; isPalindrome; binary search.
  - procedural: no return result (function returns void). The task is accomplished during the recursive calls.
    - Example: Sierpinski fractals.

- Recursion
  - exhaustive
  - non-exhaustive: stops early